The role of the fundamental constants

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Menu

- 1 – Fundamental constants (or fundamental units) via deformation theory
  c, , ℓ or τ
- 2 – How many fundamental constants (units) ?
- 3 – Dimensionless constants
- 4 – Fundamental constants or fundamental parameters?
- 5 – Time variation of the fine structure constant (α_{EM})
  Experimental situation.

1 - Fundamental constants (units) via deformation theory

- All theories have parameters.
  A stable theory is one that does not change qualitatively by a small change of parameters.
- Deformation theory (of algebras) not only is the theory of stable (rigid) theories, it also is the theory that identifies the fundamental (dimensionful) constants.
- *The fundamental constants play*

  First act : From Galilean to Lorentzian relativity
  c (the finite speed of light)

  Second act : From classical to quantum mechanics
  h (the elementary action)

  Third act : From relativistic quantum mechanics to O(4,1) or O(3,2)
  ℓ or τ (fundamental length or fundamental time)

First act : From Galilean to Lorentzian relativity

Galilean algebra

$$\begin{aligned}[J_i, J_j] &= i\epsilon_{ijk}J_k \\ [J_i, K_j] &= i\epsilon_{ijk}K_k \\ [K_i, K_j] &= 0\end{aligned}$$

J_i = generators of rotations

K_i = generators of velocity transformations

$\phi_1(K_i, K_j) = i\epsilon_{ijk}J_k$ and $\phi_1 = 0$ otherwise, is a 2-cocycle that is not a 2-coboundary $\Longrightarrow$ the second cohomology group does not vanish

