Networks: Structure and dynamics

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Contents

1 - Networks are everywhere

- Extended dynamical systems
  (Celular automata, coupled maps, neural networks)
- Metabolic processes of living beings
- Protein-protein networks
- Gene expression and regulation
- Social, economic and political networks
- The internet

- Most studies deal with networks as statistical objects, less attention has been paid to the dynamical phenomena taking place in the networks or to the behavior of the evolving networks as dynamical systems

2 - Structure parameters. Network types

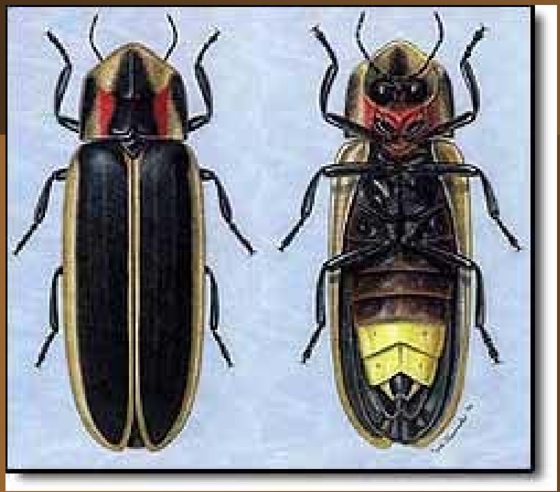
- Path length (*L*)
- Clustering coefficient (*C*)
- Degree and degree distribution (k), (P(k))

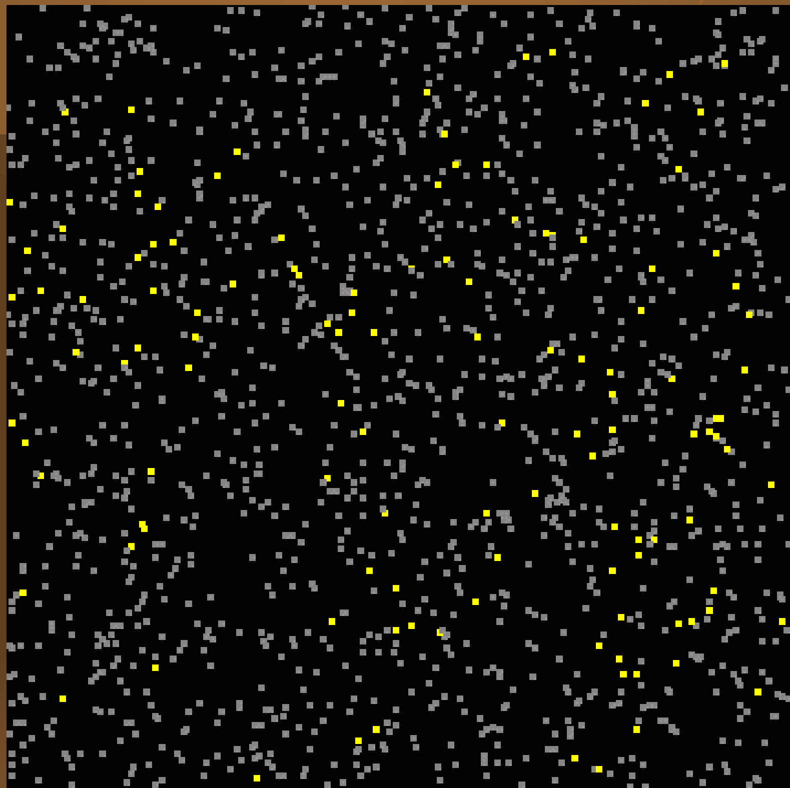
- Network types
  Ordered (*high L, high C*)
  Random (*low L, low C*)
  Small-world (*low L, high C*)
  Scale-free ($P(k) \sim k^{\gamma}$)

- Network growth
  Preferential attachment (Barabási, Cameo, etc)
  Node duplication

3 - Dynamics: an example. Synchronization and beyond

- Synchronous flashing of fireflies, cells, fads, ...





3 - Dynamics: an example. Synchronization and beyond

- Synchronization
  (Classical mathematical example: the Kuramoto model)
  A similar, discrete-time oscillators model :

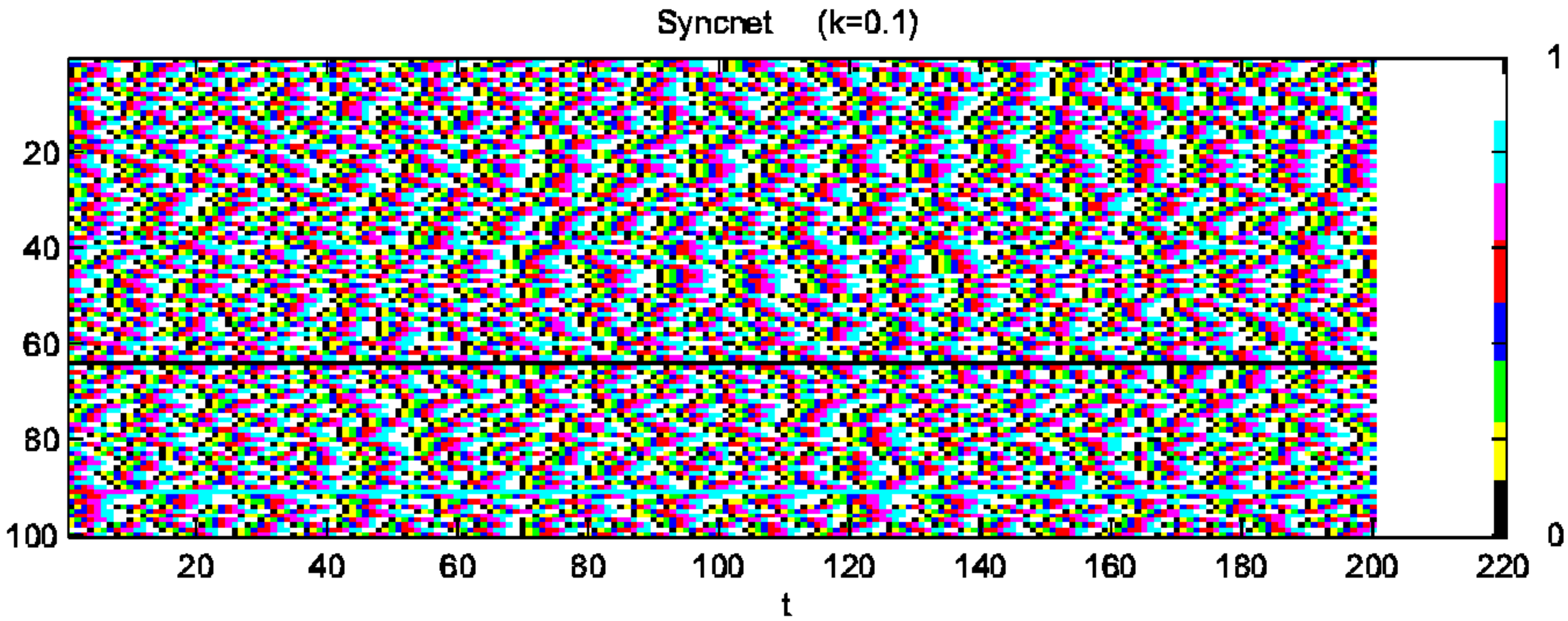
$$x_i(t+1) = x_i(t) + \omega_i + \frac{k}{N-1}\sum_{j=1}^{N} f_\alpha(x_j - x_i)$$

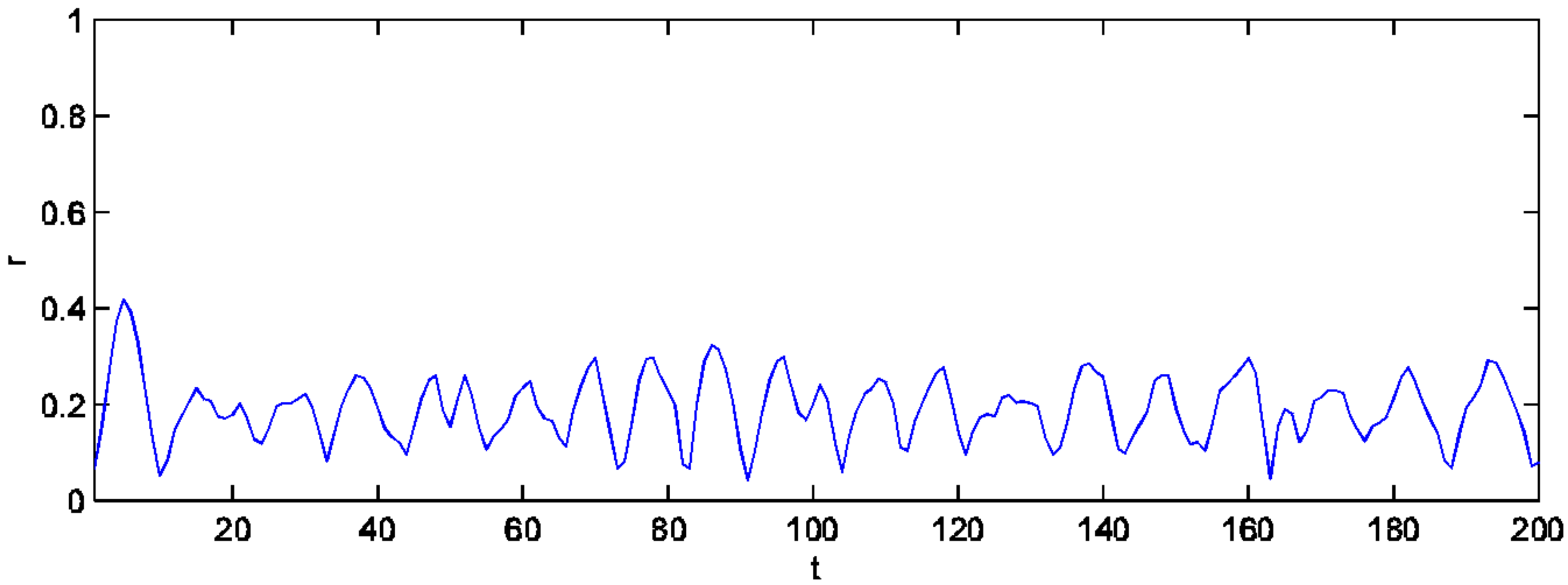
$$p(\omega) = \frac{\gamma}{\pi\left[\gamma^2 + (\omega - \omega_0)^2\right]}$$

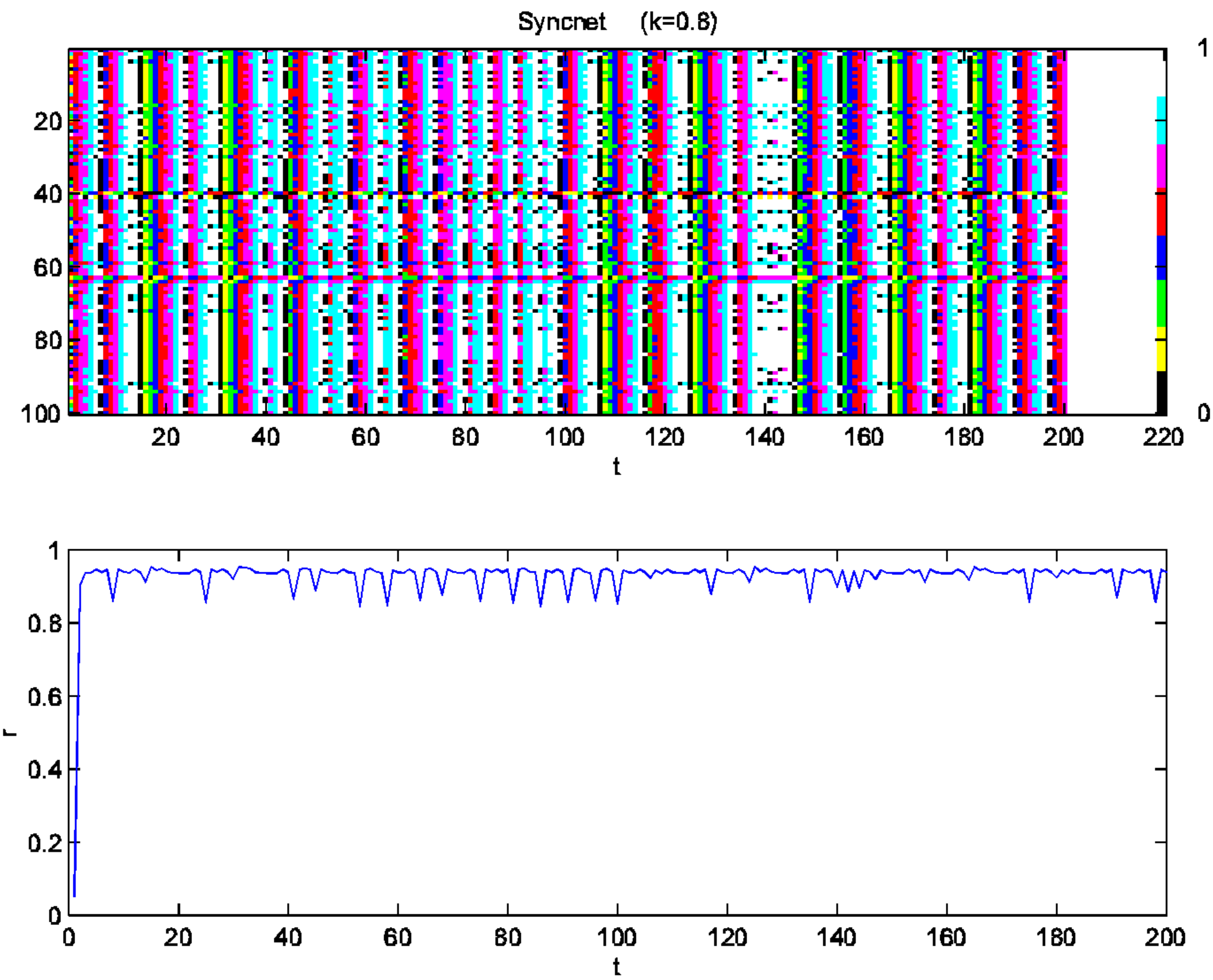
$$f_\alpha(x_j - x_i) = \alpha(x_j - x_i)\,(mod 1)$$

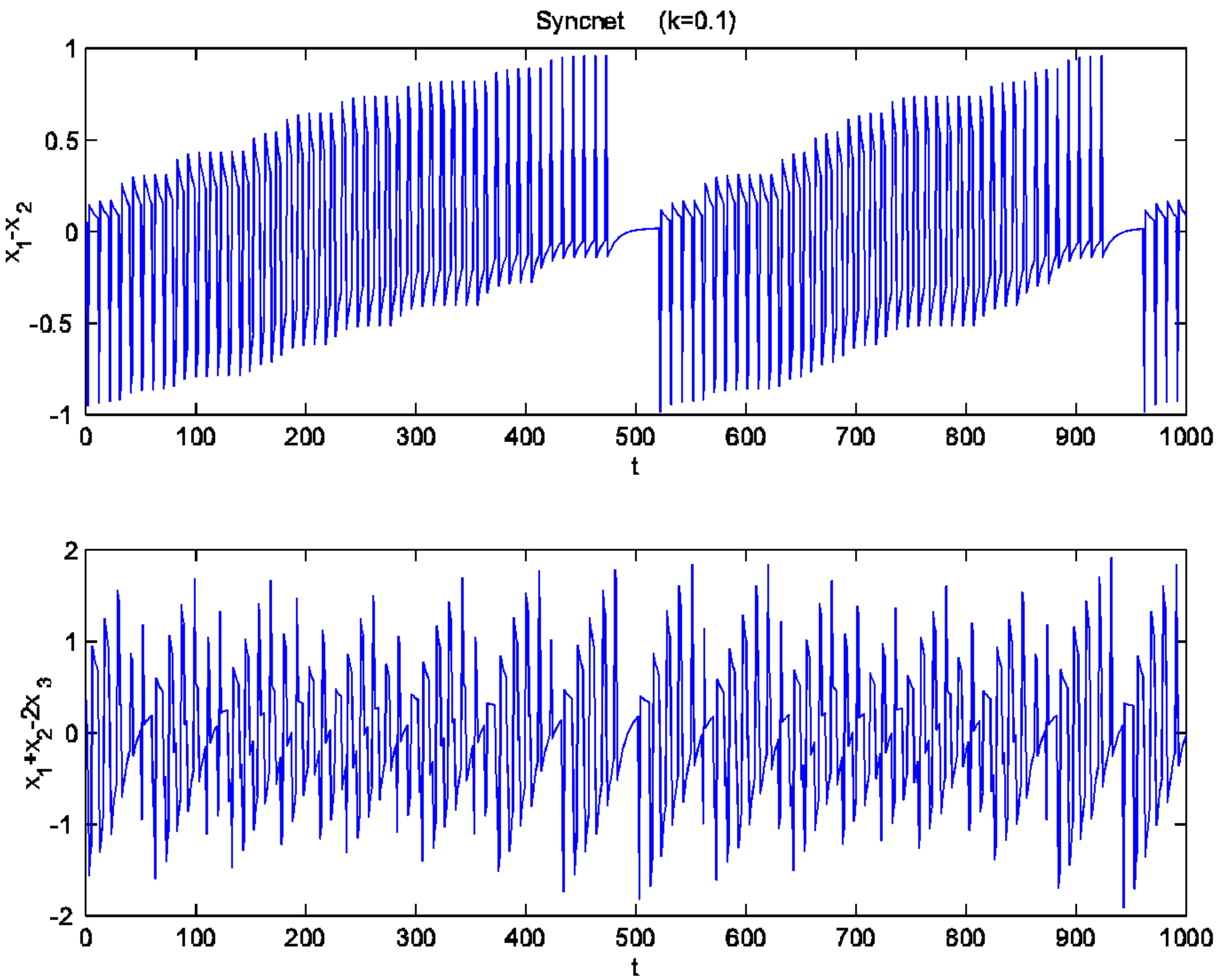
- Order parameter

$$r(t) = \left|\frac{1}{N}\sum_{j=1}^{N} e^{i2\pi x_j(t)}\right|$$


Syncnet (k=0.1)
20
40
60
80
100
20
40
60
80
100
120
140
160
180
200
220
t
1
0


1
0.8
0.6
0.4
0.2
0
r
20
40
60
80
100
120
140
160
180
200
t


Syncnet (k=0.8)
t
r
t


Syncnet (k=0.1)
x1-x2
t
x1+x2-2x3
t
1
0.5
0
-0.5
-1
2
1
0
-1
-2
0
100
200
300
400
500
600
700
800
900
1000

