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Portfolios and the market geometry

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HIGHLIGHTS

- A geometric analysis of stock returns shows that the market systematic information lies in a space of small dimension.
- We have explored subspaces of this space to find out the relative performance of portfolios.
- These portfolios are formed from companies that have the largest projections in each one of the subspaces.
- It was found that the best performance portfolios are associated with some of the small eigenvalue subspaces.
- This is found to occur in a systematic fashion over an extended period (1990–2008).

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ABSTRACT

A geometric analysis of return time series, performed in the past, implied that most of the systematic information in the market is contained in a space of small dimension. Here we have explored subspaces of this space to find out the relative performance of portfolios formed from companies that have the largest projections in each one of the subspaces. As expected, it was found that the best performance portfolios are associated with some of the small eigenvalue subspaces and not to the dominant dimensions. This is found to occur in a systematic fashion over an extended period (1990–2008).

1. Introduction

Modern portfolio theory (MPT), as first developed by Markowitz in the 50's [1,2] and further refined by other authors (see for example Ref. [3]), is a set of financial tools which attempts to maximize the expected portfolio return for a certain amount of portfolio risk or minimize the risk for a given expected return. MPT is a form of diversification which, under certain assumptions and definitions of risk and return, finds the best possible diversification strategy. The function to be maximized may, for example, be the portfolio Sharpe ratio

$$SR = \frac{\sum_i x_i \mu_i - R_f}{\sum_i \sum_j x_i x_j \sigma_{ij}} \tag{1}$$

where x_i denotes the proportion of the portfolio invested in the asset i, μ_i the expected return of the asset, σ_{ij} the covariance of assets i and j and R_f the risk-free rate.

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Mean–variance optimization is not the only general criterion for portfolio construction. Maximizing the Sharpe ratio is essentially a static choice in a single time period. If instead, one is concerned with multiperiod performance with cumulative results, the relevant variables are different from the case where gains and losses are not reinvested. In this case the strategy that asymptotically provides the largest cumulative return is geometric mean maximization (GMM). GMM proposed for portfolios by Latane [4–6] traces its origin to a similar gambling strategy discussed by Kelly [7] or even to a much earlier paper of Daniel Bernoulli [8] in 1738. Neglecting higher order moments of the fluctuations of the return around its mean, GMM amounts to maximize the function

$$GM \approx \exp\left\{\ln\left(1+\sum_i x_i\mu_i\right) - \frac{\sum_i\sum_j x_i x_j \sigma_{ij}}{2\left(1+\sum x_i\mu_i\right)^2}\right\} - 1 \tag{2}$$

which allows to compare the different roles played by covariances in (1) and (2).

GMM has been criticized by Samuelson [9] who, although acknowledging that GMM maximizes the terminal wealth for sufficiently long multi-periods it may fail to maximize the expected utility. Nevertheless and although not widely used, GMM has some appeal because investors, when receiving their periodic financial statements, may be less concerned with the Sharpe ratio of their investments than with the rate at which their invested capital grows.

Evolution in the concepts of portfolio theory (two fund and one fund theorems, etc.) led to the "capital asset pricing model" (CAPM) which parametrizes the expected return of an asset as

$$\mathbb{E}(R_i) = \alpha_i + \beta_i(\mathbb{E}(R_M) - R_f). \tag{3}$$

$\mathbb{E}(R_M)$ being the expected value of the market portfolio and R_f the risk-free rate. Therefore, when designing a portfolio, α is the skill required to choose individual assets that will outperform the market and β is the return from exposure to the overall market. Instead of exposure to the overall market, for example via an index fund, the return may be enhanced by tracking particular performing asset classes, as in the now fashionable "smart beta" approach [10].