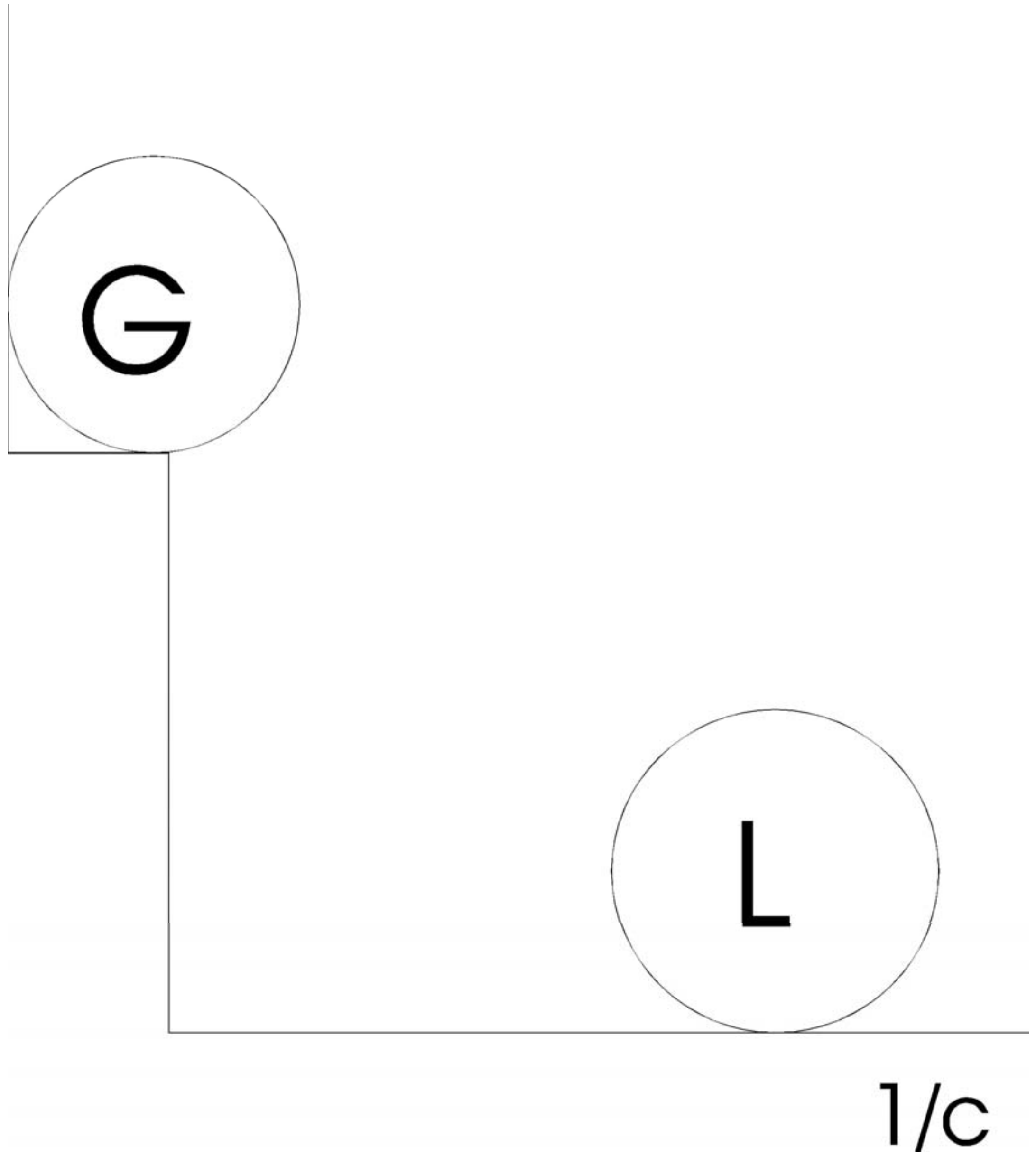
One-parameter deformation

$$[K_i, K_j] = -i\frac{1}{c^2}\epsilon_{ijk}J_k \tag{1}$$

$\Longrightarrow$ **Lorentzian algebra**

The Galilean algebra is an isolated point. Lorentzian algebra is rigid (stable). The deformation parameter $\left(\frac{1}{c^2}\right)$ is a fundamental constant.

First act : From Galilean to Lorentzian relativity



First act : From Galilean to Lorentzian relativity

- Reference:

  L. D. Faddeev ; in *Frontiers in Physics, High Technology and Mathematics*, Cerdeira, Lundqvist (Eds.) pp. 238-246, World Scientific 1989

Second act : From classical to quantum mechanics

Poisson algebra (in phase space)

$$\{f, g\} = \sum_i \frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}$$

in local coordinates.

Let $W = T^*M = R^{2n}$,

$\omega = \sum_{1 \le i,j \le 2n} \omega_{ij} dx^i \wedge dx^j = \sum_{1 \le i \le n} dx^i \wedge dx^{i+n}$

$$P^r(f, g) = \sum_{i_1 \cdots i_r, j_1 \cdots j_r} \omega^{i_1 j_1} \cdots \omega^{i_r j_r} \partial_{i_1} \cdots \partial_{i_r}(f) \partial_{j_1} \cdots \partial_{j_r}(g)$$

$P^3(f, g)$ is a non-trivial 2-cocycle

Existence of non-trivial deformations have been proved in general

If W is finite-dimensional, for a flat Poisson manifold they are all equivalent to Moyal bracket

$$[f, g]_M = \frac{2}{\hbar} sin(\frac{\hbar}{2} P)(f, g) = \{f, g\} - \frac{\hbar}{4.3!} P^3(f, g) + \cdots$$

Second act : From classical to quantum mechanics

Correspondence with quantum mechanics formulated in Hilbert space obtained by the Weyl quantization prescription.