4 - Dynamical tools

- Differential dynamics tools

  * Describing dynamics by global functions

    - Construction of multistable systems
    - Construction of invariant measures
    - Necessary conditions for cycles
    - Evolving networks as dynamical systems

  * Conditions for multistability

  ("*Tools for network dynamics*", cond-mat/0304640, to appear in IJBC)

- Ergodic tools

Differential dynamics. A theorem and examples

$$\frac{dx_i}{dt} = a_i(x_i)\left\{ b_i(x_i) - \sum_{j=1}^{n} W_{ij} f_j(x_j) \right\}$$

$$\begin{aligned}
W_{ij} &= W_{ij}^{(S)} + W_{ij}^{(A)} \\
W_{ij}^{(S)} &= \frac{1}{2}\left(W_{ij} + W_{ji}\right) \\
W_{ij}^{(A)} &= \frac{1}{2}\left(W_{ij} - W_{ji}\right) \\
V^{(S)} &= -\sum_{i=1}^{n} \int^{x_i} b_i(\xi_i) f_i'(\xi_i) d\xi_i \\
&\quad + \frac{1}{2} \sum_{j,k=1}^{n} W_{jk}^{(S)} f_j(x_j) f_k(x_k) \\
H &= \sum_{i=1}^{n} \int^{x_i} \frac{f_i(\xi_i)}{a_i(\xi_i)} d\xi_i
\end{aligned}$$

Differential dynamics. A theorem and an example

Theorem. If $a_i(x_i)/f_i^{'}(x_i) > 0 \; \forall x, i$ and $W_{ij}^{(A)}$ has an inverse, then

$$\overset{\bullet}{x}_i = \overset{\bullet}{x}_i^{(G)} + \overset{\bullet}{x}_i^{(H)}$$

$$\overset{\bullet}{x}_i^{(G)} = -\frac{a_i(x_i)}{f_i^{'}(x_i)}\frac{\partial V^{(S)}}{\partial x_i} = -\sum_j g_{ij}(x)\frac{\partial V^{(S)}}{\partial x_j}$$

$$\overset{\bullet}{x}_i^{(H)} = -\sum_j a_i(x_i) w_{ij}^{(A)}(x) a_j(x_j)\frac{\partial H}{\partial x_j} = \sum_j \Gamma_{ij}(x)\frac{\partial H}{\partial x_j}$$

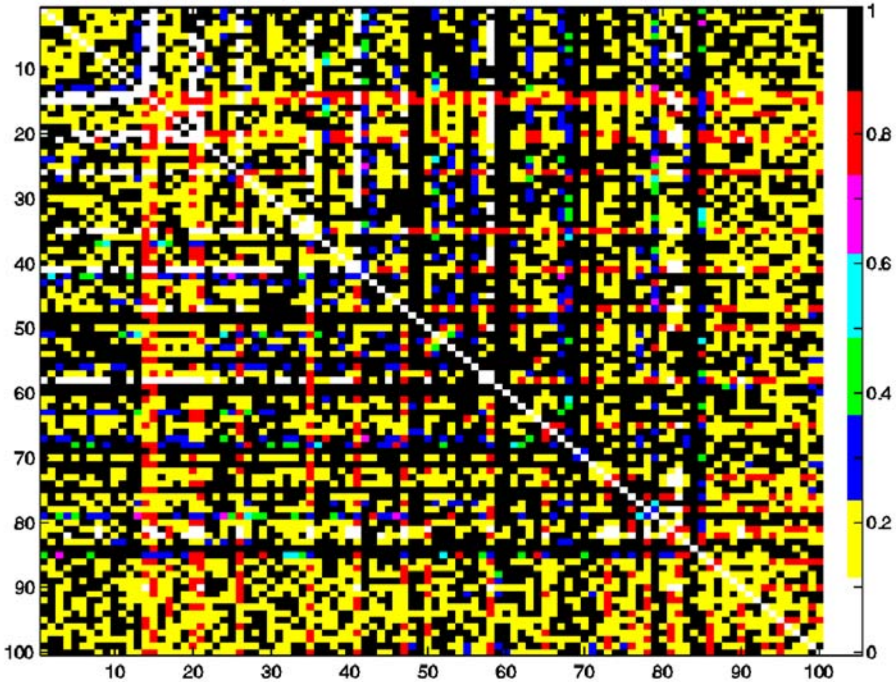
$$g_{ij}(x) = \frac{a_i(x_i)}{f_i^{'}(x_i)}\delta_{ij}$$

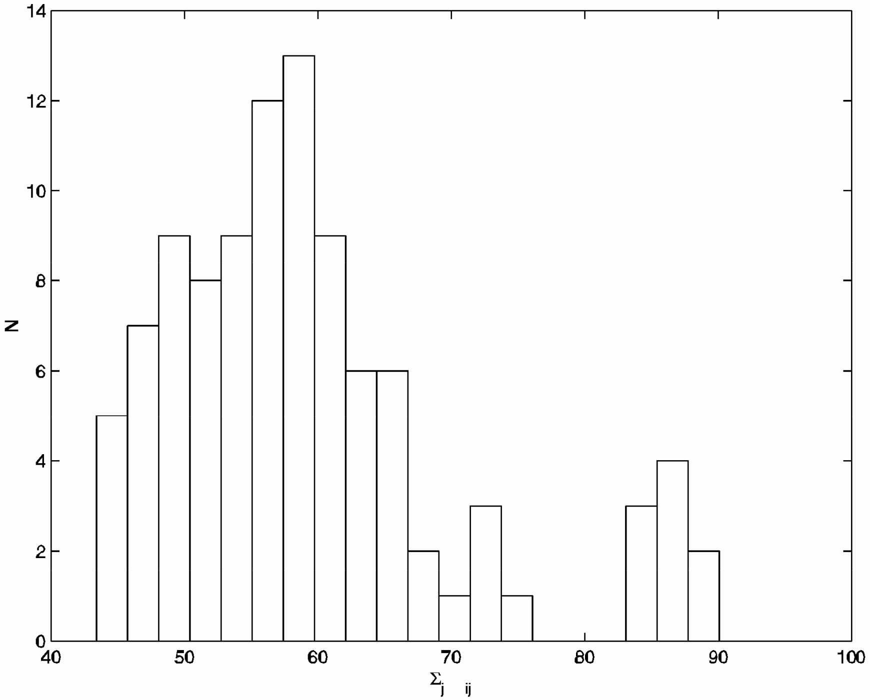
$$\omega_{ij}(x) = -a_i(x_i)^{-1}\left(W^{(A)-1}\right)_{ij}(x) a_j(x_j)^{-1}$$

An evolving network. Preferential attachment

$$V_1(\{W\}) = \alpha \sum_{i<j} W_{ij}^2 (W_{ij}-1)^2 + \beta \sum_{i\neq j\neq l} (W_{ij}-1)^2 W_{jl}^2$$

$$\frac{dW_{ij}}{dt} = -\frac{\partial V_1}{\partial W_{ij}}$$





Invariant measure by small random perturbations

Let $dx_i = X_i(x)dt + \varepsilon\sigma(X)dW(t)$

$$X(x) = -\nabla_{(g)} V(x)$$

$\nabla_{(g)} =$ gradient in the metric

$$ds^2 = \sum a_{ij}(x)\, dx_i dx_j$$

$$a_{ij}(x) = \left(\sigma(x)\,\sigma^*(x)\right)^{-1}_{ij} = g_{ij}(x)$$

$$\rho^\varepsilon(x) = C_\varepsilon \exp\left(-2\varepsilon^{-2} V(x)\right)$$

$\rho^\varepsilon(x)$ is the density of the invariant measure

Other examples :

- Node duplication

$$V_2(\{W\}) = \alpha \sum_{i<j} W_{ij}^2 (W_{ij}-1)^2 + \beta \sum_{i<j} \sum_{k \neq i,j} \frac{1}{|i-j|} (W_{ik}^2 + W_{jk}^2)((W_{ik}-1)^2 + (W_{jk}-1)^2)$$

- The corruption network

$$V(x_i) = \alpha \sum_i x_i^2 (x_i-1)^2 + \beta \sum_i \theta(\sum_j W_{ij} x_j - m)(x_i-1)^2$$

Ergodic tools. Exponents and entropies

- Invariant measures and ergodic parameters

$$I_F(\mu) = \lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} F\left(f^n x_0\right)$$

- Lyapunov and conditional exponents

From the k x k and (n-k)x(n-k) blocks of the Jacobian

Eigenvalues of the limits

$$\lim_{n\to\infty} \left(D_k f^{n*}(x) D_k f^n(x)\right)^{\frac{1}{2n}}$$

$$\lim_{n\to\infty} \left(D_{m-k} f^{n*}(x) D_k f^n(x)\right)^{\frac{1}{2n}}$$

Or

$$\lim_{n\to\infty} \frac{1}{n} \log \|D_k f^n(x) u\| = \xi_i^{(k)}$$

$$0 \neq u \in E_x^i / E_x^{i+1}$$

E_x^i is the subspace spanned by the eigenstates corresponding to eigenvalues $\leq \exp(\xi_i^{(k)})$

5 - Structures and structure-generating mechanisms

- Structure index

$$S = \frac{1}{N}\sum_{i=1}^{N_+}\left(\frac{\lambda_0}{\lambda_i} - 1\right)$$

diverges whenever a Lyapunov exponent approaches zero from above (points where long time correlations develop)

- Self-organization (partitions $\Sigma_k = R^k \times R^{m-k}$)

$$I_\Sigma(\mu) = \sum_{k=1}^{N}\{h_k(\mu) + h_{m-k}(\mu) - h(\mu)\}$$

$$h_k(\mu) = \sum_{\xi_i^{(k)}>0} \xi_i^{(k)};\, h_{m-k}(\mu) = \sum_{\xi_i^{(m-k)}>0} \xi_i^{(m-k)};\, h(\mu) = \sum_{\lambda_i>0} \lambda_i$$

- Self-organization concerns the dynamical relation of the whole to its parts. Therefore, $I_{\Sigma}(\mu)$ is a measure of dynamical self-organization

- Also, it is a measure of apparent dynamical freedom (or apparent rate of information production).

- These parameters characterize the dynamics of multiagent networks.

- Also, provide insight on the relation between dynamics and the topology of the network (the small world phase, for example)

Examples :