Many criticisms have been raised against all these portfolio strategies. These include the fact that financial returns do not follow Gaussian distributions and that correlations between asset classes are not fixed but can vary depending on external events. These methods also assume that the problem of utility optimization is identical to mean–variance or geometric-mean optimization, whereas investor utilities may be sensitive to higher moments of the distributions. Furthermore, there is growing evidence that investors are not rational, markets are not efficient and variance, used as a proxy for risk does not explain the low volatility anomaly nor is it a "coherent risk measure" [11,12]. Better estimates of risk are obtained by entropic [13] or iso-entropic measures [14].

The probability beliefs of investors do not necessarily match the true distribution of returns, nor the historical data is a good predictor of future behavior in times of financial turbulence. Ambiguity plays an important role in portfolio selection [15] and in many cases it is more profitable to explore local trends in the market [16–18].

Looking for trends or structures in the market as a guide for investment is already implicit in the classical "arbitrage pricing theory" (APT) which holds that the expected return of a financial asset can be modeled as a linear function of various macro-economic factors, sensitivity to changes in each factor being represented by a factor-specific beta coefficient. Hence, in APT each investor holds a portfolio with its own particular set of betas, as opposed to the identical "market portfolio" of the "capital asset pricing model" (CAPM). In APT, however, the number and nature of the factors is not revealed, nor how do they change over time and among economies.

Our approach to market analysis and portfolio design also follows a search for the structures (or factors), endogenous or exogenous, that at particular time periods, subsume the dynamics of the market. For this purpose a geometric approach is used, with a market metric constructed from the return time series of each company [19,20]. By a technique similar to classical multidimensional scaling, each company is mapped onto a point in Euclidean space and then the shape of the resulting "cloud" is analyzed. It turns out that, up to very small deviations, the market is mostly concentrated in a low dimensional subspace. For example, performing this analysis for the SP500 companies the most relevant subspace has six dimensions. Furthermore the companies populate an elongated ellipsoid with faster decreasing axis. The principal axes of the ellipsoid behave like market factors, the return behavior of the companies being ruled by the projections on these axes. For coherence with the geometrical interpretation we call the axes directions "market dimensions" rather than factors.

The relevance of the geometrical analysis for portfolio design is the following: given the elongated nature of the "market cloud" and the fast decrease of the eigenvectors for the other dimensions, it is clear that the bulk of the market is concentrated along the first, or the first and the second dimensions. Therefore choosing a portfolio with the companies that have the largest projections along the leading dimensions should be identical to a market portfolio. The other dimensions would correspond to the classes of assets that either outperform the market or underperform the market. Then searching, from recent historical data, for the dimensions that outperform the market, better portfolios may be constructed. In a sense our methodology provides a concrete prescription to look for the enhanced asset classes in the "smart beta" approach.

In the remaining of the paper, after a very brief review of the market geometry approach, we show numerical evidence for our conjecture that the asset classes that outperform the market are located along secondary market dimensions.

2. The market geometry: a short review

Correlations in return fluctuations of market securities play an important role in the analysis of market structure [21]. The quantity

$$d_{kl} = \sqrt{2(1 - C_{kl})} \tag{4}$$

where C_{kl} is the correlation coefficient of two (return) time series $\overrightarrow{r}(k)$ and $\overrightarrow{r}(l)$

$$C_{kl} = \frac{\left\langle \overrightarrow{r}(k)\overrightarrow{r}(l)\right\rangle - \left\langle \overrightarrow{r}(k)\right\rangle\left\langle \overrightarrow{r}(l)\right\rangle}{\sqrt{\left(\left\langle \overrightarrow{r}^{\,2}(k)\right\rangle - \left\langle \overrightarrow{r}(k)\right\rangle^2\right)\left(\left\langle \overrightarrow{r}^{\,2}(l)\right\rangle - \left\langle \overrightarrow{r}(l)\right\rangle^2\right)}} \tag{5}$$

($r_t(k) = \log(p_t(k)) - \log(p_{t-1}(k))$), has been shown [22] to satisfy all the metric axioms. Hence it may be used as a basis to develop a geometrical analysis of the market structure. Such an analysis has been performed in Ref. [19]. Given a matrix of distances, obtained from (4), for a set of N time series in a time window, one obtains coordinates in a Euclidean space of dimension $d \le N - 1$ compatible with these distances.

