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The Quantum Ultimatum Game

R. Vilela Mendes[1]

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A quantum version of the ultimatum game is studied. Both a restricted version with classical moves and the unitary version are considered. With entangled initial states, Nash equilibria in quantum games are in general different from those of classical games. Quantum versions might therefore be useful as a framework for modeling deviations from classical Nash equilibrium in experimental games.

KEY WORDS: Quantum games; social norms; ultimatum.

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1. THE ULTIMATUM GAME

Classical game theory is a mathematical framework for decision problems with rational rules and rational players. It is used in many situations in economics, social sciences, communication and biology. An important item, leading to a solution, is the notion of non-cooperative *Nash equilibrium*. Given a payoff matrix P, a strategy vector x^* is a Nash equilibrium if no player can improve his payoff by changing his strategy, when the strategies of the other players are fixed. That is, denoting by P_i and x_i the payoff and strategy of player i

$$P_i \left(x_1^*, x_2^*, \ldots, x_i^*, \ldots, x_j^*, \ldots, x_N^* \right) \geq P_i \left(x_1^*, x_2^*, \ldots, x_i, \ldots, x_j^*, \ldots, x_N^* \right) \tag{1}$$

for all $x_i \neq x_i^*$, this condition holding for all players.

The Nash equilibrium solutions correspond to the purely self-interested attitude where each player tries to maximize his gains regardless of

[1]Complexo Interdisciplinar, Universidade de Lisboa, Av. Gama Pinto 2, 1649-003 Lisboa, Portugal, E-mail: vilela@cii.fc.ul.pt

what happens to the other players. It is the rational expectations attitude of what has been called the *Homo oeconomicus,* a notion which is at the basis of many theoretical economics constructions. It is therefore important to check the applicability of such notion in human societies. Experiments have been carried out and the problem is that in many cases, when played by human players, games have outcomes very different from the Nash equilibrium points. An interesting case is the *ultimatum game.* [1–5] A simplified version of this game is the following: One of the players (the *proposer*) receives 100 coins which he is told to divide into two non-empty parts, one for himself and the other for the other player (the *responder*). If the responder accepts the split, it is implemented. If the responder refuses, nothing is given to the players. Consider, for example, a simple payoff matrix corresponding to two different proposer offers

	R_0	R_1
P_0	$\vert 00\rangle$ a, c	$\vert 01\rangle$ $0, 0$
P_1	$\vert 10\rangle$ b, b	$\vert 11\rangle$ $0, 0$

(2)

with $a \gg c$, $a + c = 2b$ (for example $a = 99, c = 1, b = 50$). For future reference the players' moves are labeled $|\cdot\cdot\rangle$.

It is clear that the unique Nash equilibrium is $|00\rangle$, corresponding to the greedy proposal (a, c). However, when the game is played with human players, such greedy proposals are most often refused, even in one-shot games where the responder has no material or strategic advantage in refusing the offer.

In this paper a quantum version of the ultimatum game is discussed. Both a restricted and the full unitary and trace-preserving versions are considered in Sections 2 and 3. Then, a final 'remarks' section discusses the sociological context of deviations from Nash equilibrium and how (and why) the notion of entanglement might play a role in the modeling of sociological situations.

2. THE QUANTUM ULTIMATUM GAME. RESTRICTED VERSION

Quantum games[6,7] not only enlarge the space of strategies but also provide a compact way to code for the environment where the players' moves take place. The basic difference between a classical and a quantum game is that whereas in a classical game the players' strategies are coded in a discrete set or a simplex (in the case of mixed strategies), in a quantum game they are coded as vectors in a Hilbert space $\mathcal{H}$. In the ultimatum example (with the payoff matrix (2)) the two options ($|0\rangle$ or $|1\rangle$) of each player are a basis for two two-dimensional linear spaces $\mathcal{H}_P$ and $\mathcal{H}_R$. The space of the game is then the four-dimensional tensor space $\mathcal{H}=\mathcal{H}_P \otimes \mathcal{H}_R$, with basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. In the classical case, the game outcome ψ is one of these four states (for pure strategies) or a point in the simplex defined by these points (for mixed strategies). In the quantum case, the outcome of the game may be any linear combination with unit norm

$$|\psi\rangle = c_{00}\,|00\rangle + c_{01}\,|01\rangle + c_{10}\,|10\rangle + c_{11}\,|11\rangle \tag{3}$$

$|c_{00}|^2 + |c_{01}|^2 + |c_{10}|^2 + |c_{11}|^2 = 1$.

