

# Mathematics and the subprime crisis: Lessons for the next stages

Rui Vilela Mendes  
Academia das Ciências de Lisboa  
<http://label2.ist.utl.pt/vilela/>

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# The financial crisis that started in 2007

- Talking at a meeting in January 2011, Nout Wellink, the chairman of the Basel Committee on Banking Supervision, pointed out the following as the main causes of the crisis:
- ① The financial crisis was triggered primarily by excess global liquidity, too much leverage, too little capital of insufficient quality and inadequate liquidity buffers;
- ② It was made worse by a **procyclical deleveraging process** and the **interconnectedness** among systemically important financial institutions that were considered too-big-to-fail;
- ③ A number of other factors also played a major role. These include major shortcomings in **risk management**, corporate governance, market transparency, compensation practices and the quality of supervision. Risk management and **supervision** failed due to an overly narrow, firm-specific focus and an insufficient understanding of how broader system-wide risks could play out under stress.

# Financial crisis: Was the mathematics to blame ?

- Many people thought so

**Le Monde.fr**

*Crise financière: la faute aux mathématiques?*



*Subprime crisis*

*Did rocket scientists destroy world markets ?*

- **blogbosteur.kaywa.com**

*Il faut démissionner Nicole El Karaoui*



# Financial crisis: blame the mathematicians?

- The mathematics was there.
- But nobody was paying attention, because it was profitable to do so, in the short term.
- Three examples:
  - 1 Risk measures
  - 2 The multiplicative effect (*or sensitive dependence on initial conditions*)
  - 3 Statistical arbitrage

# Risk measures, penalty terms and the subprime

- First example: *Risk measures and penalty terms*
- **Theorem for convex measures (Föllmer and Schied)**

Let  $M$  be set of all probability measures on  $\Omega$  (finite)  $\rho$  is a convex risk measure iff there is a penalty function  $\alpha : M \rightarrow (-\infty, \infty]$

$$\rho(X) = \sup_{Q \in P} (E_Q[-X] - \alpha(Q))$$

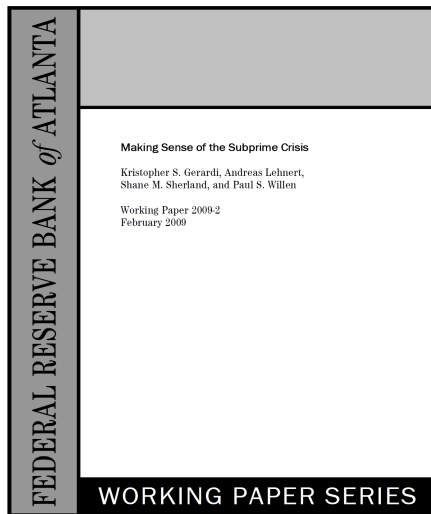
$\alpha$  convex, lower semicontinuous and  $\alpha(Q) \geq -\rho(0)$

(<http://label2.ist.utl.pt/vilela/Cursos/Risk.pdf>)

- In a convex risk measure (not VaR!), the first term represents the **maximal expected loss in the scenario**  $Q$  and  $\alpha(Q)$  accounts for the **probability of the scenario**.
- Whereas the calculation of  $E_Q[-X]$  is a simple exercise in stochastic analysis, estimation of  $\alpha(Q)$  involves many factors which are frequently not taken into account. *For example, if the historical data that is being used does not contain unfavorable events, it is tempting (or profitable) to say that melt-downs are quite improbable.*

# Was the crisis predictable?

- Intrigued why the experts of the rating agencies had rated AAA the "toxic" products and had not predicted the crisis, 3 economists of the Federal Reserve of Atlanta, made an extensive analysis of their reports of the years before the crisis
- Most experts reported that a small fall in the price of the houses would lead to disaster, but assigned a small (*penalty*) probability to that event.