The coordinates $\overrightarrow{x_i}(j)$ in Euclidean space of the j-company are computed from the distances $d_{i,j}$ through the following algorithm:

$$\begin{aligned} \overrightarrow{x_1} &= \{0\} \\ \overrightarrow{x_2} &= \{d_{1,2}, 0\} \end{aligned} \tag{6}$$

then, when $i > 2$ and $j < i - 1$, the coordinate $x_i(j)$ is given by

$$x_i(j) = \frac{d_{i,j}^2 + x_{j+1,j}^2 - d_{i,j+1}^2 + \Sigma_{z=1}^{j-1}(x_i(z) - x_{j+1}^2(z)) - \Sigma_{z=1}^{j-1}(x_i(z) - x_j^2(z))}{2x_{j+1,j}} \tag{7}$$

and when $j = i - 1$

$$x_i(j) = x_i(i-1) = \sqrt{d_{i,1}^2 - \Sigma_1^{i-2}(x_{i,j}^2)}. \tag{8}$$

If $x_i(i-1)$ so obtained is smaller than a small quantity ε no new dimension is added and the company is projected on the already obtained subspace. This Euclidean embedding algorithm is different from classical multidimensional scaling in that, by allowing for the ε-fuzziness in the definition of the hyperplanes, it leads in general to a smaller embedding dimension d.

For the chosen time window the returns of the companies are now represented by a set $\{x_i\}$ of points in Euclidean space. Assigning to each point a mass proportional to the market capitalization, the center of mass $\overrightarrow{R}$ and the center of mass coordinates $\overrightarrow{y}(k) = \overrightarrow{x}(k) - \overrightarrow{R}$ are obtained. Then the tensor

$$T_{ij} = \Sigma_k \overrightarrow{y_i}(k)\overrightarrow{y}_j(k) \tag{9}$$

is diagonalized to obtain the set of its eigenvalues and normalized eigenvectors $\{\lambda_i, \overrightarrow{e}_i\}$. The eigenvectors $\overrightarrow{e}_i$ define the characteristic dimensions of the set of stocks. The same analysis is performed for random and time permuted data and the relative behavior of the eigenvalues is compared. This comparison allows to distinguish random effects from those arising from characteristic market structures. Having carried out this analysis for the companies in the Dow Jones and SP500 indexes [19,20] the following conclusions were obtained:

1—the eigenvalues decrease very fast and soon become indistinguishable from those obtained from random data. It means that the systematic information related to the market structure is contained in a reduced subspace of low dimension. From the extensive amount of data that was analyzed (ranging from 70 to 424 stocks) one concludes that the dimension d of this subspace is at most six.

2—the characteristic dimensions in the reduced subspace do not in general correspond to the traditional industrial sectors, mixing companies of different sectors, thus showing the interlocked nature of the market. The characteristic dimensions provide a natural basis for a model of market factors.

3—carrying out the geometric analysis over many different periods, some noticeable differences were found between business-as-usual and crisis periods. During market crisis there is a contraction of volume in the reduced space. It corresponds to a greater synchronization of the market fluctuations. In addition, whereas the geometric "market cloud" of points in business-as-usual periods looks like a smooth ellipsoid, during some crisis it displays distortions, which may be detected by computing higher moments of the distribution [23]. Whether these distortions appear sufficiently ahead of the crisis to act as precursors is still an open question.

3. Portfolios and characteristic market dimensions: An empirical study

As discussed in the introduction, the main question addressed in this paper is to find out whether the geometric market structure and the characteristic dimensions have any bearing on the construction of portfolios. To explore this issue the following experimental approach was used.