The game is set up in the following way: An initial vector $|\phi\rangle$ defines the *game environment*. Then the players apply their *allowed moves* to the $|\phi\rangle$ state transforming it into some other state $|\psi\rangle$ (the *game outcome*). The payoffs of the players are then computed by projection on the basis states $|\langle ij|\,\psi\rangle|^2$, weighted by the entries of the classical payoff matrix, namely

$$\begin{aligned} \mathbb{P}_P &= a\,|c_{00}|^2 + 0\,|c_{01}|^2 + b\,|c_{10}|^2 + 0\,|c_{11}|^2 \\ \mathbb{P}_R &= c\,|c_{00}|^2 + 0\,|c_{01}|^2 + b\,|c_{10}|^2 + 0\,|c_{11}|^2 \end{aligned} \tag{4}$$

$\mathbb{P}_P$ and $\mathbb{P}_P$ being the payoffs of proposer and responder. Despite its usual connotation as being related to quantum physics, quantum decision algorithms are just another computational scheme. In short, it is just another way for decision making in probabilistic computation. I refer to the excellent paper by Bernstein and Vazirani[8] for a discussion of quantum complexity theory stripped from its quantum physics overtones.[2]

Besides the rule (4) for computing payoffs, the game also requires a specification of the *allowed moves* and the *game environment*. The allowed moves are the transformations with which each player may act on the vectors of his part of the space, that is, transformations of the proposer in $\mathcal{H}_P$ and transformations of the responder in $\mathcal{H}_R$. Here I consider two cases: the *classical moves* and the *unitary moves*.

In the classical moves case, the players are allowed to apply the matrices $M_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $M_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ or probabilistic combinations thereof

[2]For a shorter version of some of the main ideas see ref. 9.

(for mixed strategies). That is, they are allowed to make permutations of their basis states. These are called classical moves because they coincide with the classical operations in the discrete set of classical outcomes. However, here, the effect of these moves may be different because they are now operating in the full Hilbert space.

In the unitary case, each player is allowed the full set of unitary matrices in his part of the space. This is not yet the most general case, that being the full set of completely positive trace-preserving maps.(10,11)

The restriction to classical moves might be the most appropriate one for human decision problems, because it is not clear how to interpret general unitary or trace-preserving operations in terms of human decisions. I will concentrate in this case, which I call a restricted quantum game (RQG).[3] The unitary version of the game is discussed in Section 3.

The $|\phi\rangle$ state defines the game environment. For the RQG there are three different types of $|\phi\rangle$ states.

(i) $|\phi\rangle = |i\rangle \otimes |j\rangle = |ij\rangle$, $i, j = 0$ or 1. In this case $|\phi\rangle$ is one of the basis states of $\mathcal{H} = \mathcal{H}_P \otimes \mathcal{H}_R$. By the classical moves the players may always convert $|\phi\rangle$ into any one of the basis states. Therefore, the unique Nash equilibrium is $|00\rangle$ and this game coincides with the classical game with $\mathbb{P}_P = a$, $\mathbb{P}_R = c$.

(ii) $|\phi\rangle$ is a *factorized state*, but not one of the basis states. Factorized states are states that may be written as a product

$$|\phi\rangle = \{a_0\,|0\rangle + a_1\,|1\rangle\} \otimes \{b_0\,|0\rangle + b_1\,|1\rangle\} \tag{5}$$

Let $(\mu, 1-\mu)$ and $(\nu, 1-\nu)$ be the probabilities for proposer and responder to use moves M_0 and M_1. Then, their payoffs are, respectively

$$\begin{aligned} \mathbb{P}_P &= \mu\,(a-b)\left(|a_0|^2 - |a_1|^2\right)\left(\nu\,|b_0|^2 + (1-\nu)\,|b_1|^2\right) \\ &\quad + \nu\left(a\,|a_1|^2 + b\,|a_0|^2\right)\left(|b_0|^2 - |b_1|^2\right) + \left(a\,|a_1|^2 + b\,|a_0|^2\right)|b_1|^2 \\ \mathbb{P}_R &= \mathbb{P}_P\,\{a \rightarrow c\} \end{aligned} \tag{6}$$

the notation $a \rightarrow c$ meaning that $\mathbb{P}_R$ is obtained from $\mathbb{P}_P$ by the replacement of the payoff a by c.