# Was the crisis predictable?

In the Atlanta paper, instead of the language of convex measures, the authors consider the probability of foreclosures decomposed into

$$\frac{df}{dt} = \frac{df}{dp} \frac{dp}{dt}$$

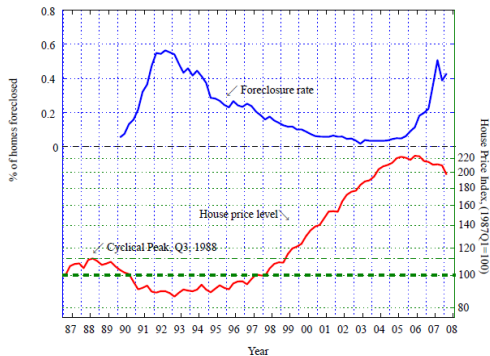
$\frac{df}{dp}$  being the sensitivity of foreclosures to price (HPA) and  $\frac{dp}{dt}$  the time variation of the price of houses. Their conclusion is that the estimation of  $\frac{df}{dp}$  was correct but not of  $\frac{dp}{dt}$  (which is equivalent to the effect of the penalty term).

Name	Scenario	Probability
(1) Aggressive	11% HPA over the life of the pool	15%
(2) [No name]	8% HPA over the life of the pool	15%
(3) Base	HPA slows to 5% by year-end 2005	50%
(4) Pessimistic	0% HPA for the next 3 years, 5% thereafter	15%
(5) Meltdown	-5% for the next 3 years, 5% thereafter	5%



# Was the crisis predictable?

- The evidence against a small penalty term: The housing bubble



It should have been clear after 2001 that the probability of a downturn in HPA was high.

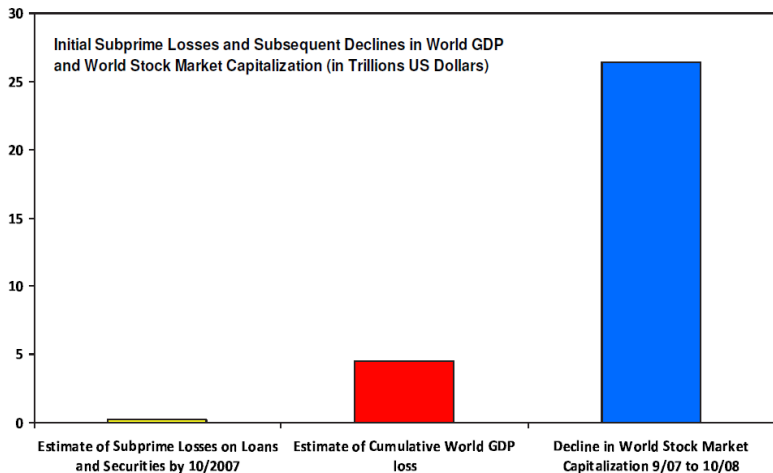
- In addition, how could inflation-adjusted prices continue to rise when real incomes of most Americans, especially at the bottom, continued to fall?

# Was the crisis predictable?

- A small value was assigned to the *penalty term*. **Why ?**
- **Conflict of interests:** The SEC recognition of the main 4 NRSRO's agencies (Fitch, Moody's, S&P and Dominion) together with the recommendations of Basel II, put them in the center of the financial world. However their clients are exactly the creators of the securities. If one agency does not provide favorable reports, look elsewhere.
- **A recent proof:** In February 2011, Redwood Trust put in the market the second (since the crisis) mortgage-based private-label security in the USA. Redwood asked both Fitch and Moody's for ratings, but only published Fitch's report. Later on, perhaps to assert its credibility, Moody's published its report which was quite negative.
- If the client is the buyer of the securities, a conflict of interest of opposite sign may also occur. A recent proposal is the creation of government-sponsored clearing agencies. Will it ever work ?
- By the way, after their deplorable performance in the subprime, the rating agencies have now become great experts of the "sovereign debt". Why? To serve their clients, the lenders, of course.

# The multiplicative effect: The mathematics of sensitive dependence

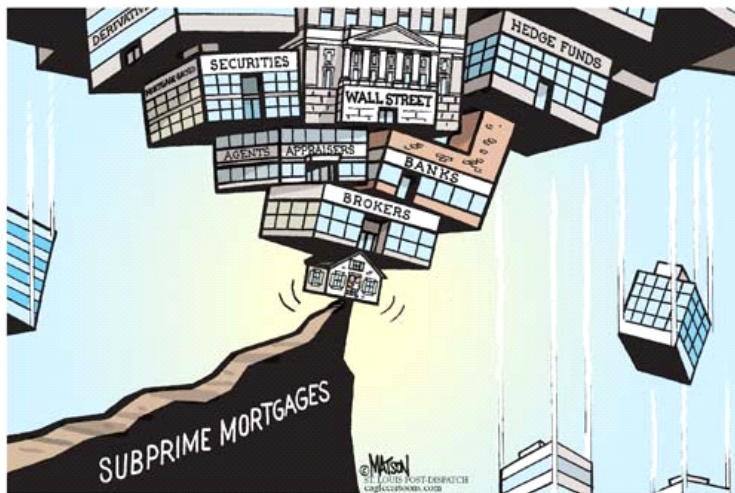
The subprime was worth less than 5% of the American market. However:



# The mathematics of sensitive dependence

- *As the subprime default rates increased, the prices of the mortgage securities dropped.*  
*The involved banks and other institutions had **to write off billions in asset values**, seek **large capital infusions** and banks drastically **reduced lending***  
*Reduced lending by banks affects all corporations, even those not involved in the mortgage-backed market, because of the **equity-to-debt ratio and operations financing**.*  
*Hedge funds, losing money in the subprime market, started **selling positions in other markets**. **Stocks dropped**.*
- In this way a sector that is less than 5% of the American market originated a global crisis
- Here again the mathematics is well-known. It is not stochastic analysis but the *mathematics of non-linear dynamics*.

# The mathematics of sensitive dependence



"I THOUGHT WE WERE JUST BUYING A HOUSE!"

Even when the mathematics is simple people does not want to pay attention. Why?

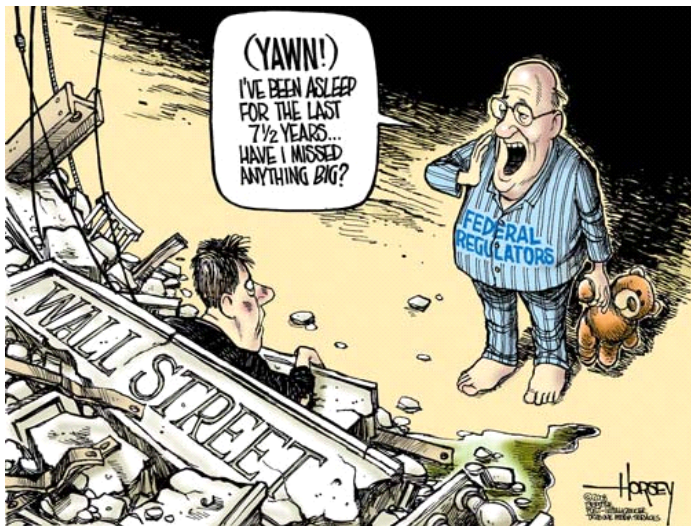
- The 1999 Markopoulos exposition to the SEC and his letter in 2005 about Bernard Madoff

## **The World's Largest Hedge Fund is a Fraud**

- From **statistical arbitrage** mathematics it was evident that Madoff could not generate the stable 12% average annual return unless he was using **insider-trading** or it was a **Ponzi scheme**
- Although some large brokerage institutions (Goldman Sachs, for example) stayed away from any deals with Madoff, he continued to attract a lot of investment in Europe and the USA. Even in a Ponzi scheme the first investors (or those that think they are special) hope to make a profit. The SEC had no timely action.

# Regulation and innovation

A lack of market regulation has also been blamed



- Banks, contrary to other financial institutions (shadow banking), are subjected to supervision and regulation: 1988 BIS Accord (Basel I), 1996 Amendment to BIS Accord, 1999 Basel II, now Basel III and Solvency II.
- That however did not discouraged them (or even encouraged them) to find clever ways to write-off risks from their balance books.
- Conclusion: **No amount of regulation will prevent innovation designed to evade it. Therefore regulation should be as dynamic as the innovation. Is this possible or desirable?**
- The differences between the Basel III and Solvency II rules have already started to be explored by the insurance companies, although these rules will not become mandatory before 2013 and 2016. *The mismatch of time scales between regulation and innovation.*
- What is the mathematics? It is the same mathematics that is used by mathematical biologists to describe for example the adaptation of microorganisms to antibiotics.



- **For the market practitioners:** The mathematics is there. It is only a question of paying attention (*but perhaps it is more profitable, in the short term, not to pay attention*)
- **For the mathematicians:** To understand the economic environment, there is more than stochastic analysis and derivative pricing.

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