- Fully coupled system

$$x_i(t+1) = (1-c)\, f(x_i(t)) + (c/(N-1)) \Sigma_{k\neq i} f(x_k(t))$$

$$f(x)=2x \pmod 1$$

c = 0.495

c = 0.51

Fig. 2

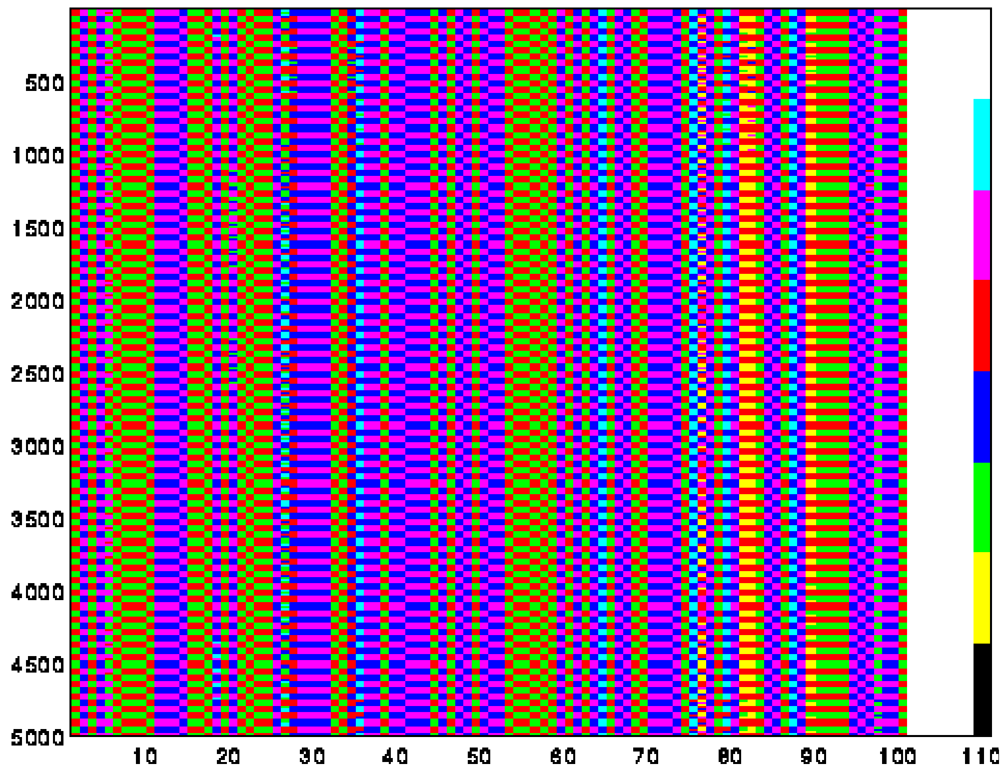
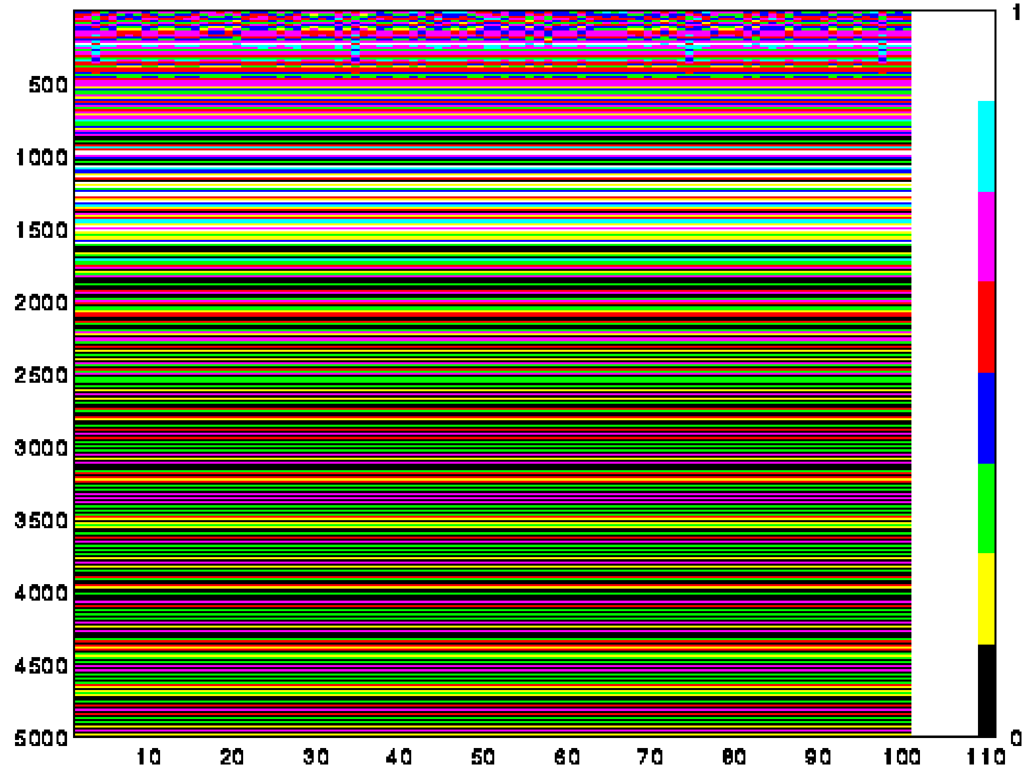
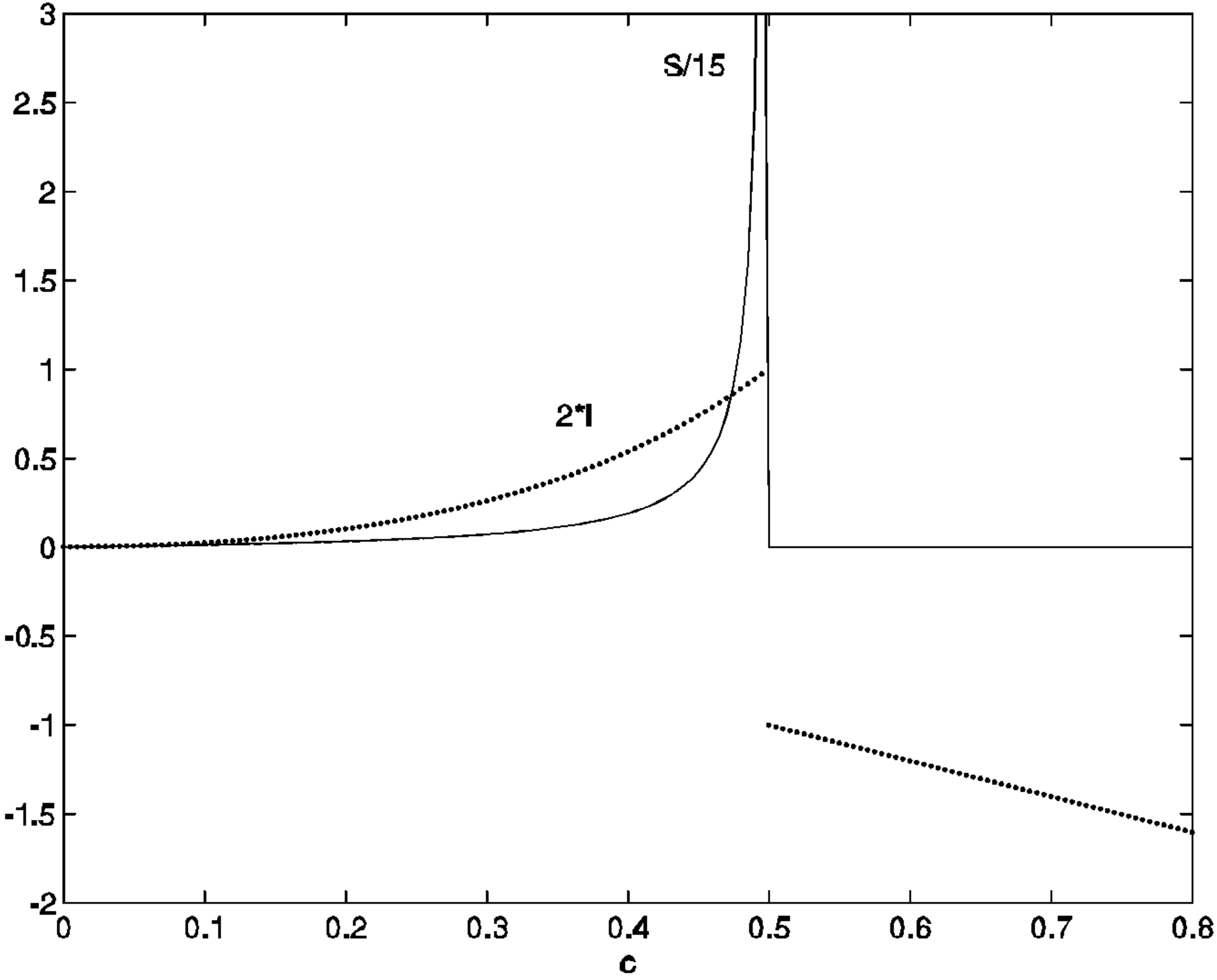


Fig. 3

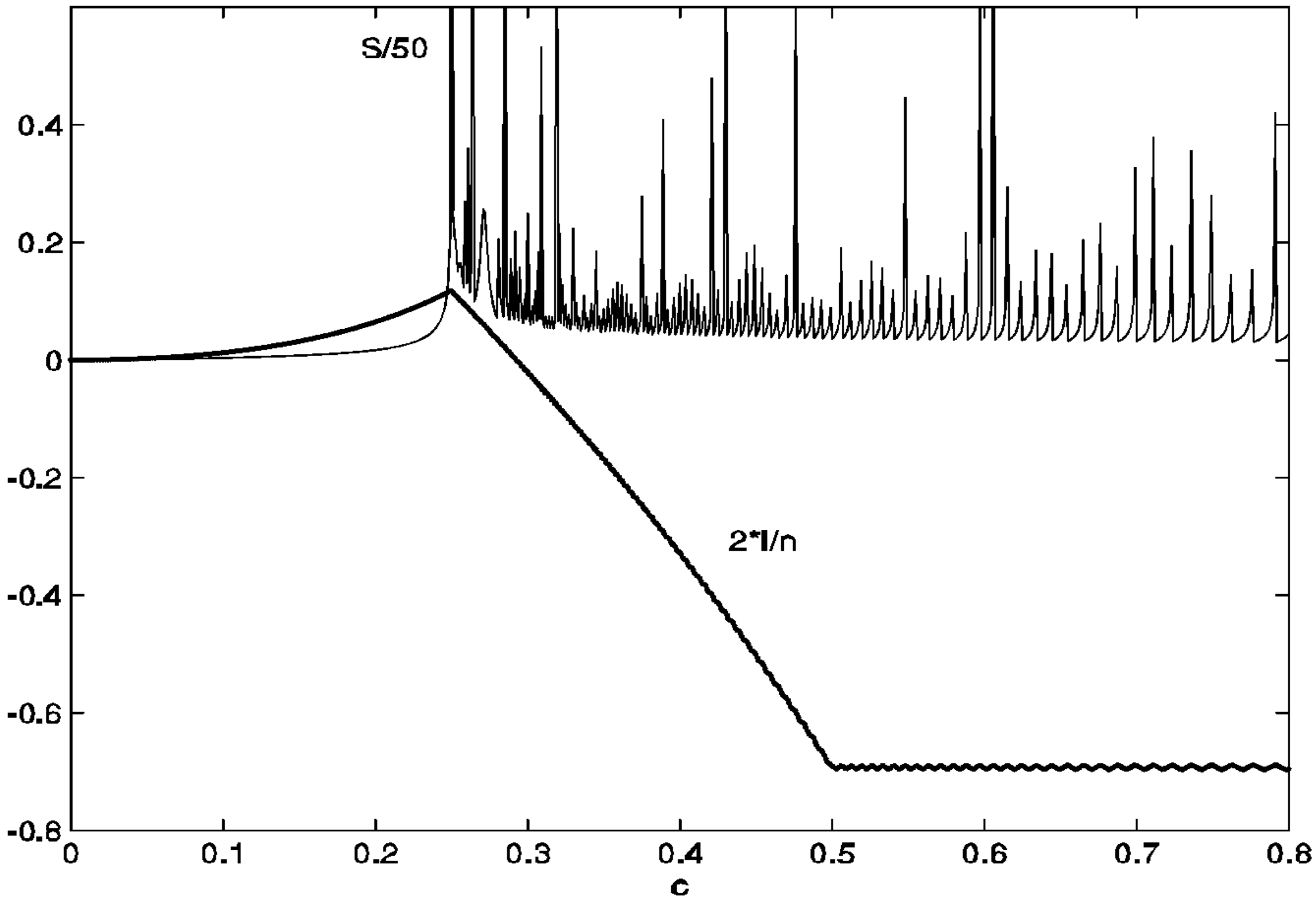


Fully coupled system. Structure and self-organization indexes



Nearest-neighbor coupling

- $x_i(t+1) = (1-c)\, f(x_i(t)) + (c/2)\,(\, f(x_{i+1}(t)) + f(x_{i-1}(t))\,)$



6 - Back to synchronization

- Synchronization and dynamical correlations
  Discrete-time oscillators model :

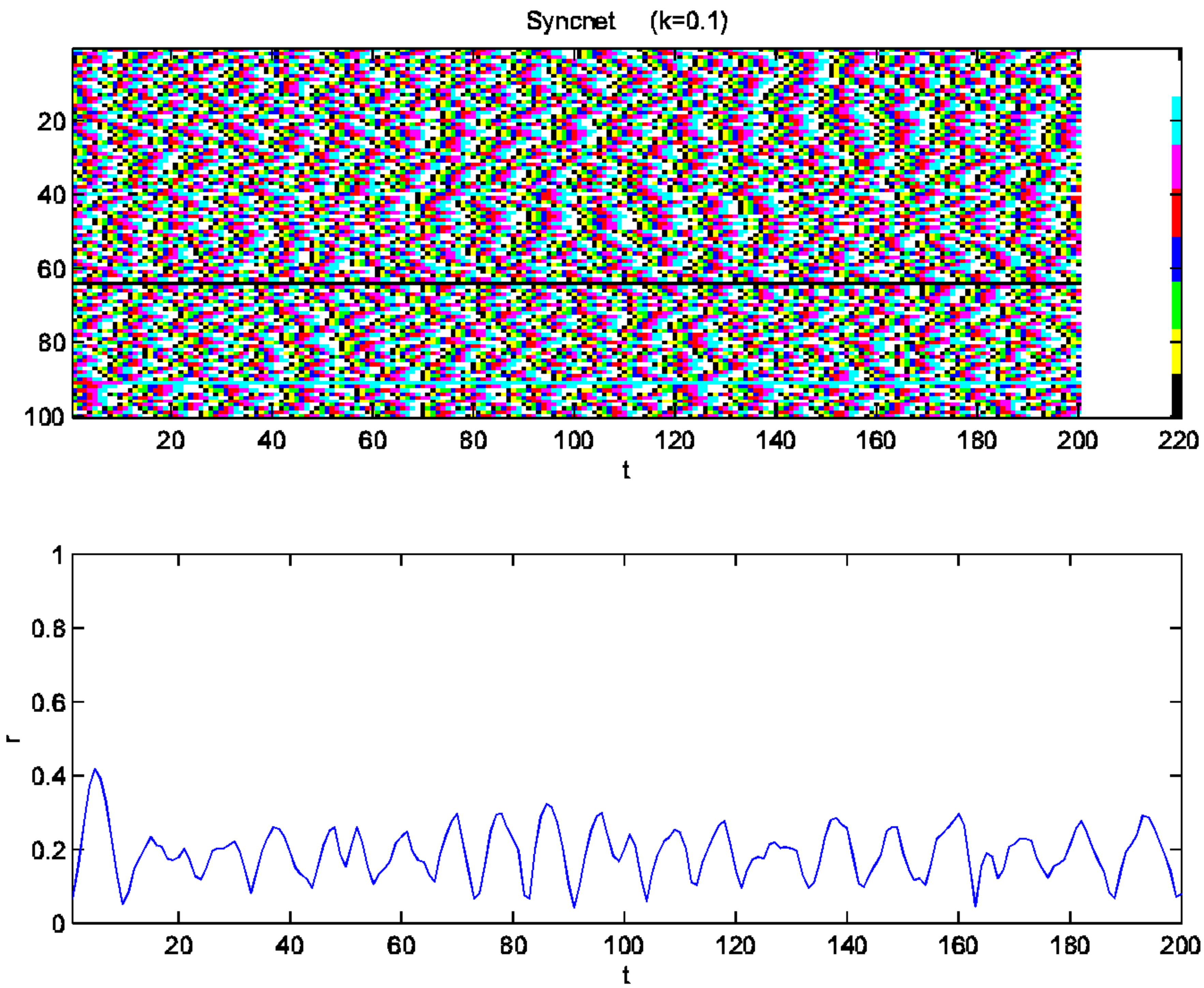
$$x_i(t+1) = x_i(t) + \omega_i + \frac{k}{N-1}\sum_{j=1}^{N} f_\alpha(x_j - x_i)$$

$$p(\omega) = \frac{\gamma}{\pi\left[\gamma^2 + (\omega - \omega_0)^2\right]}$$

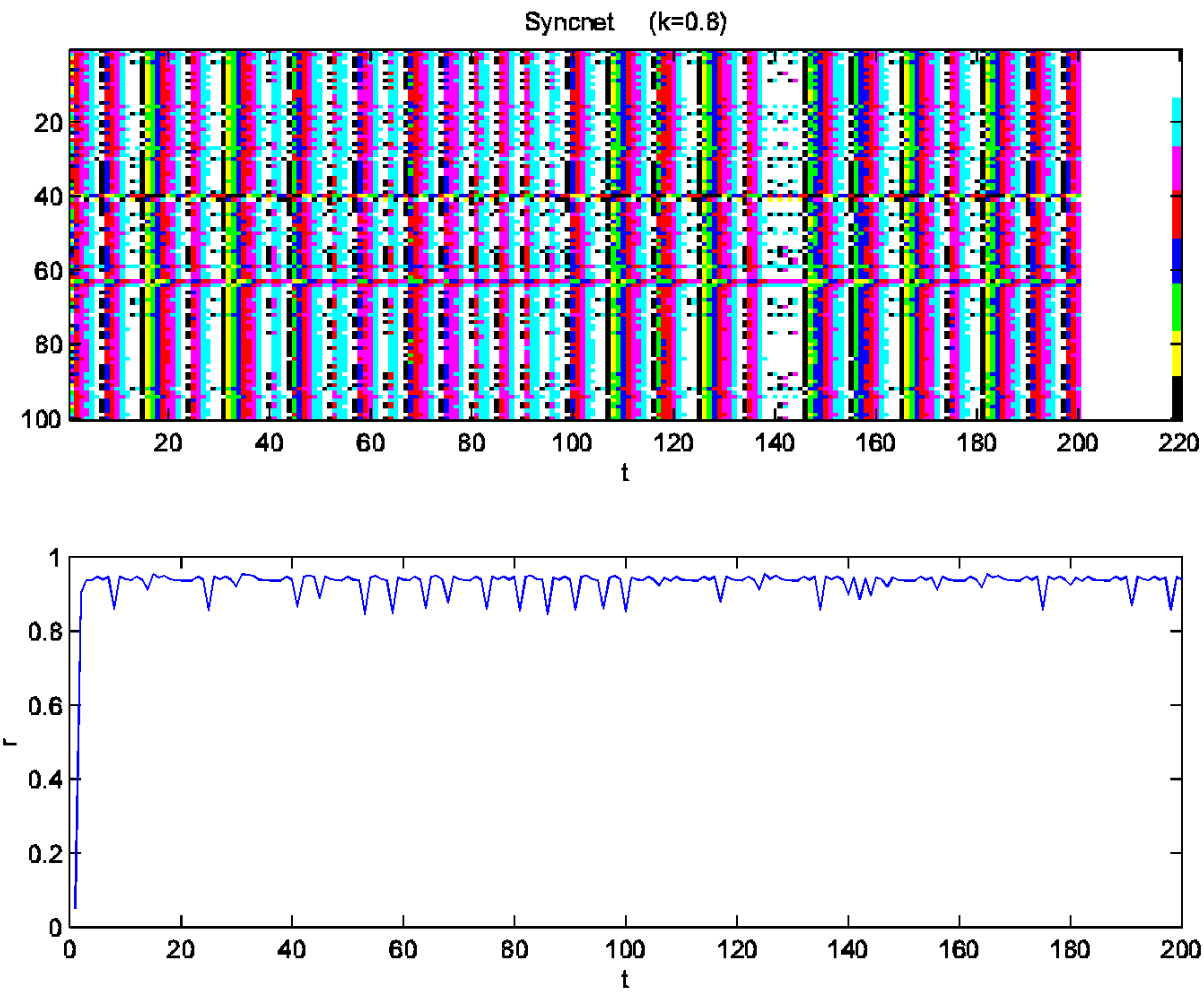
$$f_\alpha(x_j - x_i) = \alpha(x_j - x_i)(mod1)$$

- Order parameter

$$r(t) = \left|\frac{1}{N}\sum_{j=1}^{N} e^{i2\pi x_j(t)}\right|$$



Syncnet (k=0.1)
t
r
t



Syncnet (k=0.8)
t
r
t

- The Lyapunov spectrum controls the dynamical self-organization of the system.