From (6), by maximizing $\mathbb{P}_P$ in μ for fixed ν and then $\mathbb{P}_R$ in ν for that same μ, one easily concludes that there is, in all cases, a unique Nash equilibrium for pure strategies. The values of the Nash equilibrium $\mu's$ and $\nu's$ are listed in the following table:

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[3]A restricted quantum game might also be described in purely classical terms. However, the quantum version may provide a more compact coding of the game environment, in particular by the use of an entangled initial state.

	$\lvert b_0\rvert^2 > \lvert b_1\rvert^2$	$\lvert b_0\rvert^2 < \lvert b_1\rvert^2$
$\lvert a_0\rvert^2 > \lvert a_1\rvert^2$	$\mu = 1, \nu = 1$	$\mu = 1, \nu = 0$
$\lvert a_0\rvert^2 < \lvert a_1\rvert^2$	$\mu = 0, \nu = 1$	$\mu = 0, \nu = 0$

(7)

the equilibrium payoffs being

$$\begin{aligned} \mathbb{P}_P &= \left\{ a\,|a_{\max}|^2 + b\left(1 - |a_{\max}|^2\right)\right\} |b_{\max}|^2 \\ \mathbb{P}_R &= \mathbb{P}_P\,\{a \to c\} \end{aligned} \tag{8}$$

$|a_{\max}|^2$ and $|b_{\max}|^2$ being the larger of $|a_i|^2$ and $|b_i|^2$.

One sees that, although obtained for pure strategies, the payoffs are already substantially different from those of the classical game.

(iii) Finally, a distinct situation is obtained when $|\phi\rangle$ is an *entangled state*. An entangled state is a state that cannot be expressed in the factorized form (5). A general state

$$|\phi\rangle = c_{00}\,|00\rangle + c_{01}\,|01\rangle + c_{10}\,|10\rangle + c_{11}\,|11\rangle$$

is entangled if and only if

$$|c_{00}|^2\,|c_{11}|^2 + |c_{01}|^2\,|c_{10}|^2 - 2Re\left\{c_{00}c_{10}^* c_{11} c_{01}^*\right\} \neq 0 \ . \tag{9}$$

Here I analyze the cases

$$|\phi_1\rangle = \alpha_1\,|00\rangle + \beta_1\,|11\rangle$$

and

$$|\phi_2\rangle = \alpha_2\,|01\rangle + \beta_2\,|10\rangle$$

which for $\alpha_i, \beta_i \neq 0$ satisfy (9). Furthermore, consider that neither coefficient is too small, more precisely[4]

$$|\alpha_i|^2\,,\,|\beta_i|^2 > \frac{c}{b+c} \ .$$

[4]When this condition is not satisfied, that is, when the ϕ-states are very close to a basis state, there are pure strategy solutions.

For the ϕ_1-state, assuming as before that $(\mu, 1-\mu)$ and $(\nu, 1-\nu)$ are the probabilities for proposer and responder to use moves M_0 and M_1, the payoffs are

$$\begin{aligned} \mathbb{P}_P &= \mu\,(a-b)\left(\nu - |\beta_1|^2\right) + \nu\left(b\,|\alpha_1|^2 - a\,|\beta_1|^2\right) + a\,|\beta_1|^2 \\ \mathbb{P}_R &= \mathbb{P}_P\,\{a \rightarrow c\} \end{aligned} \tag{10}$$

Let $\nu > |\beta_1|^2$. Then the proposer best reply is $\mu = 1$ but then, for that μ, the responder best reply is $\nu = 0$, which contradicts $\nu > |\beta_1|^2$. The conclusion is that there is no equilibrium in pure strategies. However, there is an equilibrium for mixed strategies at

$$\begin{aligned} \mu &= \tfrac{b|\alpha_1|^2 - c|\beta_1|^2}{b-c}, \\ \nu &= |\beta_1|^2, \end{aligned} \tag{11}$$

with payoffs

$$\begin{aligned} \mathbb{P}_P &= |\alpha_1|^2\left(1 - |\alpha_1|^2\right)(a+b), \\ \mathbb{P}_R &= |\alpha_1|^2\left(1 - |\alpha_1|^2\right)(c+b). \end{aligned} \tag{12}$$