A specific time interval, herein called *the past*, is used to construct the effective dimensions of the market. For each dimension, portfolios are formed with the companies that along this dimension have projections above a certain threshold. They are called *dominant* for that dimension. Then the behavior of these portfolios is followed for a later time interval, called *the future*. Afterwards a new dimension analysis is performed using the data of the period called "the future" and the portfolio is adjusted accordingly. Portfolios were also formed mixing dominant companies in several dimensions. Carrying out this analysis for the data of 23 years we find, as expected (see the introduction), that portfolios corresponding to the largest eigenvalue dimensions tend to perform poorly, whereas it is some of the smaller eigenvalues portfolios that perform better.

The data that was analyzed is from a set of 343 NYSE stocks for a time period from 1990 to 2012. Once the characteristic market dimensions (for the time interval called *the past*) and the reduced 6-dimensional subspace are identified, the first step consists in determining the amount of each stock to be included in each one the dimension-portfolios. Once this is done, the performance of the portfolios is followed for the time period called *the future* and is compared with the performance of the S&P500 index. Afterwards the data of *the future* is used to redo the geometrical analysis and new dimension-portfolios (using the accumulated capital from the previous periods) are formed which are then followed for an equal period etc. For the results presented in this paper *past* and *future* are six months periods.

To obtain the contribution of each stock i to a particular d-dimensional subspace Ω, we compute the ratio between the projection of the stock in that subspace and the projection in the whole market space.

$$f(i, \Omega) = \frac{\sqrt{\sum_{\alpha \in \Omega} x(i, \alpha)^2}}{\sqrt{\sum_{\alpha \in R^{N-1}} x(i, \alpha)^2}}.$$

The inclusion of the stock i in the d-dimensional portfolio depends on the value of $f(i, \Omega)$, which is required to be greater than an appropriate threshold (labeled *Fact.* in the figures). The weight of stock i in the portfolio is proportional to $f(i, \Omega)$. At time zero the portfolio is normalized using the value of an index (Dow Jones or SP500) at that time.

On the final day of each time period of 6 months, the ratio D_Ω of the portfolio value $P_t(\Omega)$ to the reference index value on that day $P_t(Idx)$ is computed:

$$D_\Omega = \frac{P_t(\Omega)}{P_t(Idx)}.$$

3.1. One-dimensional portfolios

Figs. 1–3 show the evolution of six 1-dimensional portfolio for each one of characteristic market dimensions as well as the simultaneous evolution of the S&P500 index for a period from 1990 to 2012. These plots were built adjusting each six months the corresponding portfolios. The average number of companies in each portfolio ranges from 7 to 42 out of 343. The dimensions are labeled 1–6 from the largest to the smallest eigenvalue in the market space.

Observing the results in Fig. 1 it seems obvious that building the portfolio on the first dimension – the one associated with the largest eigenvalue – yields negative results when compared to the evolution of the index.

However, when the sixth dimension is chosen there is a very significant gain of 267.87% (Fig. 2). The fourth dimension is also related to gains (Fig. 3).

In general terms, it is clear that, the dominant dimensions are either similar or worse than the market, whereas the securities that outperform the market should be looked for in some of the non-dominant dimensions.

3.2. Higher dimensional portfolios

Figs. 4 and 5 show the performance of multi-dimensional portfolios as compared with the evolution of the S&P500 index. These plots were also built from a varying number of stocks, adjusted at each 6-month period from 1990 to 2012.

The negative character of the first dimension is evident even in situations where more than one dimension is involved, as shown in Fig. 5 for portfolios built on the subspace [1, 2, 3, 4, 5 and 6]. (with threshold 0.5).

In contrast, 2 and 4-dimensional portfolios containing the fourth and sixth dimensions seem to have consistently higher values as compared to the index, as shown in Fig. 4.

As mentioned before, the most interesting results were obtained from the sixth dimension, for which the gain factor is 267.87%. A similar performance (267.76%) is naturally obtained for a portfolio built on the subspace [4 and 6].