- In this case

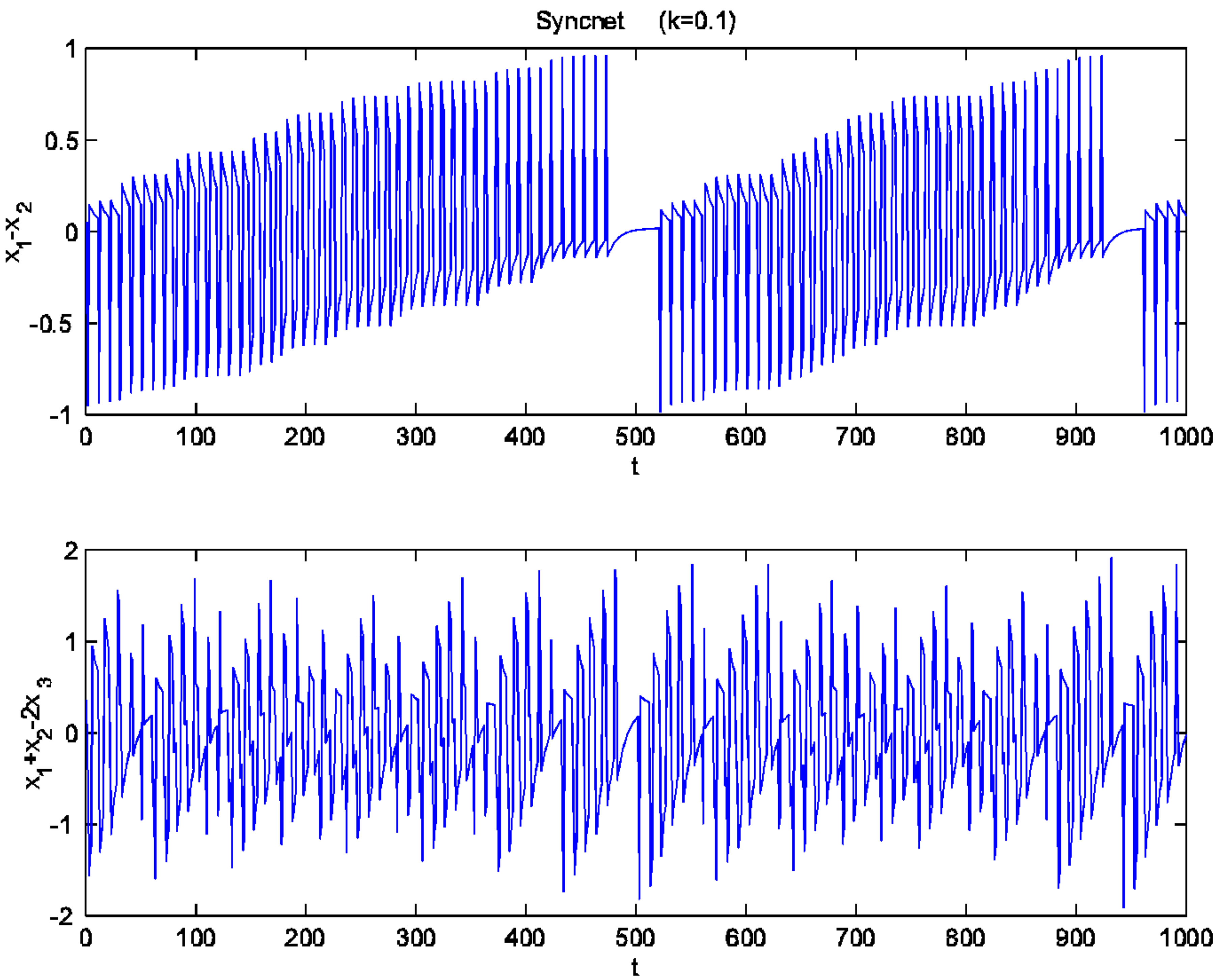
$$\lambda_1 = 0 \quad \text{and}$$

$$\lambda_i = \log(1 - \alpha\lambda k(N/N-1)) \qquad (N-1) \text{ times}$$

N-1 contracting directions for $k \neq 0$

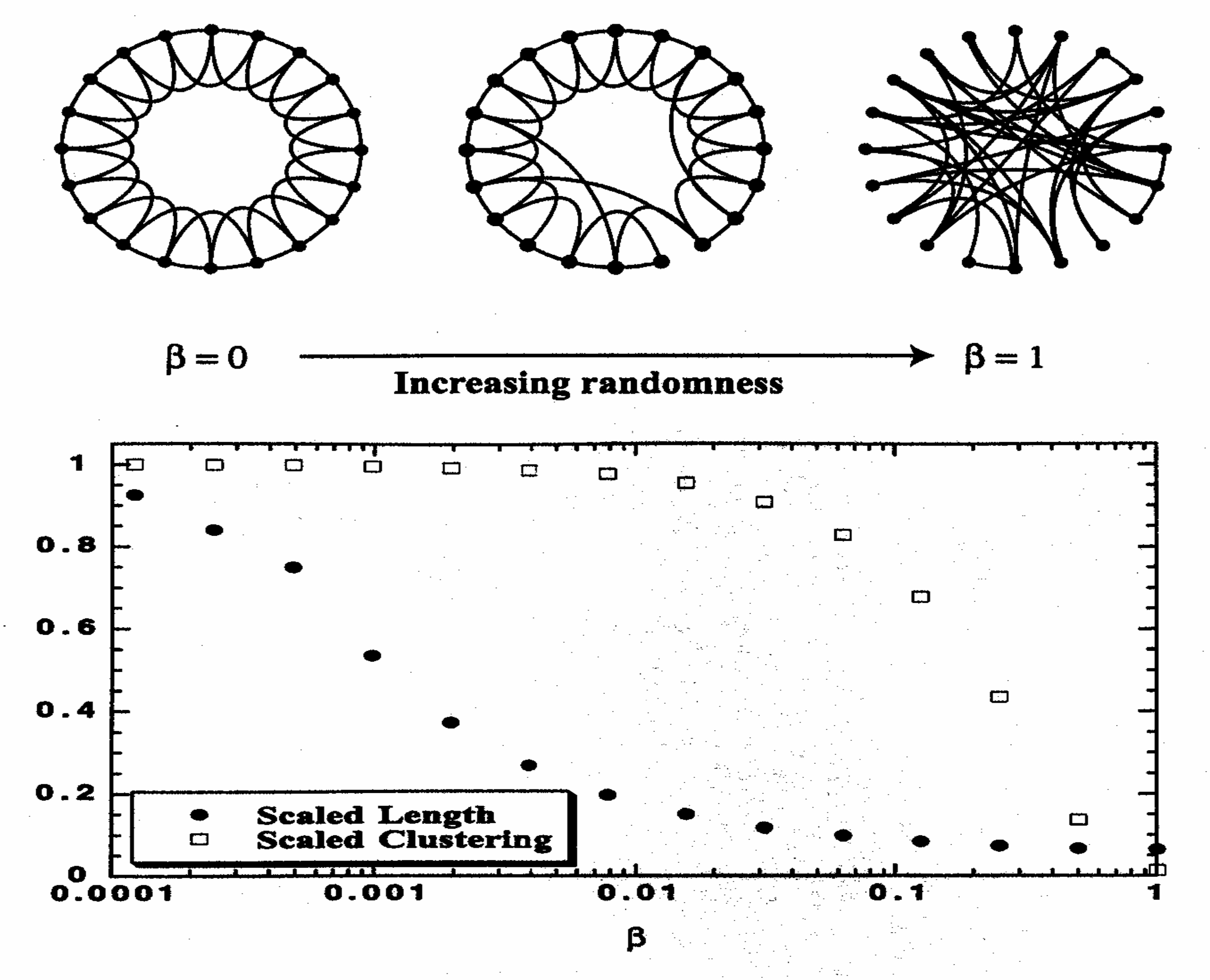
"One-dimensional" system !

- $\Rightarrow$ strong dynamical correlations even before synchronization


Syncnet (k=0.1)
x1-x2
t
x1+x2-2x3
t
0
100
200
300
400
500
600
700
800
900
1000
1
0.5
0
-0.5
-1
2
1
0
-1
-2

7.Structure versus dynamics.The small world phase

◆



Define a dynamical system on the network nodes

- $x_i(t+1) = \Sigma_{k=1}^{N}\ W_{ik}\ f(x_k(t))$

  $f(x)=\alpha x \pmod 1$

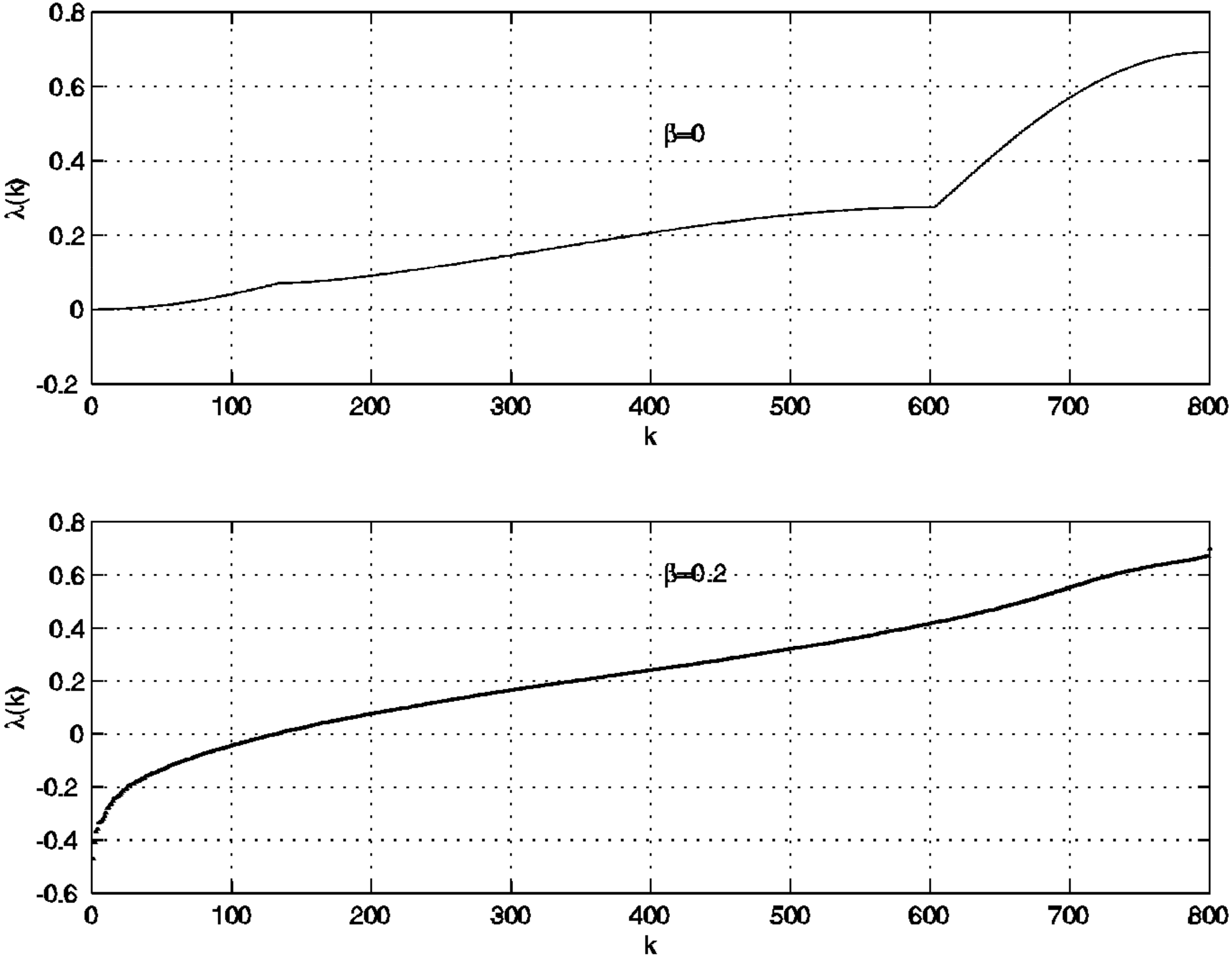
$$W_{ik} = \begin{cases} 1-\dfrac{n_v(i)}{2v}c & \text{if } i = k \\ \dfrac{c}{2v} & \text{if } i \neq k \text{ and } k \in n_v(i) \\ 0 & 0 \text{ otherwise} \end{cases}$$

- $D_\beta = -\ \Sigma_{\lambda_i<0}\ \lambda_i$

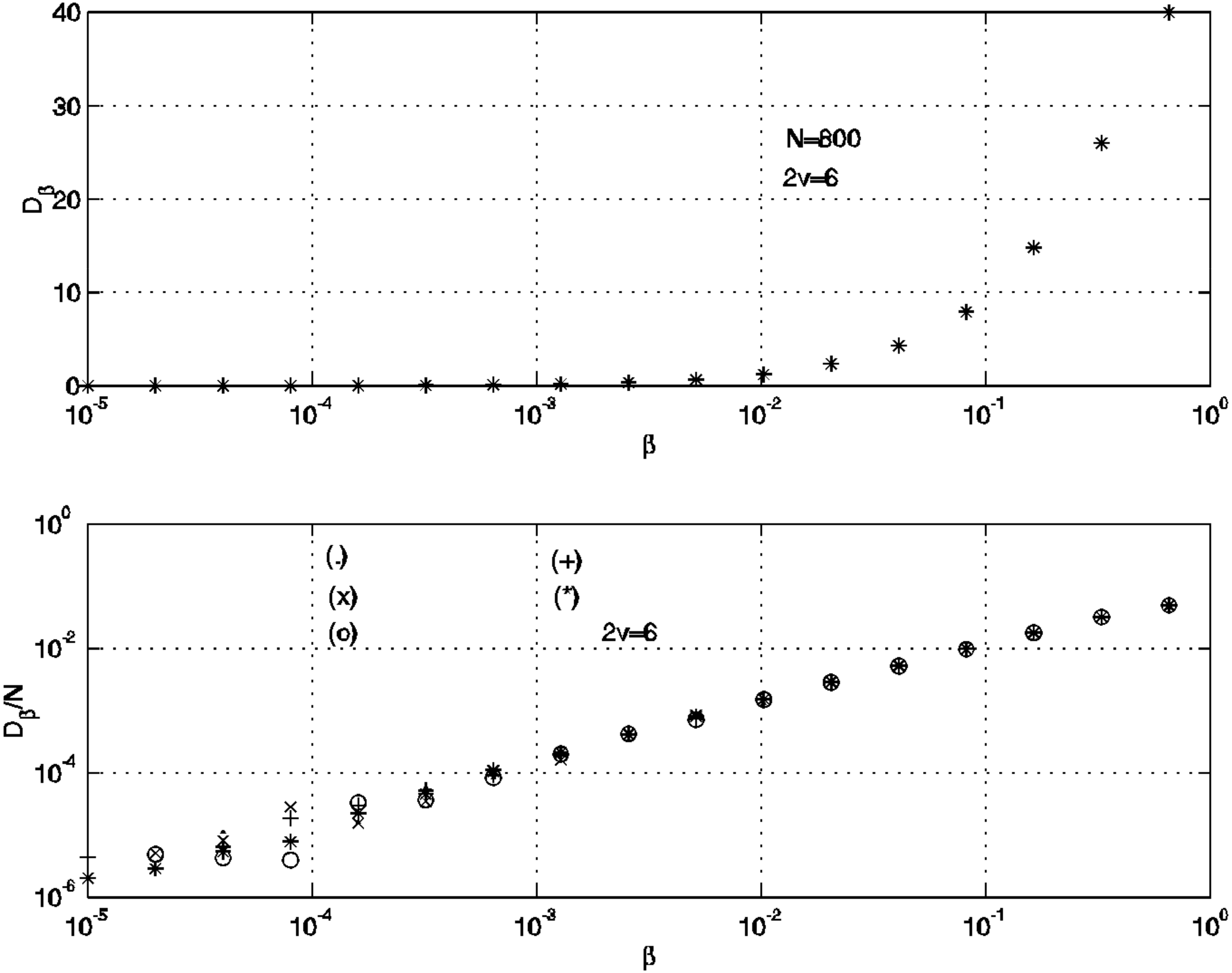
  $D_\beta = c\ N\ (\beta - \beta_{c1})^{\eta}$ $\qquad \beta_{c1} < 10^{-5}$ $\qquad \eta = 1.01 \pm 0.06$