For the ϕ_2-state the analysis is identical, with mixed strategy equilibrium at

$$\begin{aligned} \mu &= \tfrac{b|\alpha_2|^2 - c|\beta_2|^2}{b-c}, \\ \nu &= |\alpha_2|^2, \end{aligned} \tag{13}$$

and payoffs

$$\begin{aligned} \mathbb{P}_P &= |\alpha_2|^2\left(1 - |\alpha_2|^2\right)(a+b), \\ \mathbb{P}_R &= |\alpha_2|^2\left(1 - |\alpha_2|^2\right)(c+b). \end{aligned} \tag{14}$$

Already with the simple payoff matrix (1) a range of different equilibrium points is obtained. A even wider range of possibilities and payoff structures may be obtained by increasing the number of possible proposer offers. For the restricted quantum game, the three situations discussed above appear quite different from one another. In all cases the self-interest mechanism of payoff maximization leads to a solution, but the solution strongly depends on the game environment coded by the ϕ-state.

A practical question is how to relate the coding ϕ-state to the deviations from classical Nash equilibria in experimental games. That is, how do the players' environment constraints (or preferences) might be related to particular structures of the initial state. To relate and quantify the players' preferences to the mathematical properties of the game model, it is convenient to use not a particular ϕ-state but general properties of these states.

From the examples that were studied one infers two general properties. For factorized states, one has a *measure of uncertainty* for proposer and responder given by

$$\begin{aligned} \mathbb{S}_P &= -\left|a_0\right|^2 \log\left|a_0\right|^2 - \left|a_1\right|^2 \log\left|a_1\right|^2 \\ \mathbb{S}_R &= -\left|b_0\right|^2 \log\left|b_0\right|^2 - \left|b_1\right|^2 \log\left|b_1\right|^2 \end{aligned}$$

and the compound uncertainty $\mathbb{S}_P + \mathbb{S}_R$. Even if in the model the solution corresponds to an equilibrium pure strategy, the setting-up of this game environment is equivalent to a compulsion to fluctuating decisions by the players.

Whereas in the factorized states, each player has no effect on measurements made on the space of the other player, that is not the case for entangled states. Therefore, the constraints imposed by an entangled game environment are much stronger. Entanglement (for a two-player game) is quantified by the entropy of the reduced density matrix.(12) Namely, one computes

$$\lambda_\pm = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4\left|c_{00}\right|^2\left|c_{11}\right|^2 - 4\left|c_{01}\right|^2\left|c_{10}\right|^2 + 8Re\left\{c_{00}c_{10}^{*}c_{11}c_{01}^{*}\right\}}$$

and then the *entanglement measure* is

$$\mathbb{E} = -\lambda_+ \log\lambda_+ - \lambda_- \log\lambda_-$$

In conclusion: what this framework suggests is that the players uncertainties $(\mathbb{S}_P, \mathbb{S}_R)$ and the entanglement measure $(\mathbb{E})$ are quantities that might be used to quantify, at least, some of the deviations from classical Nash equilibrium in experimental games.

3. THE UNITARY AND TRACE-PRESERVING GAME

In a *unitary game*, the players would be allowed to operate on the ϕ-state with the whole set of unitary transformations in $\mathcal{H}_P$ and $\mathcal{H}_R$, which may be parametrized by matrices

$$\begin{pmatrix} e^{i(\alpha+\beta)/2}\cos\frac{\theta}{2} & e^{i(\alpha-\beta)/2}\sin\frac{\theta}{2} \\ e^{i(\beta-\alpha)/2}\sin\frac{\theta}{2} & e^{i(\alpha+\beta)/2}\cos\frac{\theta}{2} \end{pmatrix} \tag{15}$$

and global phase transformations.

If ϕ is a factorized state, the proposer may always, by an unitary operation, transform it to a state $|0\rangle\,|\zeta\rangle$ and then the best reply of the responder leads to $|0\rangle\,|0\rangle$. The conclusion is that, for factorized ϕ-states, the unitary game is equivalent to the classical game.