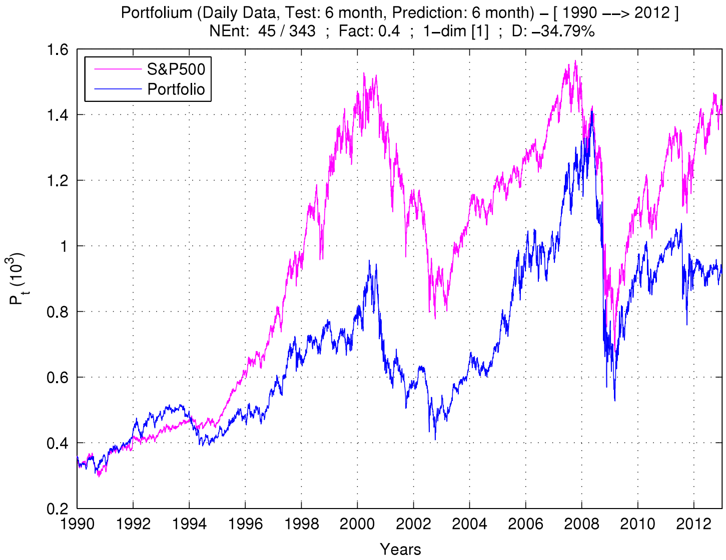

Fig. 1. Dimension-1 portfolio (with threshold 0.4) compared with S&P500.

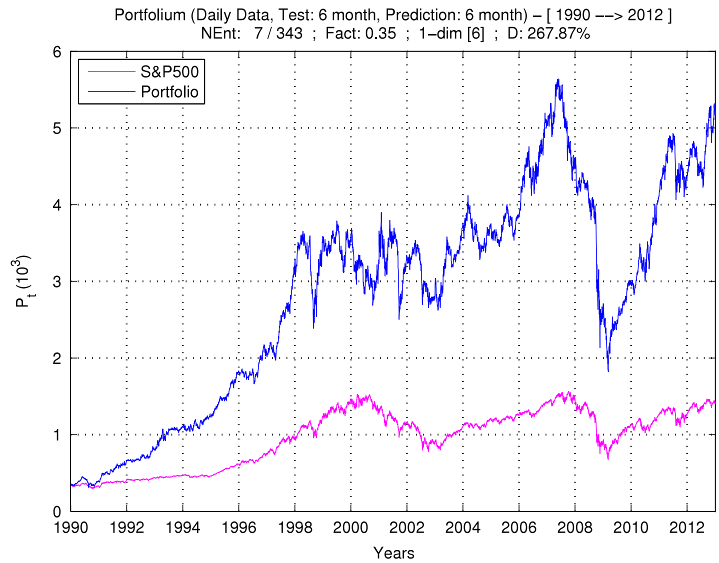

Fig. 2. Dimension-6 portfolio (with threshold 0.35) compared with S&P500.

4. Conclusions

1—the results here obtained confirm that the geometrical analysis initiated before [19,20] does indeed provide relevant information on the relatively low dimensional structures that control the overall market behavior.

2—the rapid decrease of the axes lengths in the enveloping market "ellipsoid" implies that portfolios formed with companies in the leading dimensions are identical to (or worse) as compared to the market portfolio. Securities that either outperform or underperform the market portfolio should be found along the non-dominant dimensions.

As an illustration for the interested reader, we have included in the Appendix the structure of some portfolios relative to the first semester of 2008. In particular a portfolio associated with the leading dimension, that underperforms the market, and two portfolios (dimensions 6 and 4 plus 6) that outperform it.

3—there is no systematic way to a-priori determine which of the non-leading dimensions outperforms or underperforms the market. They have to be experimentally found from the recent historical data and the portfolios adjusted accordingly.