- $C_\beta = \left|\dfrac{h_0^* - h_0}{h_\beta^* - h_\beta}\right|; \quad h_\beta^* = \sum_{i=1}^{N}\left(\dfrac{1}{d_i}\sum_{\lambda_\beta^*>0}\lambda_\beta^*(j)\right); \quad h_\beta = \sum_{\lambda_\beta>0}\lambda_\beta(j)$

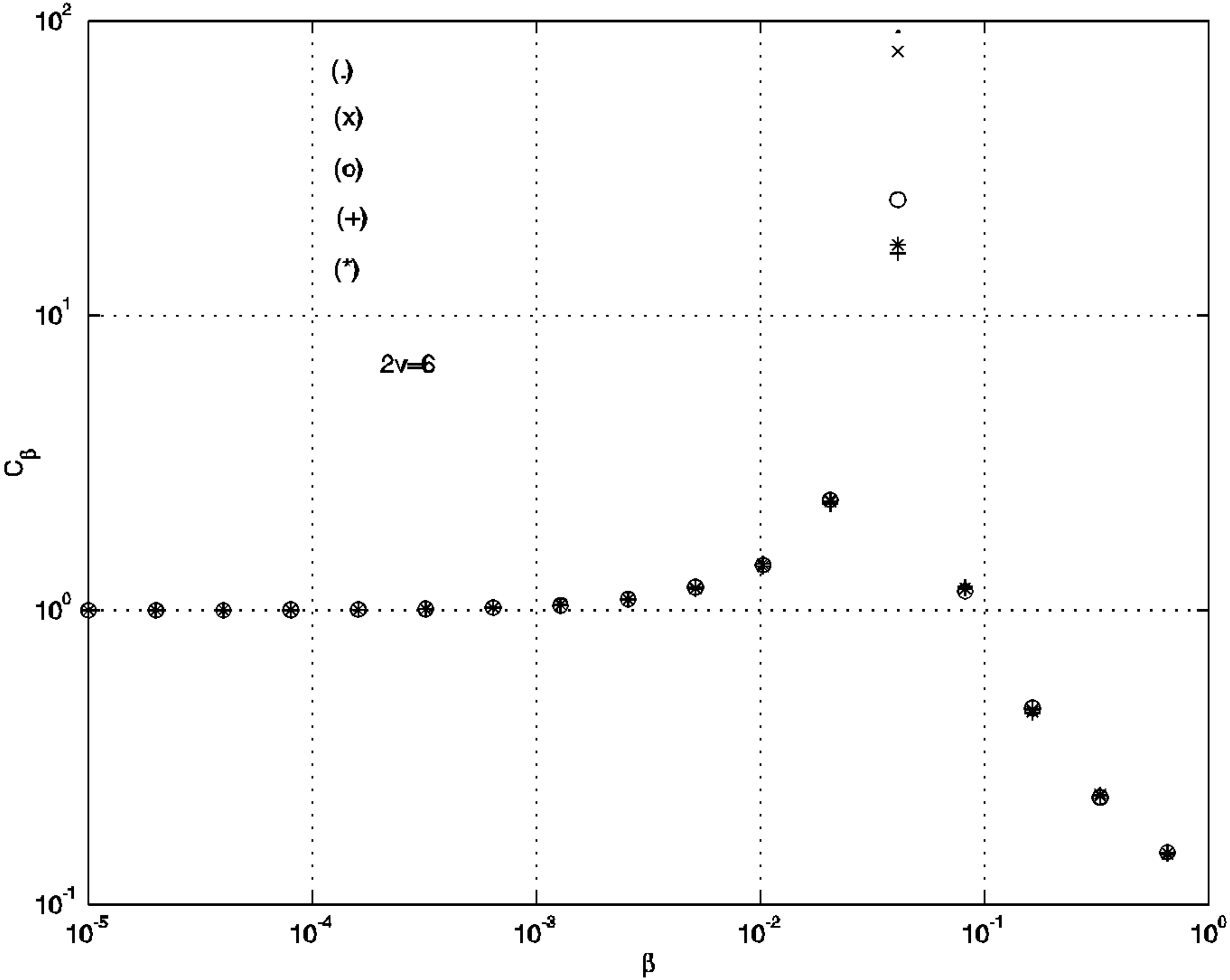
  $\beta_{c2} = 0.04$ $\qquad C_\beta \sim |\ \beta - \beta_{c2}\ |^{-\delta}$ $\qquad \delta_1 = 1.14 \quad \delta_2 = 0.93$



β=0
β=0.2
λ(k)
k



N=600
2v=6
D_β
β
(.)
(x)
(o)
(+)
(*)
2v=6
D_β/N
β



10^2
10^1
10^0
10^{-1}
C_β
(.)
(x)
(o)
(+)
(*)
2v=6
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
β

8 - Ergodic theory. Beyond the classical parameters

- Lyapunov and conditional exponents and derived quantities depend on the actual (or expected) ***average*** rates of expansion
- ***Fluctuations*** of the expansion rates along the trajectories

Generalized Lyapunov exponents

$$\Lambda(\beta) = \lim_{N\to\infty} \frac{1}{\beta N} \log \int d\mu(x_0) \exp\left[\beta \sum_{n=0}^{N-1} \log \left| f'(x_n) \right| \right]$$

Dynamical Rényi entropies

$$K(\alpha) = \lim_{N\to\infty} \frac{1}{1-\alpha} \frac{1}{N} \log \sum_{i_0 \ldots i_{N-1}} \left(p(i_0 \ldots i_{N-1}) \right)^{\alpha} \qquad \Lambda(\beta) = K(1-\beta)$$

Cumulants of the Lyapunov spectrum

$$K(\alpha) \cong \sum_{s=1}^{\infty} c_s \frac{(1-\alpha)^{s-1}}{s!}$$

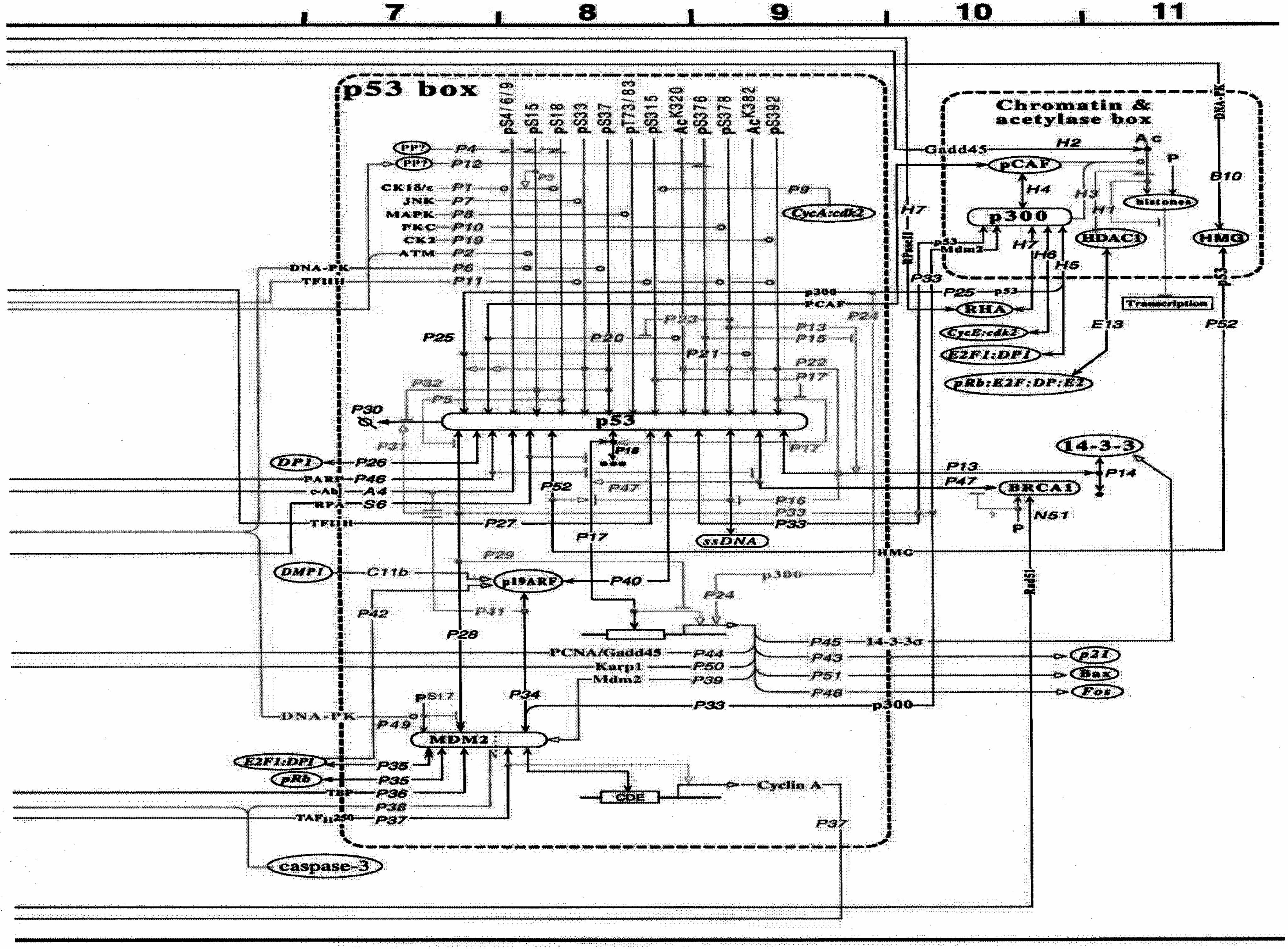
Traces of Hessian powers

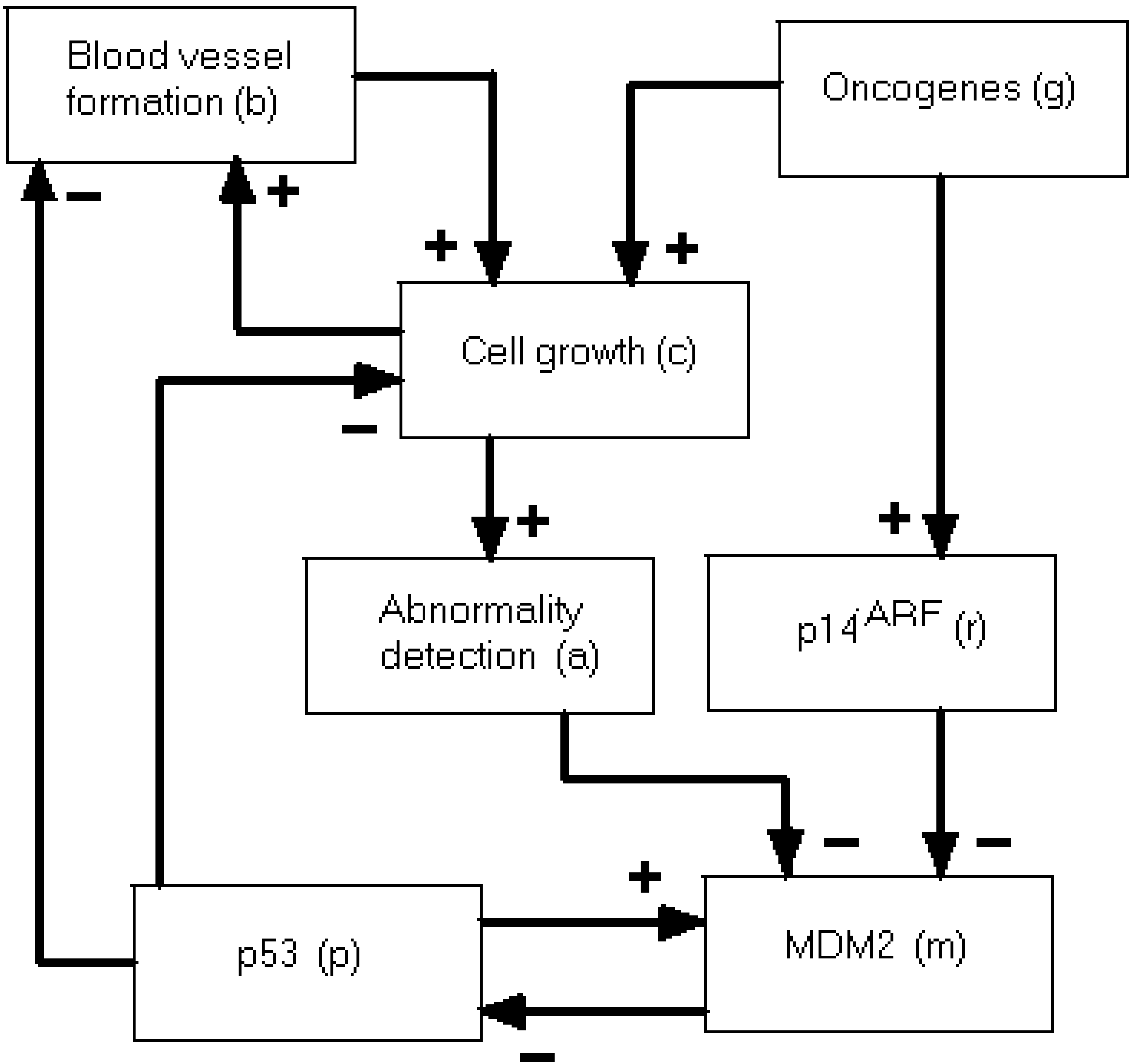
$$\frac{1}{2} H_N = \delta_{\alpha,\beta} \delta_{j,k} - (1-\delta_{k,N}) \delta_{k,j-1} \frac{\partial f^{\alpha}(x_k)}{\partial x_k^{\beta}} - (1-\delta_{j,N}) \delta_{j,k-1} \frac{\partial f^{\beta}(x_j)}{\partial x_j^{\alpha}} + (1-\delta_{j,N}) \delta_{j,k} \frac{\partial f^{\gamma}(x_j)}{\partial x_j^{\beta}} \frac{\partial f^{\gamma}(x_j)}{\partial x_j^{\alpha}}$$