The situation is different for entangled states. Let ϕ be a (maximally) entangled state

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right). \tag{16}$$

Using the parametrization (15) the payoffs are computed to be

$$\begin{array}{l} \mathbb{P}_P = \frac{1}{2}\left\{ \begin{array}{c} a\left|e^{i\frac{\beta_P+\beta_R}{2}}\cos\frac{\theta_P}{2}\cos\frac{\theta_R}{2} - e^{-i\frac{\beta_P+\beta_R}{2}}\sin\frac{\theta_P}{2}\sin\frac{\theta_R}{2}\right|^2 \\ +b\left|e^{i\frac{-\alpha_P+\alpha_R+\beta_P+\beta_R}{2}}\sin\frac{\theta_P}{2}\cos\frac{\theta_R}{2} - e^{i\frac{\alpha_P+\alpha_R+\beta_P-\beta_R}{2}}\cos\frac{\theta_P}{2}\sin\frac{\theta_R}{2}\right|^2 \end{array}\right\}, \\ \mathbb{P}_R = \mathbb{P}_P\left\{a \to c\right\}. \end{array} \tag{17}$$

Given any responder strategy, the proposer may always choose α_P, β_P to reduce his payoff to

$$\begin{aligned} \mathbb{P}_P &= \tfrac{1}{2}\left\{a\cos^2\tfrac{1}{2}\left(\theta_P+\theta_R\right) + b\sin^2\tfrac{1}{2}\left(\theta_P+\theta_R\right)\right\} \\ \mathbb{P}_R &= \mathbb{P}_P\left\{a \to c\right\} \end{aligned} \tag{18}$$

and then, choosing $\theta_P = \theta_R$, obtain his best reply. But then, the responder spoils this situation by choosing $\theta_P + \theta_R^{'} = 0$, and so on. The conclusion is that, for the unitary game with maximally entangled states there is no Nash equilibrium in pure strategies.

For mixed strategies, we no longer have unitary operations, but rather fall in the framework of completely positive trace-preserving maps.[5] In this case we have transformations

$$|\phi\rangle\,\langle\phi| \to \sum_{\mu,\nu} K_\mu^{(P)} \otimes K_\nu^{(R)}\,|\phi\rangle\,\langle\phi|\,K_\nu^{(P)\dagger} \otimes K_\mu^{(R)\dagger} \tag{19}$$

with the (Kraus) operators, satisfying

$$\sum_\mu K_\mu^\dagger K_\mu = 1. \tag{20}$$

[5] Mixed 'unitary' strategies are simply the case where the Kraus operators are proportional to a unitary one.

Choosing a basis of hermitean operators

$$\begin{aligned} K_\mu^{(P)} &= \sum_\nu k_{\mu\nu}^{(P)} \sigma_\nu \\ K_\mu^{(R)} &= \sum_\nu k_{\mu\nu}^{(R)} \sigma_\nu \end{aligned} \tag{21}$$

and defining

$$\begin{aligned} p_{\nu\nu'} &= \sum_\mu k_{\mu\nu}^{(P)} k_{\mu\nu'}^{(P)*} \\ r_{\nu\nu'} &= \sum_\mu k_{\mu\nu}^{(R)} k_{\mu\nu'}^{(R)*} \end{aligned} \tag{22}$$

the payoffs are

$$\begin{aligned} \mathbb{P}_P &= \sum_{\mu\nu\alpha\beta} p_{\mu\nu} r_{\alpha\beta} \left\{ \begin{array}{l} a \langle 00| \sigma_\mu^{(P)} \otimes \sigma_\alpha^{(R)} |\phi\rangle \langle\phi| \sigma_\beta^{(R)} \otimes \sigma_\nu^{(P)} |00\rangle \\ +b \langle 11| \sigma_\mu^{(P)} \otimes \sigma_\alpha^{(R)} |\phi\rangle \langle\phi| \sigma_\beta^{(R)} \otimes \sigma_\nu^{(P)} |11\rangle \end{array} \right\}, \\ \mathbb{P}_R &= \mathbb{P}_P \{a \to c\}. \end{aligned}$$

From (20) and (22) it follows $|p_{\nu\nu'}|^2 \leq 1$, $|r_{\nu\nu'}|^2 \leq 1$. The compactness and convexity of the sets $\{p_{\nu\nu'}\}$ and $\{r_{\nu\nu'}\}$ and the multilinearity of the payoff now implies, by Kakutani's theorem, the existence of Nash equilibria for the trace-preserving game.[11]