Corresponds to the transition from

$$[p_i, x_j] = [p_i, p_j] = [x_i, x_j] = [p_i, y_j] = 0$$

to the **Heisenberg algebra**

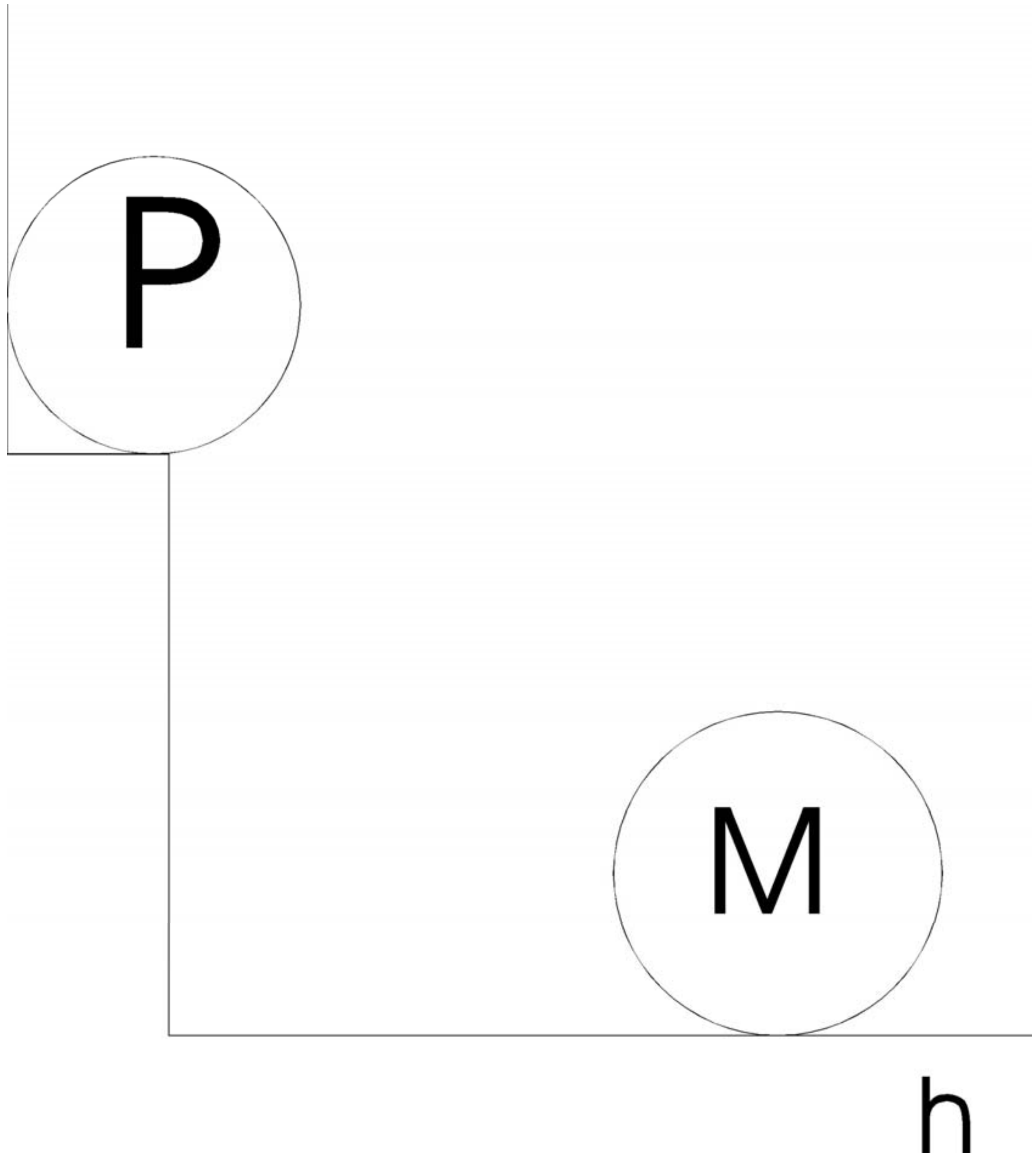
$$[p_i, p_j] = [x_i, x_j] = 0$$

$$[x_i, p_j] = i\hbar\Im\delta_{ij}$$

$\Im$ is the a trivial center of the Heisenberg algebra

The deformation parameter is $\hbar$.

Second act : From classical to quantum mechanics



Second act : From classical to quantum mechanics

- References :

  - F. Bayen, M. Flato, C. Fronsdal, C. Lichnerowicz and D. Sternheimer
    Lett. Math. Phys. 1, 521 (1977)
    Ann. Phys. 111, 61 and 111 (1978)

Third act : From relativistic quantum mechanics to O(4,1) or O(3,2)

The algebra of relativistic quantum mechanics

$$(c = \hbar = 1)$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(M_{\mu\sigma}\eta_{\nu\rho} + M_{\nu\rho}\eta_{\mu\sigma} - M_{\nu\sigma}\eta_{\mu\rho} - M_{\mu\rho}\eta_{\nu\sigma})$$

$$[M_{\mu\nu}, p_\lambda] = i(p_\mu\eta_{\nu\lambda} - p_\nu\eta_{\mu\lambda})$$

$$[M_{\mu\nu}, x_\lambda] = i(x_\mu\eta_{\nu\lambda} - x_\nu\eta_{\mu\lambda})$$

$$[p_\mu, p_\nu] = 0$$

$$[x_\mu, x_\nu] = 0$$

$$[p_\mu, x_\nu] = i\eta_{\mu\nu}\Im$$

Is it stable ?