9 - A biological network. A toy model for the p53 action

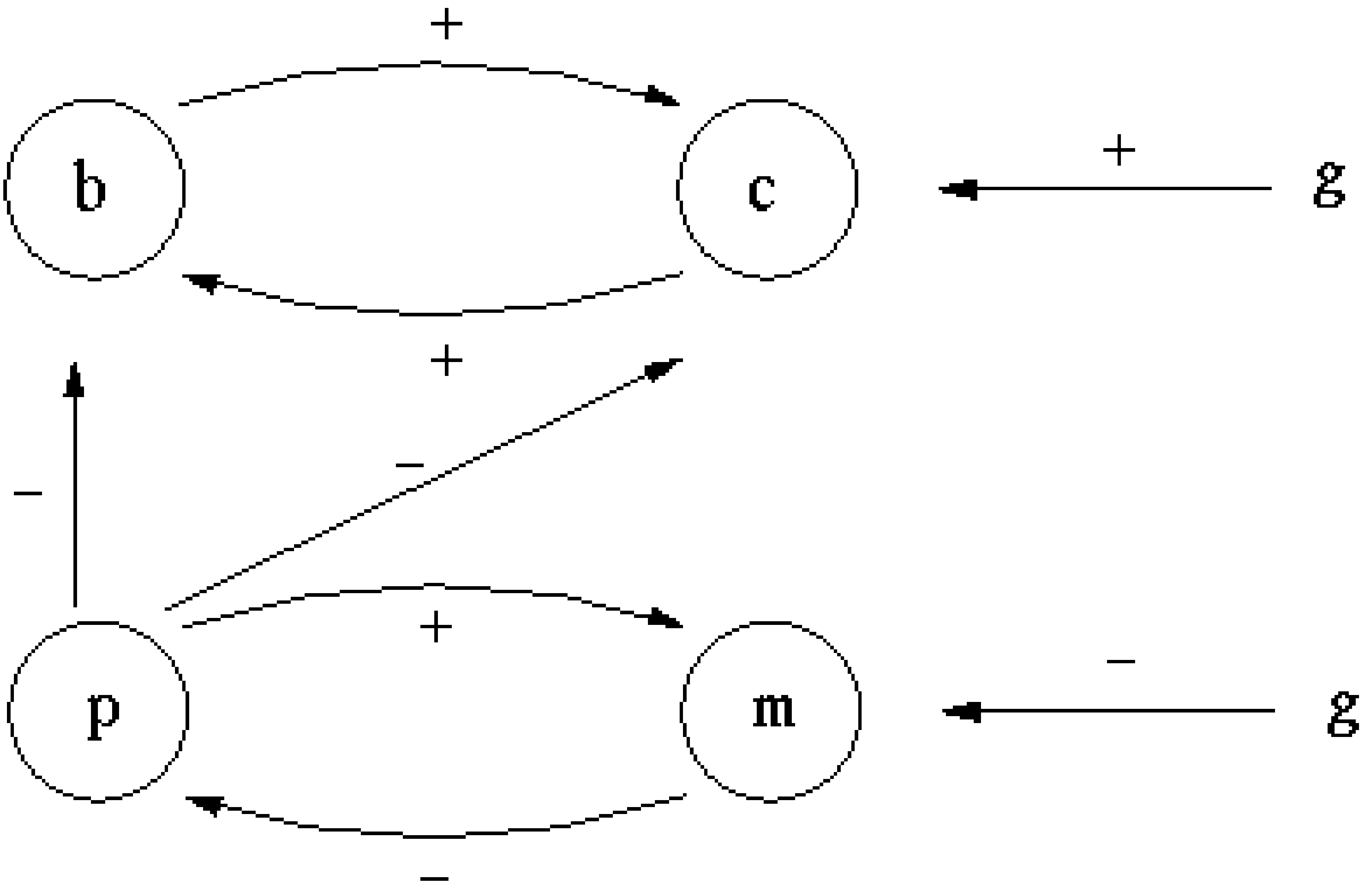
- p53 is a tumour suppressor gene. Its protein acts as an inhibitor of uncontrolled or defective cell growth
- In most tumours it is found to be mutated or inactivated by viral proteins
- In a number of cases normal p53 cannot achieve control
- Is “off” in normal circumstances. Produced at some rate but degraded by ubiquitin labelling (MDM2, ...)
- Activates its own control
- Is activated by detection of cell anomalies, DNA damage, abnormal growth signals, through inhibition of degradation

- A very complex network of interactions (K. W. Kohn, Mol. Bio. Cell 10, 2703-2734, 1999)


7
8
9
10
11
p53 box
pS4/6/9
pS15
pS18
pS33
pS37
pT73/83
pS315
AcK320
pS376
pS378
AcK382
pS392
PP?
P4
P12
CK1δ/ε
P1
P3
JNK
P7
MAPK
P8
PKC
P10
CK2
P19
ATM
P2
DNA-PK
P6
TFIIH
P11
P9
CycA:cdk2
p300
PCAF
P23
P13
P15
P24
P25
P20
P21
P22
P17
P32
P5
P30
p53
P31
P18
DP1
P26
PARP
P46
c-Abl
A4
RPA
S6
P52
P47
P16
P33
P27
ssDNA
DMP1
C11b
P29
p19ARF
P40
HMG
p300
P24
P42
P41
P28
P45
14-3-3σ
PCNA/Gadd45
P44
Karp1
P50
Mdm2
P39
P43
P51
P48
P34
P33
p300
DNA-PK
P49
MDM2
E2F1:DP1
P35
pRb
P35
TBP
P36
P38
TAFII250
P37
CDE
Cyclin A
P37
caspase-3
Chromatin & acetylase box
Gadd45
H2
Ac
P
pCAF
H4
H3
histones
H1
p300
H7
p53
Mdm2
H7
H6
H5
HDAC1
HMG
B10
DNA-PK
P33
P25
p53
RHA
CycE:cdk2
E2F1:DP1
pRb:E2F:DP:E2
E13
Transcription
P52
14-3-3
P13
P14
P47
BRCA1
N51
P
Rad51


Blood vessel formation (b)
Oncogenes (g)
Cell growth (c)
Abnormality detection (a)
p14 ARF (r)
p53 (p)
MDM2 (m)
+
−

A positive and a negative cycle in interaction



A positive and a negative cycle in interaction

- $p(t+1) = a_p\, p(t) + W_{pm}\, H(T_m - m(t))$
  $m(t+1) = a_m\, m(t) + W_{pm}\, H(p(t) - T_p) + W_{mg}\, H(T_g - g)$
  $c(t+1) = a_c\, c(t) + W_{cb}\, H(b(t) - T_b) + W_{cp}\, H(T_p - p(t)) + W_{cg}\, H(g - T_g)$
  $b(t+1) = a_b\, b(t) + W_{bc}\, H(c(t) - T_c) + W_{bp}\, H(T_p - p(t))$

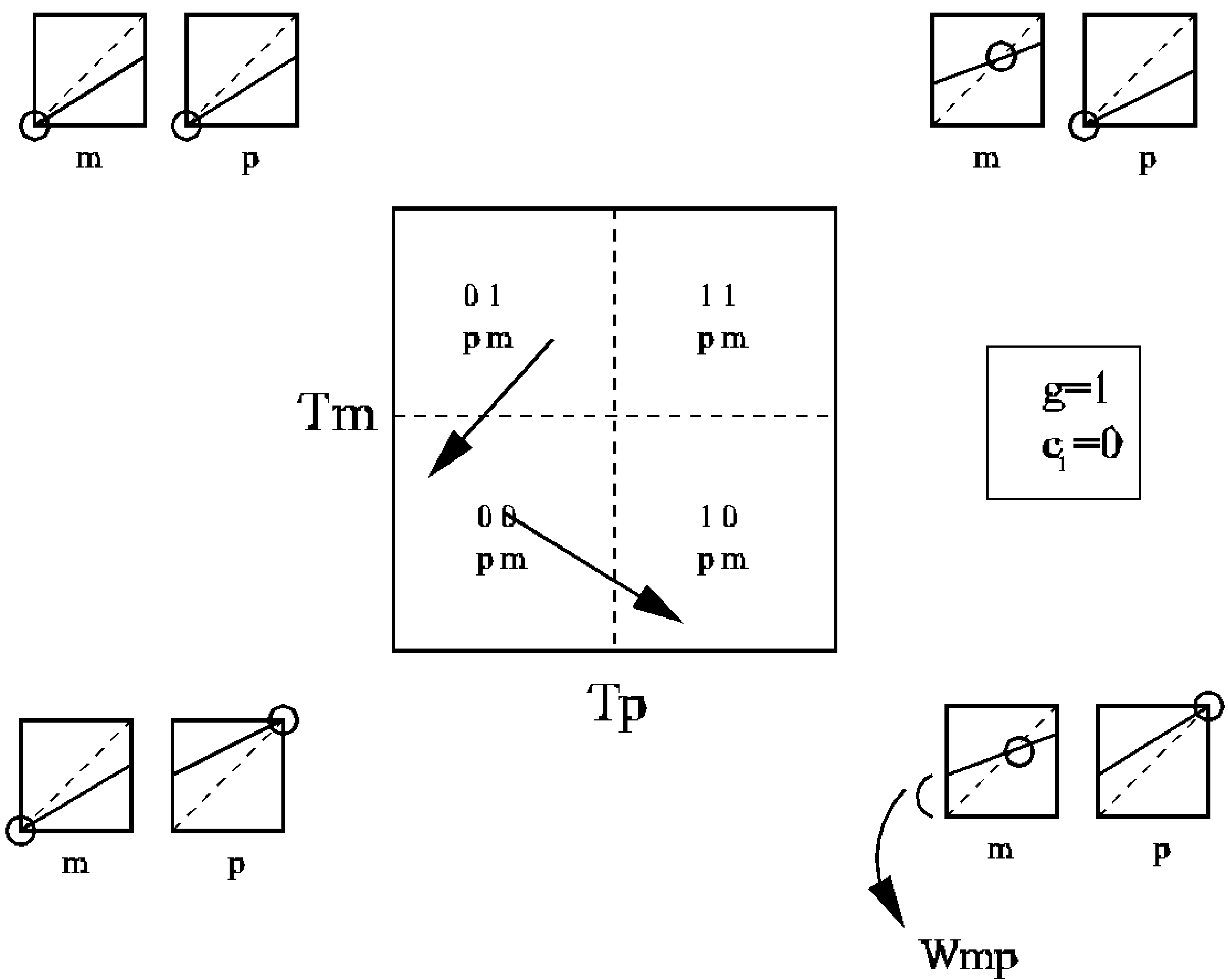
- Piecewise linear dissipative dynamics with thresholds

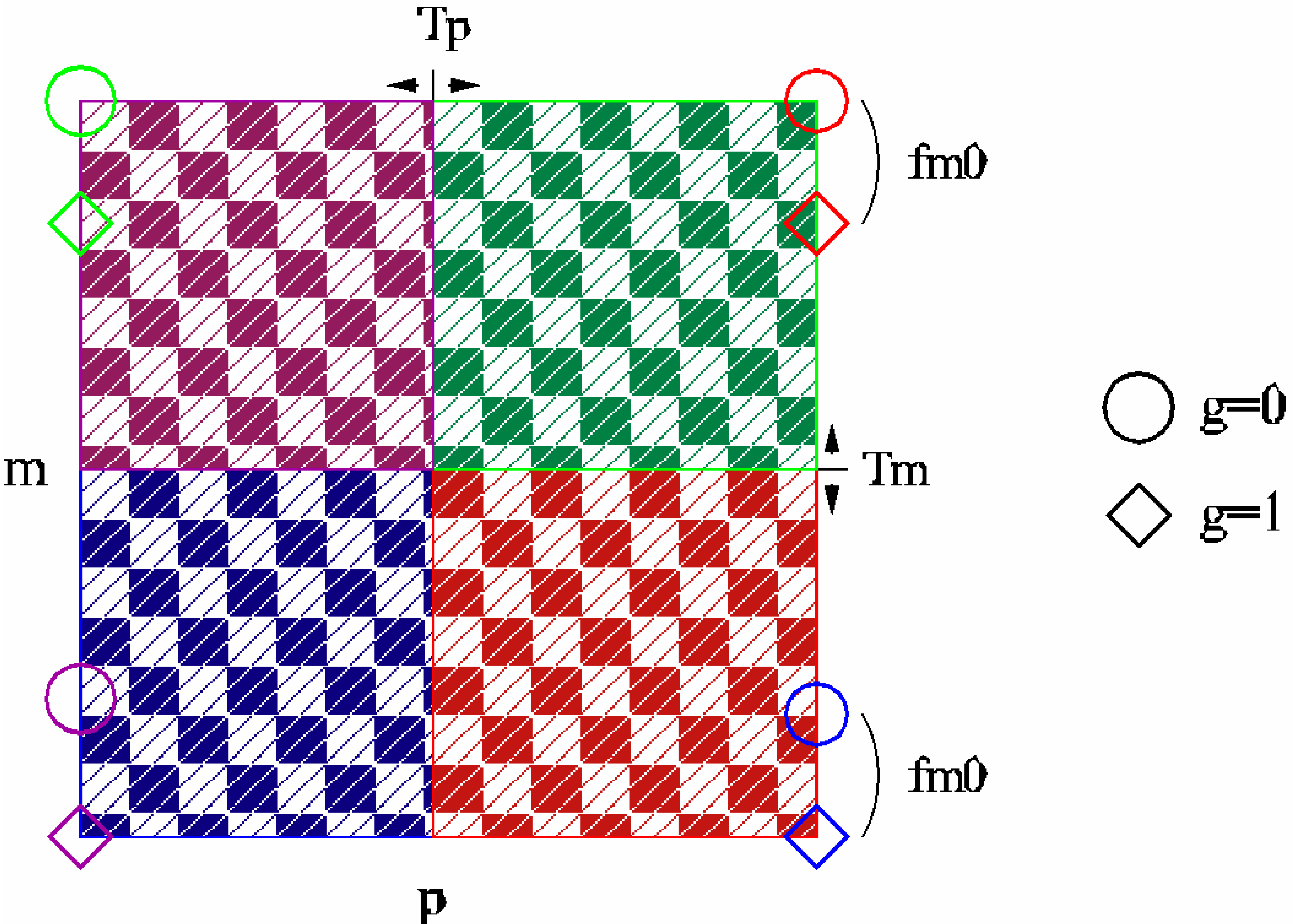
- Important quantities :

  $fmg = W_{mg} / (1 - a_m)$; $fmp = 1 - fmg$

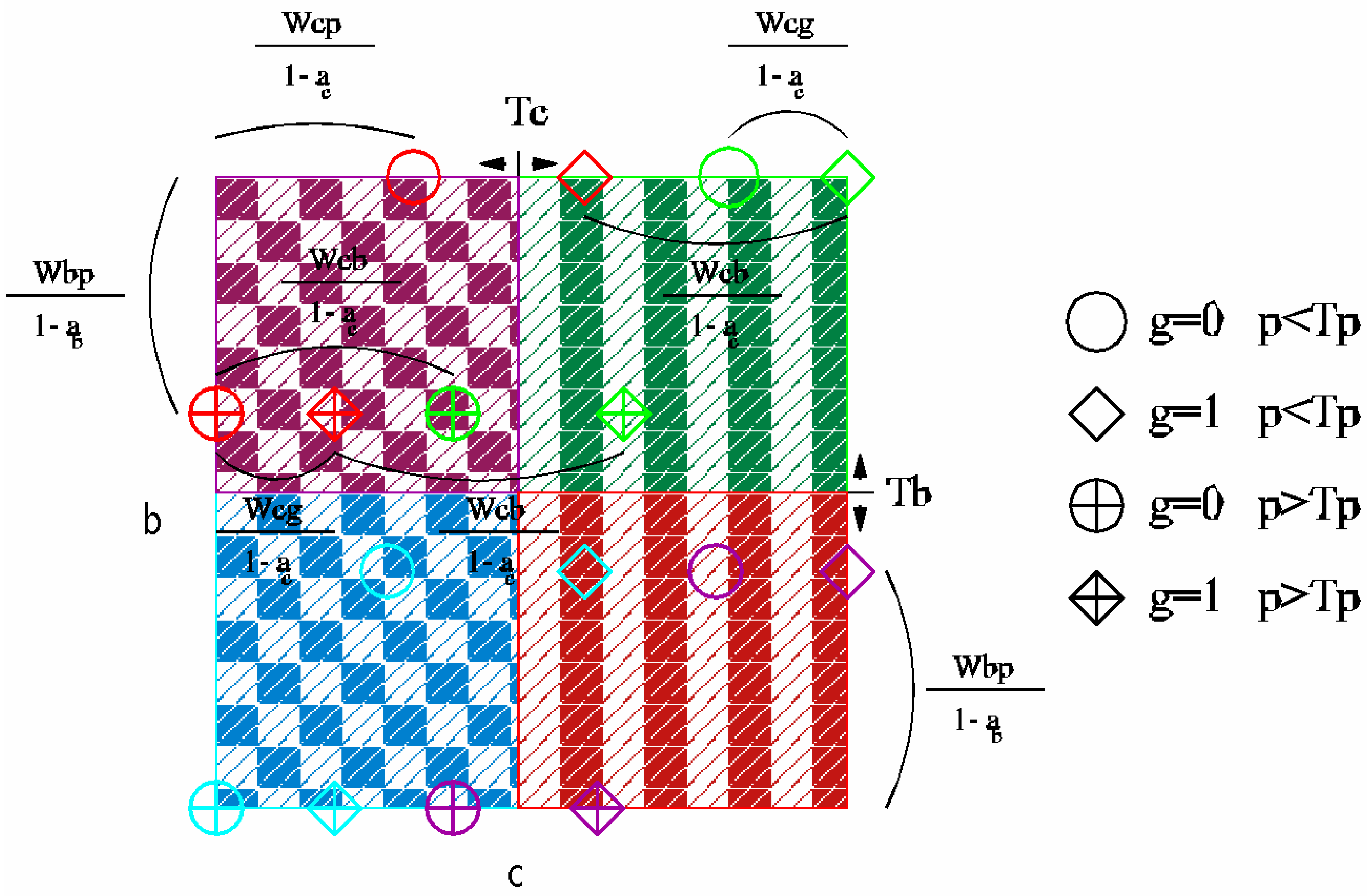
  $fcp = W_{cp} / (1 - a_c)$; $fcb = W_{cb} / (1 - a_c)$; $fcg = 1 - fcp - fcb$