4. REMARKS

The classical Nash equilibrium solution is closely related to the rational decisions notion of *Homo oeconomicus*, which is a basis of many theoretical economics constructs. Based on the observed deviations from classical Nash equilibrium in the ultimatum game and in other human situations (public goods games, etc),[13] Bowles and Gintis [14,15] developed the notion of strong reciprocity (*Homo reciprocans*[16]) as a better model for human behavior. The general notion, as stated by these authors, is that *Homo reciprocans would come to social situations with a propensity to cooperate and share but would respond to selfish behavior on the part of others by retaliating, even at a cost to himself and even when he could not expect any future personal gains from such actions*.

In addition to 'laboratory' experiments with university students and other volunteers[17–19], an 'ultimatum game experiment' was also carried out in 15 small-scale societies around the world.[20] Consistently different results were obtained in different societies and the conclusion is that *Homo oeconomicus* is rejected in all cases, the players' behavior being strongly correlated with existing social norms and market structure in their societies. Apparently, human decision problems involve a mixture of self-interest and a background of (internalized) social norms.

What is the environment, or background of social norms or preferences, that the ultimatum game is testing? Throughout many generations of interaction, evolution has created in humans cognitive heuristics for repeated play with other humans. However, striking deviations from Nash equilibria occur in one-shot games where strategic considerations, related for example to creating a reputation for future play, cannot be invoked. It might be thought that somehow the players do not take full conscience of the one-way nature of the game and just apply their learned heuristics. This hypothesis has been experimentally disproved.[13] The players do indeed understand the nature of the game. However, their inequality-aversion or sense of reciprocity triggers an emotional response that outweights self-interest considerations. It is indeed a background of social norms or social preferences that is at play, not a conscious or unconscious strategic consideration. The feature that the ultimatum game specifically tests is *negative reciprocity*. This applies to the actions of both the responder and the proposer. The responder is ready to sacrifice his own money to punish an unfair offer and the proposer, who is also aware of the negative reciprocity trait, anticipates this reaction and raises his offer accordingly.

Cooperative or coalition modalities of classical game theory do not seem appropriate, because strong deviations from classical Nash equilibrium are found in one-shot games, when one is not dealing with a direct accord or negotiation between the players. What changes is not the non-cooperative nature of the game but the background environment where it takes place. And this environment is not the result of the actions of the present players, but of many previous generations, maybe even genetically encoded in the Pleistocene.[14]

Given that *strong reciprocators* may act even at a cost to themselves, an interesting question is how this trait developed and why[14,15] (and when[21]) it is evolutionary stable. A neurological basis for this kind of behavior has been found,[22,23] which lends some support to the view that strong reciprocity might have evolved from reciprocal altruism by a reduction in discrimination.

It is then clear that, in addition to material self-interest, the utility functions determining the payoff matrix in human games must include some other components expressing sentiments like fairness, envy, pleasure, etc.

An open question is how does one code for social norms and sentiments in mathematical games. There is no claim that quantum games provide a full solution to the discrepancy between classical games solutions and actual human behavior. However, by providing a wider framework they allow for the coding of additional constraints. In particular, the notion of entangled initial states establishes an interdependence between the players' actions that might in some cases be used to code for a particular social environment. The strength of interdependence of the players' actions would then be measured by an *entanglement measure* as discussed in Section 2.

It might be argued that because humans are macroscopic objects, their environment is classical, not quantum. However, it must be pointed out that quantum concepts are not restricted to the world of microphysics. The quantum paradigm is in fact a *modality of knowledge* that applies whenever there are sets of incompatible observables (in the sense of observables that cannot be simultaneously specified). For example in Economics, *price* (in the sense of actual monetary value) and *ownership* are incompatible observables, because price is only well defined when a transaction takes place, that is, when ownership is changing.

Also, as stated before, quantum decision algorithms are just another computational scheme. They are another way for decision making in probabilistic computation.(8) Therefore there is, in principle, no reason why quantum games might not be used to code for social norms and sentiments in mathematical games. Even in cases where a classical description is possible, a quantum game framework might provide for a more compact coding, in particular exploring the entangled initial states feature.

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