Third act : From relativistic quantum mechanics to O(4,1) or O(3,2)

NO

$\exists$ a two-parameter deformation

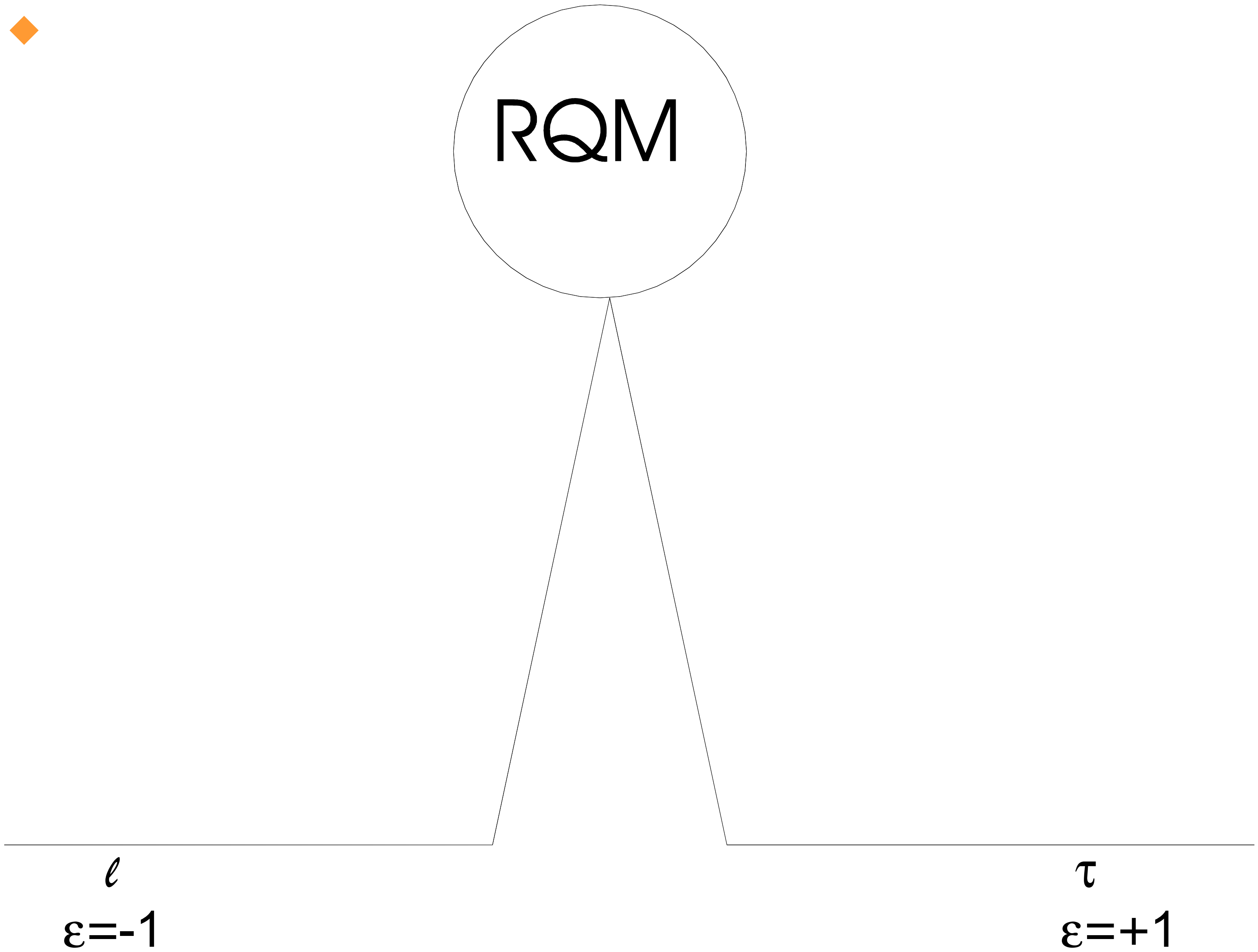
$$
\begin{aligned}
[M_{\mu\nu}, M_{\rho\sigma}] &= i(M_{\mu\sigma}\eta_{\nu\rho} + M_{\nu\rho}\eta_{\mu\sigma} - M_{\nu\sigma}\eta_{\mu\rho} - M_{\mu\rho}\eta_{\nu\sigma}) \\
[M_{\mu\nu}, p_{\lambda}] &= i(p_{\mu}\eta_{\nu\lambda} - p_{\nu}\eta_{\mu\lambda}) \\
[M_{\mu\nu}, x_{\lambda}] &= i(x_{\mu}\eta_{\nu\lambda} - x_{\nu}\eta_{\mu\lambda}) \\
[p_{\mu}, p_{\nu}] &= -i\frac{\epsilon'}{R^2}M_{\mu\nu} \qquad \longrightarrow 0 \\
[x_{\mu}, x_{\nu}] &= -i\epsilon\ell^2 M_{\mu\nu} \\
[p_{\mu}, x_{\nu}] &= i\eta_{\mu\nu}\Im \\
[p_{\mu}, \Im] &= -i\frac{\epsilon'}{R^2}x_{\mu} \qquad \longrightarrow 0 \\
[x_{\mu}, \Im] &= i\epsilon\ell^2 p_{\mu} \\
[M_{\mu\nu}, \Im] &= 0
\end{aligned}
$$

$R \longrightarrow \infty$ corresponds to the algebra of tangent space.