4—here, we have illustrated how the geometric technique may be used for the establishment of efficient long positions. Extensions of the technique for more complex portfolios involving, for example, short positions on the underperforming dimensions or derivative products is straightforward.

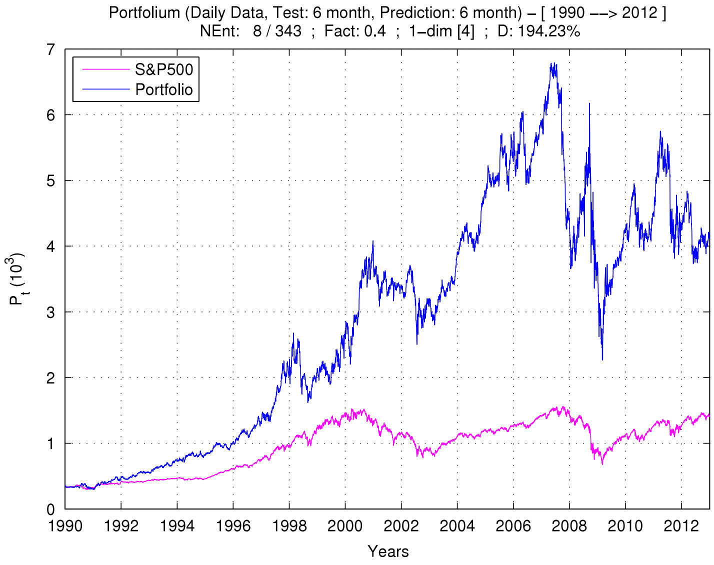

Fig. 3. Dimension-4 portfolio (with threshold 0.4) compared with S&P500.

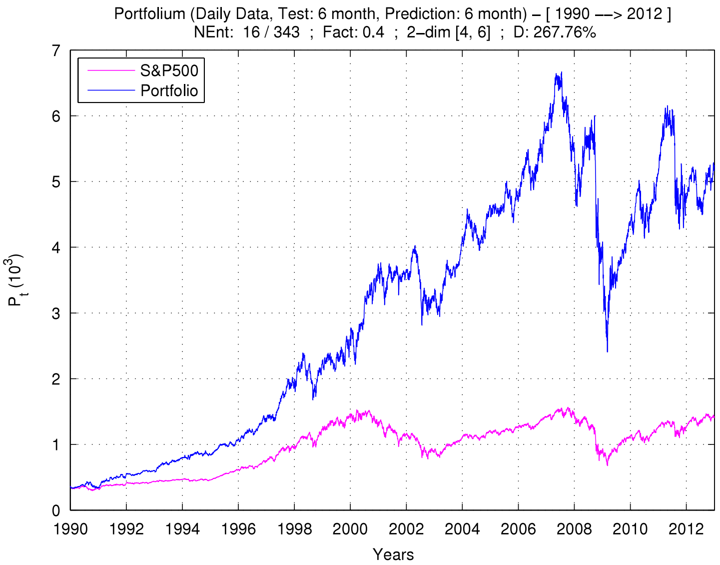

Fig. 4. Dimension-4 and 6 portfolio (with threshold 0.4) compared with S&P500.

5—the geometric analysis of the market, that is performed here, is based on the second moments of the distributions. This seems to be sufficient for the synthesis of portfolios. Skewness and kurtosis are however important for the characterization of market crisis [23].

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Appendix. Some portfolios

Here, as an illustration of the kind of results that are obtained from our geometrical analysis, we include some portfolios relative to the first semester of 2008.