  $fbc = W_{bc} / (1 - a_b)$; $fbp = 1 - fbc$


m
p
m
p
0 1
p m
1 1
p m
Tm
0 0
p m
1 0
p m
Tp
g=1
c_i =0
m
p
m
p
Wmp

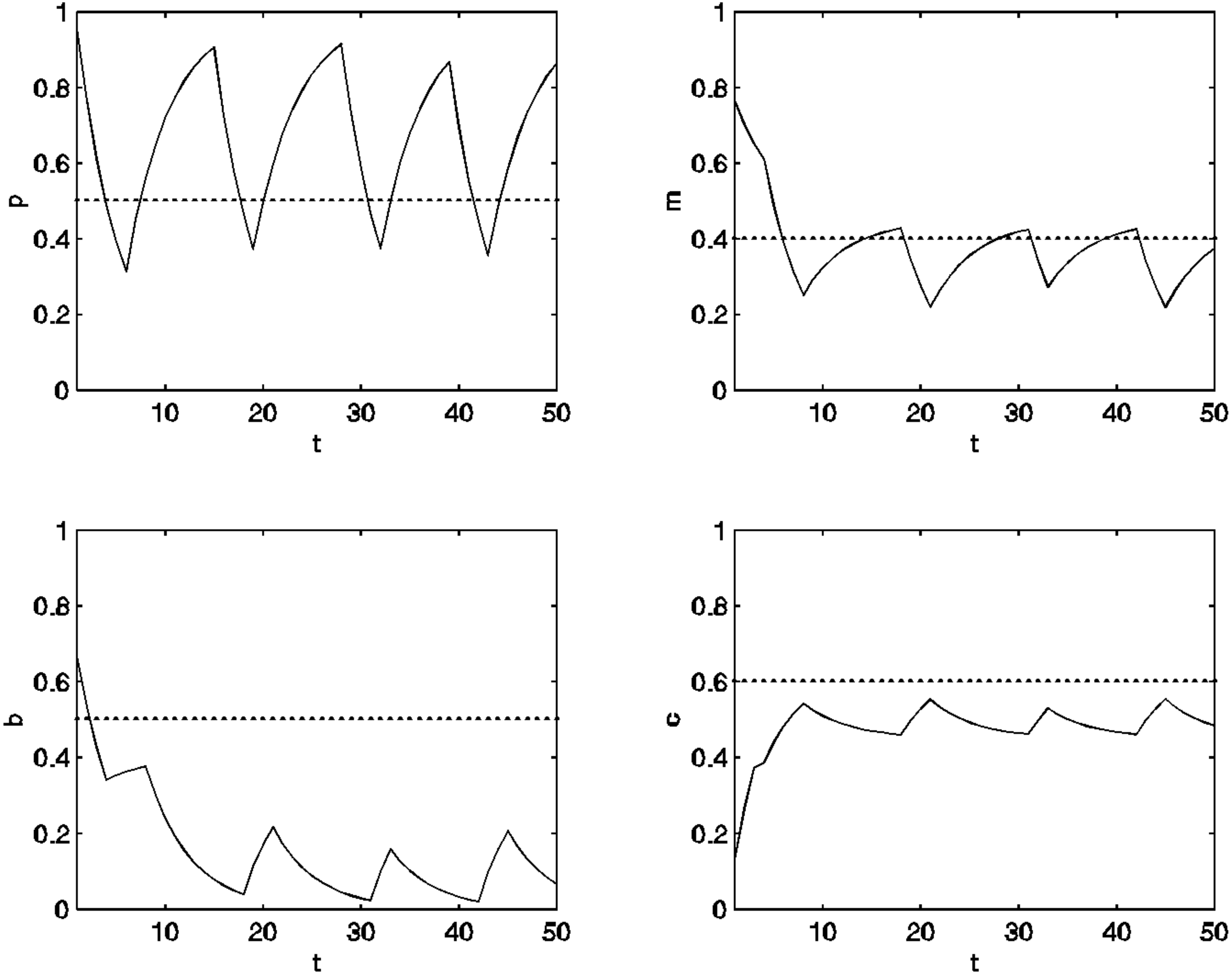


Tp
fm0
m
Tm
fm0
p
g=0
g=1

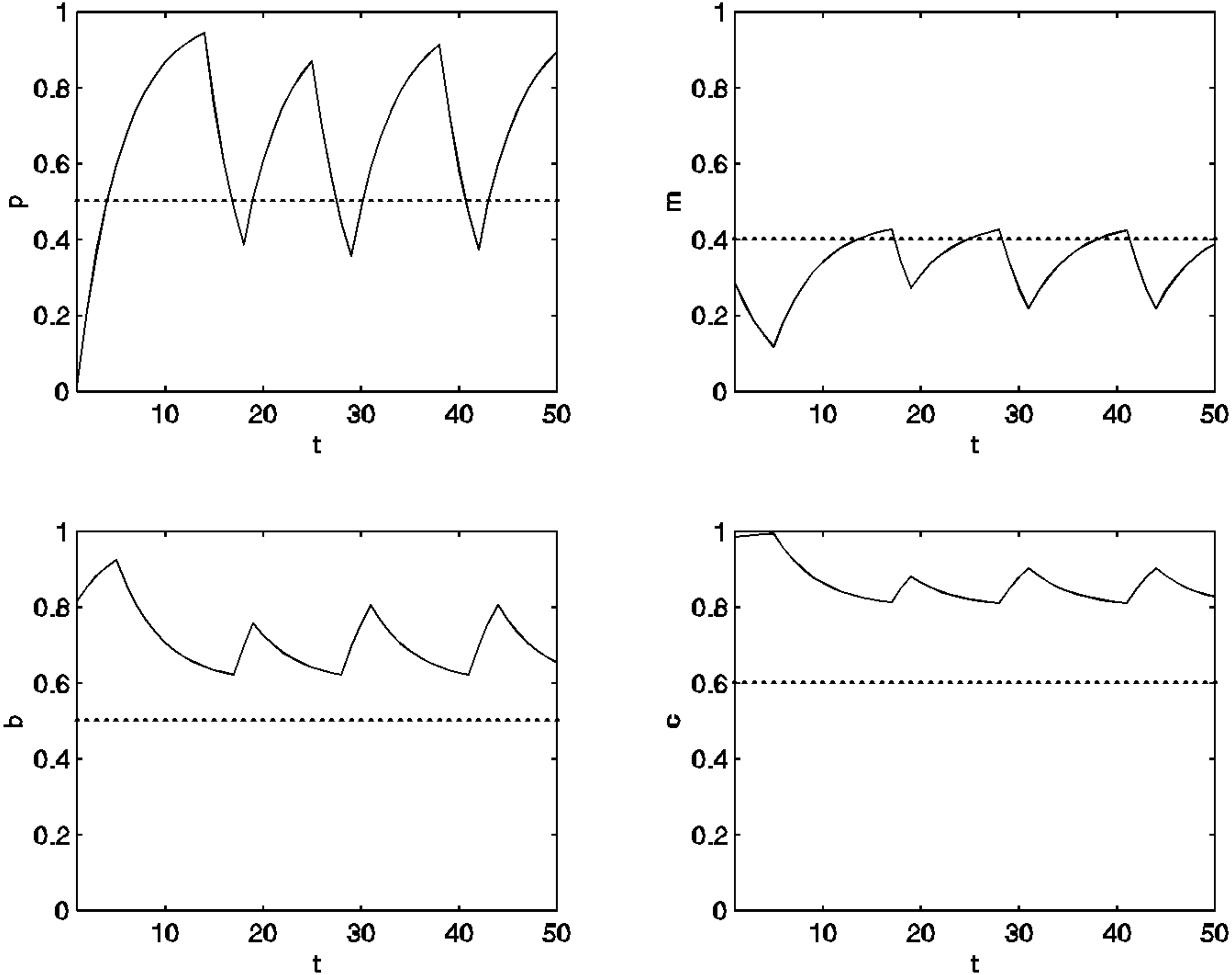


Wcp
1- ac
Wcg
1- ac
Tc
Wbp
1- ab
Wcb
1- ac
Wcb
1- ac
Tb
b
Wcg
1- ac
Wcb
1- ac
Wbp
1- ab
c
g=0 p<Tp
g=1 p<Tp
g=0 p>Tp
g=1 p>Tp

- Complete characterization of asymptotic states and coding of trajectories.
- Biological realism imposes some conditions on the threshold values : $T_m < fmg$; $T_c > fcp + fcb$; $T_b < fbp$
- Most interesting case : Expressed p53 ($p>T_p$) with oncogenes ($g>T_g$)
  - $T_c > fcg+fcb \Rightarrow$ effective control of cell growth
  - $T_c < fcg$ and $T_b < fbc \Rightarrow$ no control
  - $T_c < fcg+fcb \Rightarrow$ depends on initial conditions



p
m
b
c
t
0
0.2
0.4
0.6
0.8
1
10
20
30
40
50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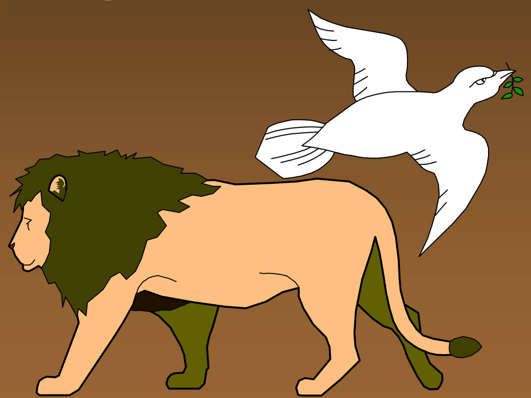
10 - Networks in evolutionary sociology. Strong reciprocity

- Nash equilibrium and experimental games
- Homo Oeconomicus rejected in all cases
- The player's behavior is strongly correlated with existing social norms in their societies and market structure
- Human decision problems involve a mixture of self-interest and a background of (internalized) social norms
- Exits Homo Oeconomicus
- Enters Homo Reciprocans (Samuel Bowles, Herbert Gintis) Strong reciprocity

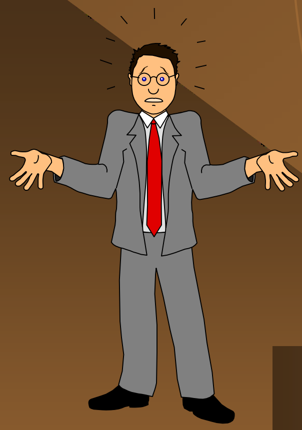
Homo reciprocans

- Homo reciprocans comes to new social situations with a propensity to cooperate and share, responds to cooperative behavior by maintaining or increasing the level of cooperation and responds to selfish free-riding behavior on the part of others by retaliating, even at a cost to himself and even when he could not expect future personal gains
- Strong reciprocity is a form of altruism in that it benefits others at the expense of the individual that exhibits this trait.



Homo reciprocans

- Monitoring and punishing selfish agents or norm violators is a costly (and dangerous) activity without immediate direct benefit to the agent that performs it
- It seems that the strong reciprocity trait could not invade a population of self-interested agents, nor be maintained in a stable population equilibrium
- Not evolutionary stable ?



Homo reciprocans. The Bowles-Gintis model

- Small hunter-gatherer bands of the late Pleistocene
- Population of size N with two species of agents:
- Reciprocators (R-agents)
- Self-interested (S-agents)
- Public goods activity: each agent can produce a maximum amount of goods q at cost b
- The benefit that an S-agent takes from shirking is the cost of effort $b(\sigma)$, σ being the fraction of shirking time
- $b(0)=b \quad b(1)=0 \quad b'(\sigma)<0 \quad b''(\sigma)>0 \quad q(1-\sigma)>b(\sigma)$
- At every level of effort, working helps the group more than it hurts the worker

Homo reciprocans. The Bowles-Gintis model

- R-agents never shirk and punish free-riders at cost $c\sigma$, the cost being shared by all R-agents
- f = fraction of R-agents
- For an S-agent the estimated cost of being punished is $s\sigma$. He chooses σ^* to minimize the function

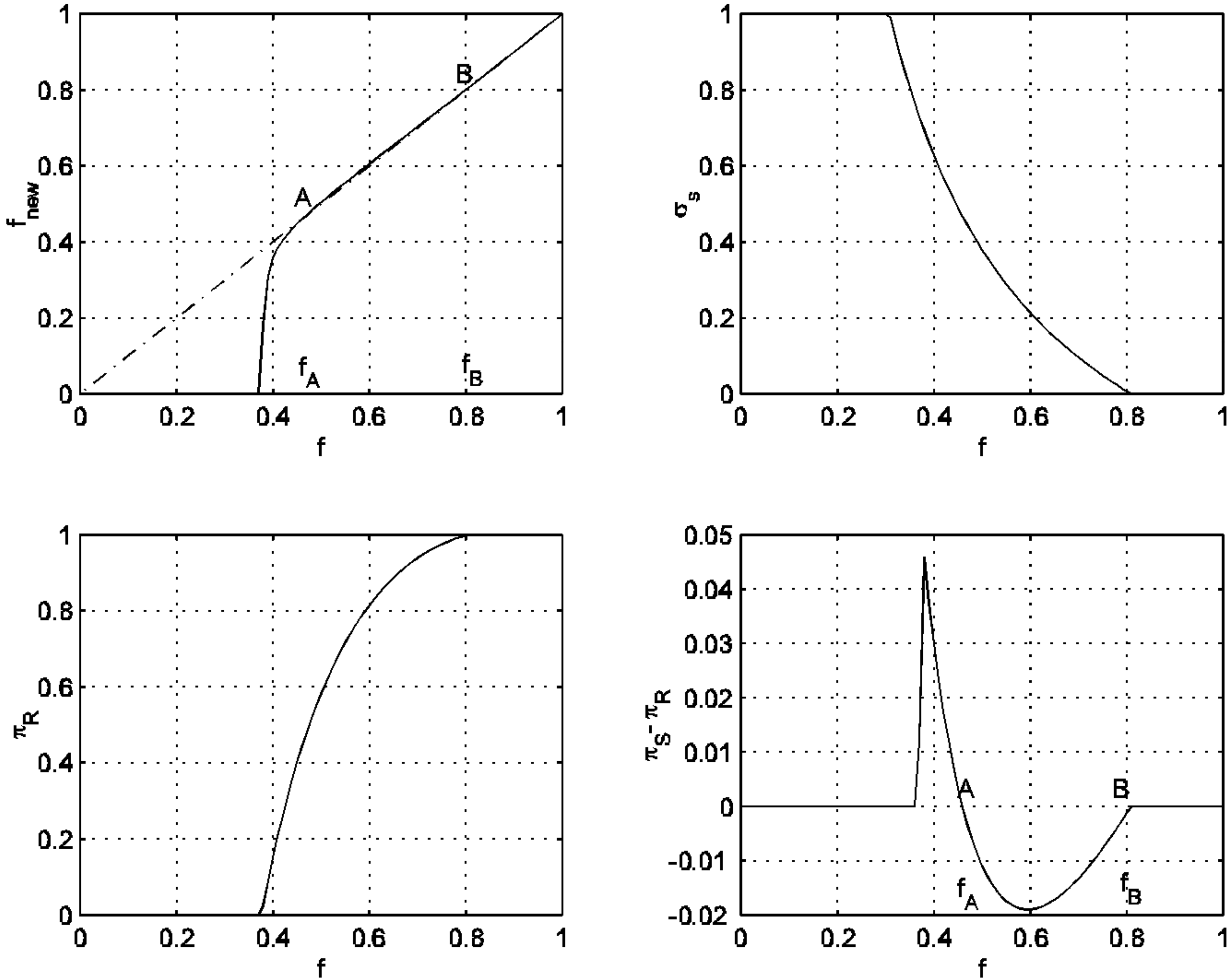
$$B(\sigma) = b(\sigma) + s f \sigma + q(1-\sigma)/N$$

- Fitness of each species :

$$\pi_S = \max(q(1-(1-f)\sigma^*)-b(\sigma^*)-\gamma f \sigma^* , 0)$$
$$\pi_R = \max(q(1-(1-f)\sigma^*)-b-c(1-f)N\sigma/(Nf) , 0)$$