$\ell-$ deformation not removable. ℓ, a fundamental length (or time), is the new deformation parameter.

$\epsilon-$ a sign.

Third act : From relativistic quantum mechanics to O(4,1) or O(3,2)



Third act : From relativistic quantum mechanics to O(4,1) or O(3,2)

- References :

  - J. Phys.: Math. Gen. 27, 8091 (1994)
  - J. Math. Phys. 41, 156 (2000)

Third act : From relativistic quantum mechanics to O(4,1) or O(3,2)

- Consequences :
- Non-commutative space-time
- Modified uncertainty principle (Phys. Lett. A290, 109 (2001))

$$\Delta x \Delta p \geq \frac{1}{2}\left|\left\langle \left(1+\ell^2 p^2\right)^{1/2} \right\rangle\right|$$

- Which sign for ε ?
- Modified density of states

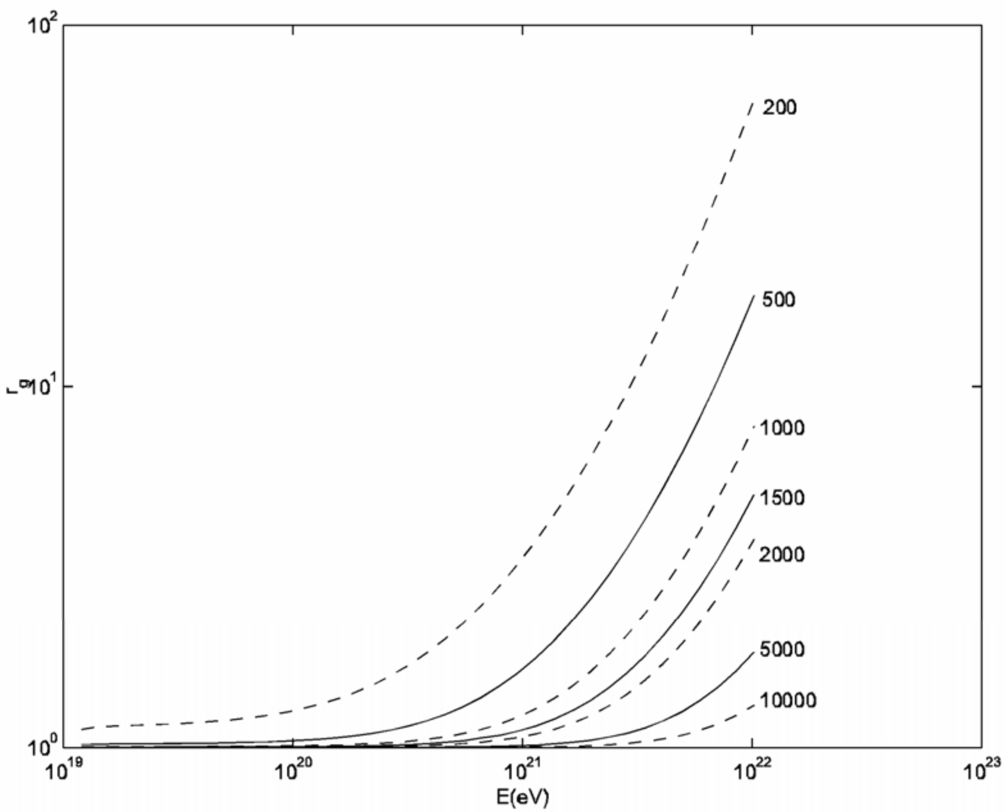
$$dn = \frac{L}{2\pi}\frac{dp}{\sqrt{1-\ell^2p^2}} \quad \text{for } \epsilon = -1$$

$$dn = \frac{L}{2\pi}\frac{dp}{\sqrt{1+\ell^2p^2}} \quad \text{for } \epsilon = +1$$

(Eur. Phys. J. C42 (2005) 445-452)

Third act : From relativistic quantum mechanics to O(4,1) or O(3,2)

- The Greisen, Zatsepin, Kuz'min (GZK) sphere
  For proton sources at cosmological distances (>100Mpc), the spectrum should have a cutoff around 10^{20}eV.
- Detection of events above this energy without plausible local sources. However, better statistics is needed (Auger ?)
- Cosmic ray lifetime increasing factor for $\varepsilon = +1$