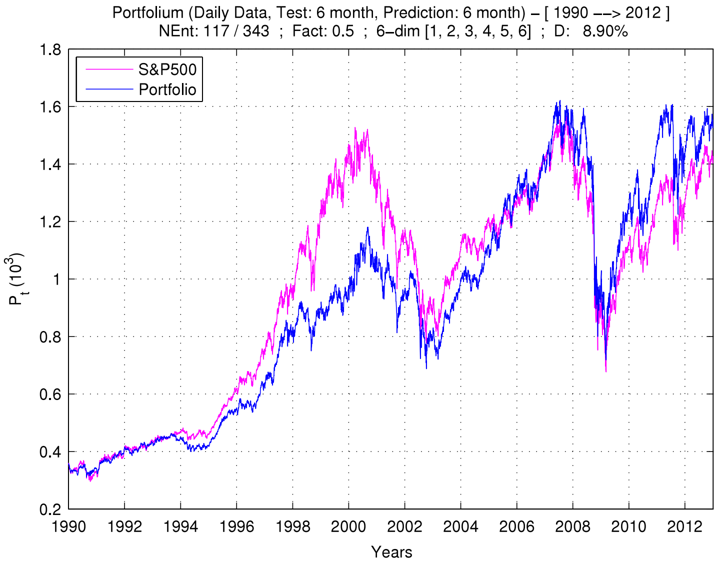

Fig. 5. Dimension-1, 2, 3, 4, 5 and 6 portfolio (with threshold 0.4) compared with S&P500.

A.1. A portfolio associated with the first (leading) dimension

The projection threshold is 0.4, contains 106 companies and it underperforms the index by −32.7%. The two columns contain the companies and their percentage in the portfolio

(1.41%)—'Apache Corp'
(1.36%)—'Devon Energy Corp'
(1.33%)—'Newfield Exploration Co'
(1.32%)—'Denbury Resources Inc'
(1.32%)—'Noble Energy Inc'
(1.30%)—'Halliburton Co'
(1.30%)—'Pioneer Natural Resources Co'
(1.29%)—'National Oilwell Varco Inc'
(1.28%)—'Freeport-McMoRan Copper & Gold Inc'
(1.28%)—'Cameron International '
(1.27%)—'Chesapeake Energy Corp'
(1.27%)—'Anadarko Petroleum Corp'
(1.27%)—'Occidental Petroleum Corp'
(1.26%)—'Cabot Oil & Gas Corp'
(1.25%)—'Murphy Oil Corp'
(1.25%)—'EOG Resources Inc'
(1.24%)—'Noble Corp'
(1.23%)—'Peabody Energy Corp'
(1.23%)—'Southwestern Energy Co'
(1.22%)—'Range Resources Corp'
(1.22%)—'Helmerich & Payne Inc'
(1.21%)—'Rowan Companies PLC'
(1.21%)—'Diamond Offshore Drilling Inc'
(1.19%)—'CONSOL Energy Inc'
(1.15%)—'FMC Technologies Inc'
(1.13%)—'Williams Companies Inc'
(1.12%)—'ConocoPhillips'
(1.12%)—'Hess Corp'
(1.12%)—'Ensco PLC'
(1.11%)—'Schlumberger NV'
(1.11%)—'Bank of America Corp'
(1.11%)—'CF Industries Holdings Inc'
(1.10%)—'Mosaic Co'
(1.10%)—'Baker Hughes Inc'