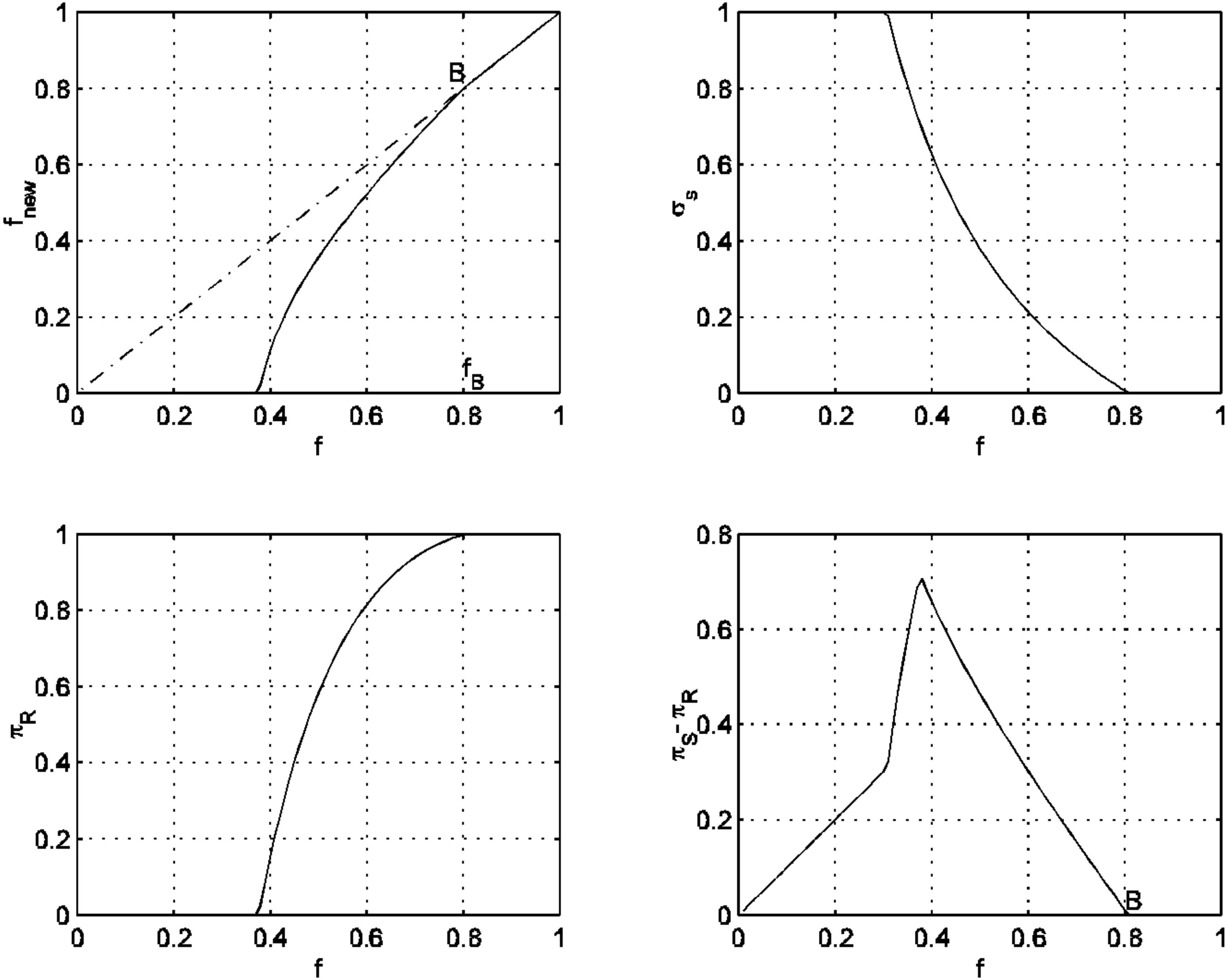
- Replicator dynamics

$$f_{new} = f \frac{\pi_R(f)}{(1-f)\pi_S + f\pi_R(f)}$$



q=2, b=1, s=2, c=0.1, γ=3.5, N=1000

Fig.1



q=2, b=1, s=2, c=0.1, γ=1, N=1000

Fig.2

Homo reciprocans. The Bowles-Gintis model

- If γ is large enough, the map has an unstable fixed point (A) and a left-stable one (B)
- Between B and f = 1 there is a continuum of marginally stable fixed points
- For smaller γ the region between A and B disappears and only the marginally stable fixed points remain
- The asymptotic behavior corresponds either to f = 0 ($\sigma^*=1$) or to f between 0 and 1 but $\sigma^*=0$
- When $f\neq 0$, reciprocators and shirkers remain in the population but shirkers choose not to shirk
- For initial f smaller than f_A the fraction of reciprocators falls very rapidly to zero

Homo reciprocans. The Bowles-Gintis model

- *Intragroup dynamics* :
  either reciprocators are eliminated from the population or they remain in equilibrium with a large number of shirkers (which do not shirk for fear of being punished)
- Intragroup dynamics cannot explain how strong reciprocity might have become a dominant trait.
- *Intergroup dynamics* :
  Only groups that contain at the start $f>f_A$ will have in the end a nonzero fitness. All others suffer a "tragedy of the commons" with final zero fitness.
  Groups with reciprocators tend to dominate and impose an above average predominance of the reciprocator trait.
- For initial f smaller than f_A the fraction of reciprocators falls very rapidly to zero

Network dependence of strong reciprocity

- What happens when, later on, the Pleistocene reciprocators and their fellow shirkers become imbedded into a larger society?
- Monitoring and punishment of shirkers by reciprocators necessarily looses its global collective nature.
- It becomes the business of the neighbors of the shirker
- Monitoring and (or) punishing free-riders requires force to insure the effectiveness of the punishment and to make the punisher safe from direct retaliation from the violator.
- Central authorities play a role in the control of serious offenses, but not so much on the day to day monitoring of public goods work

Network dependence of strong reciprocity

- Punishing a norm-violator requires a minimal social power and consensus. Punishment only if at least two neighbors agree to do so.
- R-agents and (1-f) S-agents placed at random in a network where, on average, each agent is connected to k other agents, rewired with probability β
- Each reciprocator, on detecting an S-agent, looks for another reciprocator in his own neighborhood also connected to S-agent. If he finds one, he punishes by an amount proportional to the fraction of shirking.
- The amount of work an S-agent does is inversely proportional to the number of reciprocators in his neighborhood.

Network dependence of strong reciprocity

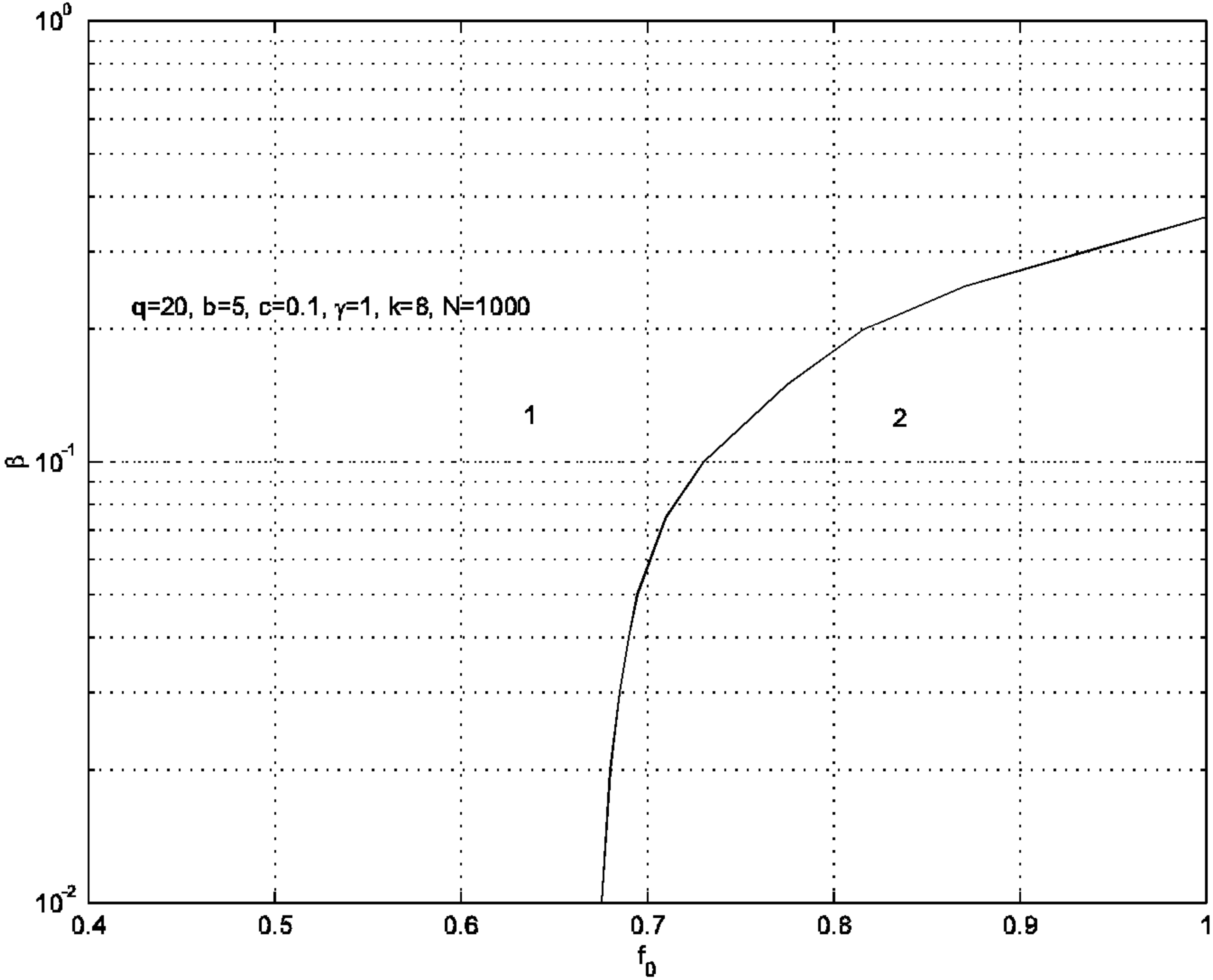
- Wk( ) = work vector
- Pu( ) = punishment vector
- Cpu( ) = cost of punishment vector
- f = fraction of reciprocators
- q = maximum amount of goods produced by each agent
- b = cost of work
- c = cost to punish
- γ = cost to be punished

Network dependence of strong reciprocity

- Average fitness of R-agents and S-agents

$$\pi_R = \tfrac{q}{N} \sum_{all} Wk(i) - \tfrac{b}{fN} \sum_{R} Wk(i) - \tfrac{c}{fN} \sum_{R} Cpu(i)$$

$$\pi_S = \tfrac{q}{N} \sum_{all} Wk(i) - \tfrac{1}{(1-f)N} \left(b \sum_{S} Wk(i) + \gamma \sum_{S} Pu(i) \right)$$



10^0
10^{-1}
10^{-2}
β
0.4
0.5
0.6
0.7
0.8
0.9
1
f_0
q=20, b=5, c=0.1, γ=1, k=8, N=1000
1
2

Fig.3

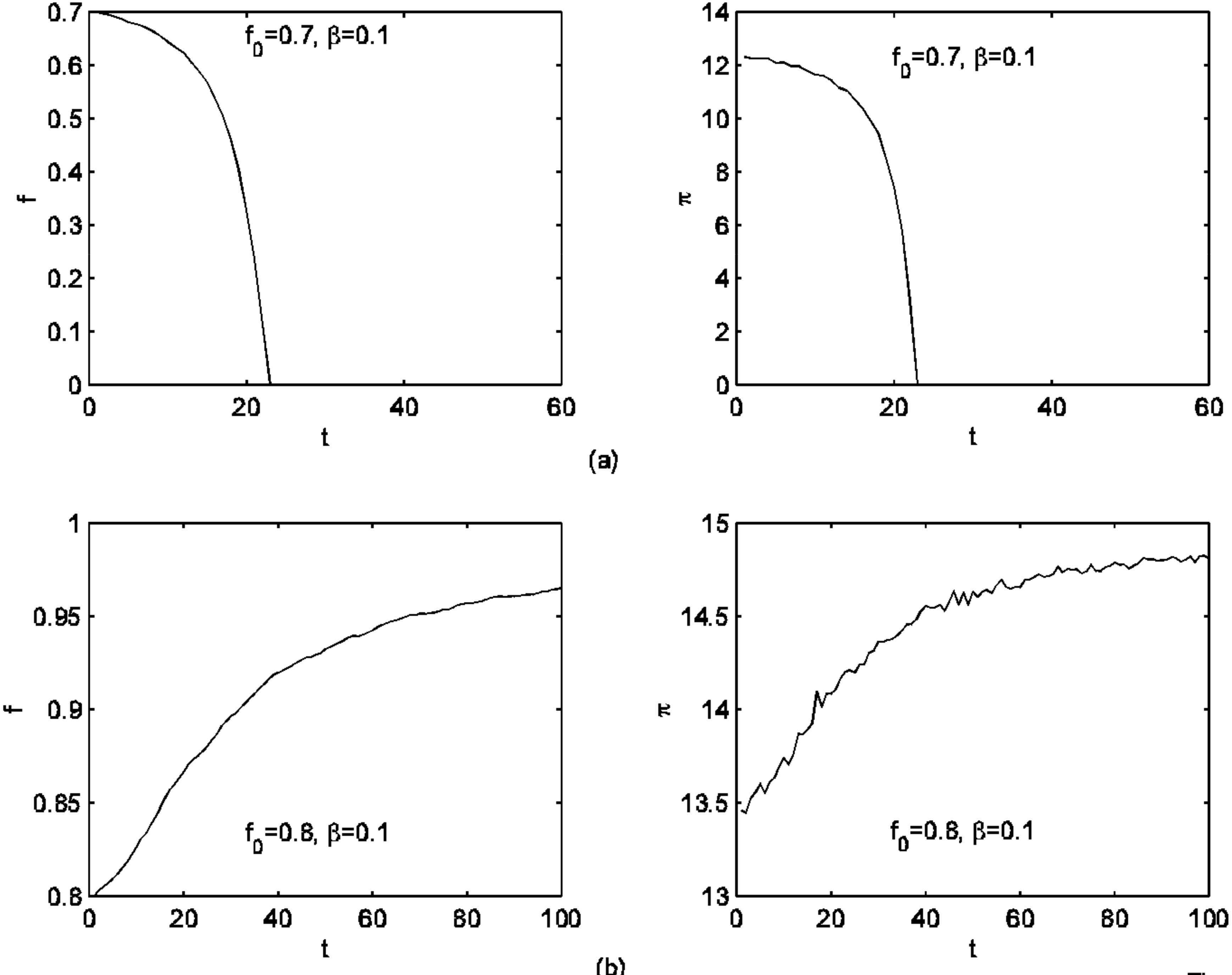


Fig.4

Network dependence of strong reciprocity

- Mean-field model

$$\pi_S = q(1-(1-f)\sigma^*(f)) - b(\sigma^*(f)) - f\gamma C_\beta(\Phi, fk)\sigma^*(f)$$

$$\pi_R = q(1-(1-f)\sigma^*(f)) - b - c(1-f)\tfrac{fk}{2} C_\beta(\Phi, fk)\sigma^*(f)$$

with σ* chosen to minimize

$$B(\sigma) = b(\sigma) + sfC_\beta(\Phi, fk)\sigma - \tfrac{q}{N}(1-\sigma)$$

Similar conclusion

- f = fraction of reciprocators
- q = maximum amount of goods produced by each agent
- b = cost of work
- c = cost to punish
- γ = cost to be punished

Conclusions

- 1 - In small groups with collective monitoring, the interplay of intra- an intergroup dynamics makes the emergence of the strong reciprocity trait a likely event.
- 2 - Self-interested (S-agents) are not completely invaded. If the social structure changes, they may be a source of instability and invade the population.
- 3 - In a large population, monitoring of the public goods behavior cannot be a fully collective activity and punishment of free-riders requires a certain amount of local consensus among reciprocators.
- 4 - The clustering nature of the society plays an important role in the maintenance and evolution of the reciprocator trait.

Conclusions

- Modern societies are "small worlds" in the sense of short path lengths but not necessarily in the sense of also maintaining a high degree of clustering.
- Therefore if the reciprocator trait has a high cultural component, it may very well happen that, eventually, we will see homo oeconomicus leaving the benches of economy classes for a life on the streets.

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