- Compatible with experiment if $\tau \geq 10^{-25}$ seconds

  Much larger than Planck scale !
  $T_P=(G\quad /c^5)^{1/2}=5.39 \times 10^{-44}$s

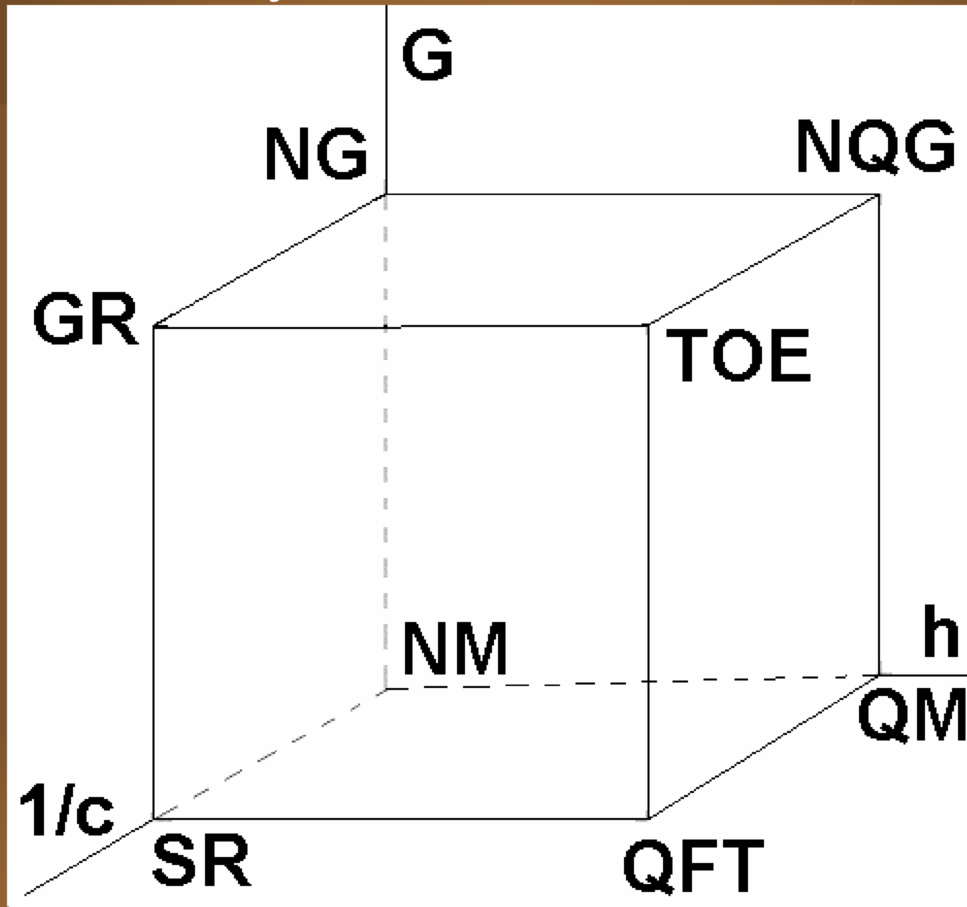
  $1/\ell$ =200-10000 Mev
  ($330-6.6 \times 10^{-26}$ s)

2 – How many fundamental units ?

- International system of units (SI)
  7 basic units (m, s, Kg, A, K, mole, cd)
  17 derived ones
- 4 out of the seven basic ones are also derived
  Electr. Current = no. of moving electrons / second
  Temperature = average energy of an ensemble of particles (up to a factor k_B=1.38x10 $^{-23}$ Joules/Kelvin)
  Mole = no. of molecules in one gram
  Candela = flux of photons
- Left with [m]=[L] , [s]=[T] , [Kg]=[M]
- Measuring velocity in units of [c]=[LT $^{-1}$] and action in units of [=h/2π] =[L^2 M T $^{-1}$] , that is :
  $c \rightarrow 1$, $\rightarrow 1$
  Then everything is expressed in units of L
- For unit of L one may use τ (when it becomes known) or [G]=[L^3 M^{-1} T^{-2}]

2 – How many fundamental units ?