(1.10%)—'Nabors Industries Ltd'
(1.09%)—'Monsanto Co'
(1.08%)—'Chevron Corp'
(1.07%)—'JPMorgan Chase & Co'
(1.06%)—'EQT Corp'
(1.04%)—'Exxon Mobil Corp'
(1.02%)—'Home Depot Inc'
(1.01%)—'Cliffs Natural Resources Inc'
(1.00%)—'Newmont Mining Corp'
(0.99%)—'US Bancorp'
(0.98%)—'BB&T Corp'
(0.96%)—'M&T Bank Corp'
(0.95%)—'AutoNation Inc'
(0.91%)—'The Hartford Financial Services Group Inc'
(0.90%)—'Joy Global Inc'
(0.89%)—'Fluor Corp'
(0.89%)—'SunTrust Banks Inc'
(0.88%)—'Lowe's Companies Inc'
(0.88%)—'American Express Co'
(0.87%)—'Archer Daniels Midland Co'
(0.86%)—'Torchmark Corp'
(0.86%)—'Macy's Inc'
(0.84%)—'Marathon Oil Corp'
(0.84%)—'TJX Companies Inc'
(0.84%)—'Praxair Inc'
(0.84%)—'PNC Financial Services Group Inc'
(0.84%)—'Wells Fargo & Co'
(0.83%)—'Best Buy Co Inc'
(0.83%)—'Lincoln National Corp'
(0.82%)—'Regions Financial Corp'
(0.82%)—'United States Steel Corp'
(0.82%)—'Starwood Hotels & Resorts Worldwide Inc'
(0.81%)—'Carnival Corp'
(0.81%)—'Limited Brands Inc'
(0.80%)—'Comerica Inc'
(0.80%)—'Alcoa Inc'
(0.79%)—'Marriott International Inc'
(0.79%)—'Allegheny Technologies Inc'
(0.77%)—'Principal Financial Group Inc'
(0.77%)—'PulteGroup Inc'
(0.74%)—'Nucor Corp'
(0.74%)—'Chubb Corp'
(0.74%)—'Capital One Financial Corp'
(0.74%)—'ONEOK Inc'
(0.73%)—'Metlife Inc'
(0.73%)—'Marsh & McLennan Companies Inc'
(0.73%)—'Morgan Stanley'
(0.73%)—'Wal-Mart Stores Inc'
(0.72%)—'Target Corp'
(0.72%)—'Kohl's Corp'
(0.72%)—'Family Dollar Stores Inc'
(0.72%)—'Leggett & Platt Inc'
(0.71%)—'First Horizon National Corp'
(0.71%)—'Lennar Corp'
(0.71%)—'FMC Corp'
(0.71%)—'Nordstrom Inc'
(0.71%)—'Genworth Financial Inc'
(0.69%)—'Darden Restaurants Inc'
(0.69%)—'Aon PLC'
(0.68%)—'Abercrombie & Fitch Co'
(0.68%)—'Walgreen Co'

(0.68%)—'Harley-Davidson Inc'
(0.68%)—'Gap Inc'
(0.67%)—'Jacobs Engineering Group Inc'
(0.67%)—'Kroger Co'
(0.67%)—'ACE Limited'
(0.66%)—'Safeway Inc'
(0.66%)—'Citigroup Inc'
(0.66%)—'Coach Inc'
(0.65%)—'NRG Energy Inc'
(0.65%)—'American International Group Inc'
(0.65%)—'D.R. Horton, Inc'

A.2. A portfolio associated with the sixth dimension

The projection threshold is 0.35, contains 5 companies and it outperforms the index by 8.47%. The two columns contain the companies and their percentage in the portfolio

(24.34%)—'PepsiCo Inc'
(21.07%)—'The Coca-Cola Co'
(19.81%)—'Iron Mountain Inc'
(17.98%)—'Assurant Inc'
(16.79%)—'AGL Resources Inc'

A.3. A portfolio associated with the fourth plus sixth subspace

The projection threshold is 0.4, contains 14 companies and it outperforms the index by 11.18%. The two columns contain the companies and their percentage in the portfolio

(8.33%)—'PepsiCo Inc'
(7.64%)—'Bristol-Myers Squibb Co'
(7.47%)—'General Dynamics Corp'
(7.29%)—'AGL Resources Inc'
(7.21%)—'The Coca-Cola Co'
(7.13%)—'Assurant Inc'
(7.04%)—'Family Dollar Stores Inc'
(7.04%)—'Dover Corp'
(6.92%)—'Parker Hannifin Corp'
(6.88%)—'Iron Mountain Inc'
(6.86%)—'Waste Management Inc'
(6.77%)—'Kohl's Corp'
(6.74%)—'Rockwell Collins Inc'
(6.68%)—'UnitedHealth Group Inc'

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