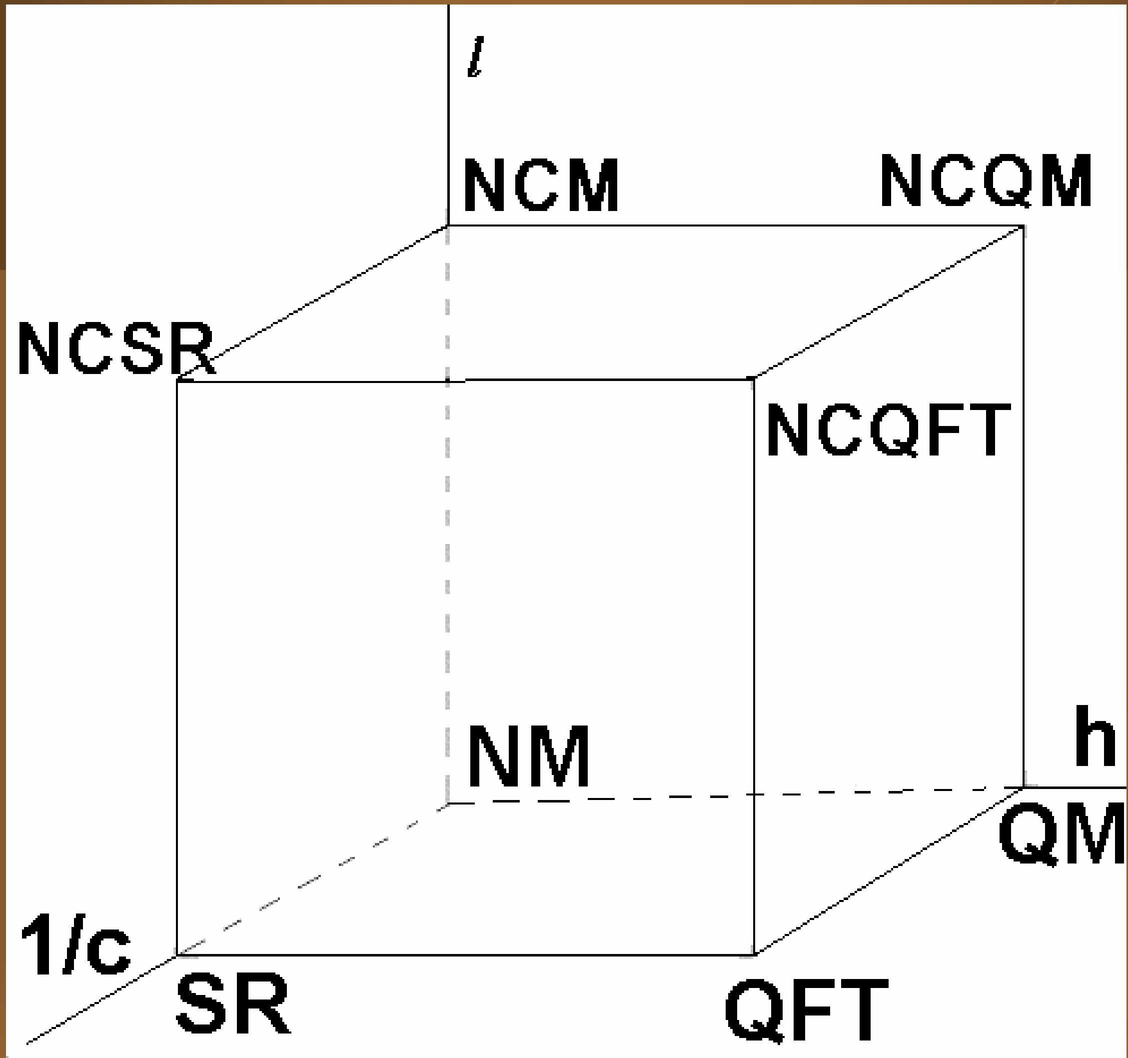
- Conclusion : 3 fundamental dimensionful constants (or units or conversion factors)
- The "Cube of Physical Theories" according to Okun



The third axis depends on dynamics

2 – How many fundamental units ?

- Another cube



All three axis are geometrical

2 – How many fundamental units ?

- However :
  At the level of QFT c and are "fundamental" in the sense that they cannot be computed in terms of more fundamental units.
  In string theory S/ $= \lambda_s^{-2} \int d(\text{Area})$. Therefore everything may be expressed in just c and λ_s
  Two fundamental (dimensionful) units (Veneziano)
- Other point of view (Duff) : G, and c are just *conversion factors* :
  Mass to length $R_S = (2Gm)/c^2$
  Energy to frequency $E = \ \omega$
  Energy to mass $E = m c^2$
  In this sense they are not more fundamental than the Boltzmann constant (Energy to temperature) $E=k_B T$
  The more different units one uses, the more different constants one needs. Only dimensionless constants are fundamental
  Conclusion : Use natural units $\Rightarrow$ *no fundamental (dimensionful) constants*
- However, all this is more an epistemological question than a physical one

2 – Is the value of the fundamental units irrelevant ?

- Yes, as far as the structure of the theories is concerned.
- No, for the type of life and universe as we know it
  Example :
  - Let , e, m_e be the same but $c=3.10^{20}$ cm/sec (10 orders of magnitude larger – closer to the Galilean limit)
  - Bohr radius ($r_B = {}^2/ m_e e^2$), Bohr energy ($E_B=e^2/r_B$) and Bohr time($/E_B$) are the same
  - Implies that chemistry and biochemistry are basically unchanged
  - However :
  Probability of emission of a photon by an excited atom depends on phase space E^2dE/c^3 . Then the time for an excited atom to radiate exceeds the life of the Universe.

…No sun or light bulb to shine, nor eyes to see it (Okun)

3 – Dimensionless constants

- Characterize the strength of the interactions

$$\alpha_{EM} = \frac{e^2}{(4\pi\varepsilon_0)\hbar c} \approx 1/137.035999$$

$$\alpha_W = \frac{G_F m_p^2 c}{\hbar^3} \approx 1.03 \times 10^{-5}$$

$$\alpha(E) = \frac{g_s^2(E)}{\hbar c}$$

$$\alpha_G = \frac{G m_p^2}{\hbar c} \approx 5 \times 10^{-39}$$

$$\mu = \frac{m_e}{m_p} \approx 5.44617 \times 10^{-4}$$

$$x = g_p \alpha_{EM}^2 \mu \approx 1.62 \times 10^{-7}$$

$$y = g_p \alpha_{EM}^2 \approx 2.977 \times 10^{-4}$$

etc.

4 – Fundamental constants or fundamental parameters ?

- Deformation theory plus robustness of the physical models suggests 1/c , h and $\tau \neq 0$.
- However, does not specify its values, nor their constancy
- Do they depend on time ? (or space ?)
- Dirac's large number hypothesis (DLNH)

  $$Gm_e m_p \,/\, e^2 \sim 10^{-40}$$

  (ratio between the gravitational and electromagnetic forces)

  $$e^2 H_0 / m_e c^3 \sim 10^{-40}$$

  (inverse of the age of the universe in atomic units)

  Does G vary as the inverse of cosmic time ?

  However DLNH is not a theory
- Theoretical motivations

  In most higher dimensional theories (Kaluza-Klein, string theories), the true constants of Nature are defined in higher dimension and the 4-dimensional effective ones depend on the expectation value of some fields, or on the compactification radus.

  Therefore if the "constants" are time (or space) – varying they provide tests of higher dimensional theories.
- It is an experimental question !

4 – Fundamental constants or fundamental parameters ?

- Measurement of a dimensionful quantity is in fact measurement of a ratio to a standard chosen as unit.
- Let X=k F(,c,e,...). Then
  $d(\ln X)/dt = d(\ln k)/dt + d(\ln F)/dt$
  To measure $d(\ln X)/dt$ by $d(\ln k)/dt$ assumes $d(\ln F)/dt=0$

  Therefore, only the measurement of the time variation of dimensionless quantities makes sense
- m_p/m_e , G , α

5 – Time variation of the fine structure constant (α_{EM}). Experimental situation

- ***Geochemical constraint***
  Oklo, a prehistoric natural fission reactor $2x10^9$ years ago (z~0.14) during $2.3x10^5$ years.
  ($^{235}U/^{238}U$~3.68%) compared to ($^{235}U/^{238}U$~0.72%) now
- $^{149}Sm/^{147}Sm$ ratio ~ 0.02 (0.9 in normal Samarium)
  Depletion of ^{149}Sm by thermal neutrons

  $$n + {}^{149}Sm \rightarrow {}^{150}Sm + \gamma$$

  Dominated by a resonance E_r~0.0973 , consequence of near cancellation of electromagnetic and strong interaction

  $$\Delta\alpha/\alpha = (0.15 \pm 1.05) \times 10^{-8}$$

5 – Time variation of the fine structure constant (α_{EM}). Experimental situation

- ***Astrophysical constraint***
  Many multiplet method: Correlating several transition lines from various species (Webb et al.)
  A lower value of in the past
  $$\Delta\alpha/\alpha = (-0.54 \pm 0.12) \times 10^{-5}$$
- Not incompatible with the geochemical constraint
  $$0.5 \leq z \leq 3$$
- However exists also a model where the enhancement of the heavy isotopes of magnesium mimics the α variation
- Results on G and m_p/m_e are also not yet conclusive
- Constants or parameters : Still an open question
  (see review papers by J.-P. Uzan)


The end

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