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ERGODIC MOTION AND NEAR COLLISIONS IN A COULOMB SYSTEM

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A many-particle system with positive and negative charged species, interacting through Coulomb forces and undergoing classical ergodic motion, is considered. The rate of close encounters of two like charged particles at low energies is shown to be controlled by three-body near collisions. The rate of near collisions, under ergodic motion, is found to be fairly large. Possible implications for the occurrence of fusion events, under non-equilibrium conditions, are discussed.

1. Introduction

Some controversial experiments[1–4] suggested the possibility that there might be conditions under which fusion reactions might occur when deuterons are absorbed to a high density into a crystal lattice. These results were received with justified reserve and it seems now clear[5,6] that the electronic clouds in the solid state lattice are not able to reduce in a significant way the static 2–body repulsive potential between a pair of deuterons. Also the original results would not be confirmed by latter experiments.[7–15]

Although the early expectations of a easy way out to the fusion problem were certainly unwarranted, it is nevertheless tempting to think that, if combined with some other mechanism, one might be able to profit from the very high deuterium densities that can be achieved inside solid matter.[16]

This combination of confinement inside a solid lattice with some additional Coulomb barrier breaking mechanism, might provide a ***hybrid fusion*** alternative to the hot plasma techniques. Along this line of thought are the accelerator experiments of heavy water droplets against TiD_2 targets[17] and the fractofusion mechanism, where one profits from the intense local electric fields associated with fracture.[18,19] An idea I have been pursuing for some time,[20] is to explore the many-body effects arising in the complex (chaotic) dynamics of charged systems. Of course, one should not expect a large enhancement of many-body effects in a lattice that is in equilibrium at low temperature and some mechanism should have to be devised to induce complex dynamics. I will come back to this point in the final section of the paper.

Many-body Coulomb systems are very complex and questions like shielding effects, thermodynamic limit and ergodicity require a delicate treatment.[21] Part of the complexity originates from the non-integrality of the many-body problem, apart from the specific aspects of the short distance singularity of the Coulomb interaction. Among the interesting questions related to the issue of complex behavior in charged systems, I will deal with the problem of near collisions at low energies for situations of ergodic motion. This seems to be a natural situation to explore to evaluate the possibility of fusion enhancement through non-equilibrium many-body effects. Under conditions of non-equilibrium, an ergodicity hypothesis might be a reasonable approximation. In any case ergodic motion provides a largely model independent framework to explore general questions and define potentially useful limiting behaviors.

Another class of complex phenomena that a Coulomb system may display are collective effects. There are however more model dependent than the realization of ergodic motion and it is not very clear how they address the basic problem of the Coulomb barrier in two-body collisions.

The quantum theory of non-periodic ergodic motion being still in its infancy,[22] it would be safer to start by studying a classical non-relativistic system of positive and negative charges interacting through Coulomb forces. Quantum corrections in an ergodic context will be dealt with elsewhere.

In Sec. 2 one defines scaled equations of motion for a dimensional phase space variables. In Sec. 3 it is proved that, under certain conditions of ergodicity and quasi-local energy balance, the rate of near collisions of two positively charged particles is dominated by 3-body effects. In Sec. 4 the results of Sec. 3 are used to estimate the near collision rates. Finally some conclusions concerning further work that is needed in this direction, the question of whether complex dynamical behavior may be induced inside solid matter at low temperature and some nuclear physics implications of 3-body effects are made.

As a conclusion to this section, I should state that, in spite of the somewhat speculative nature of both this introduction and the final remarks to the paper, all the actual results are mathematical consequences of a few simple hypothesis about the invariant measure of a many-body Coulomb dynamical system. In this sense the results could very well be presented as a mathematical exercise in classical statistical mechanics and perhaps not risk to collide with the (probably justified) prejudices that surround any reference to alternative fusion mechanisms. However, it would be unfair not to state the intriguing questions that motivated the exercise.

2. Scaled Equations of Motion

Let $(\mathbf{X}, \mathbf{V})$ and $(\mathbf{Y}, \mathbf{W})$ be the position and velocity coordinates of positive and negatively charged particles, which are denoted by the labels D and e, respectively.

For non-relativistic classical motion the equations are:

$$\frac{dX_j}{dt} = V_i \ , \qquad \frac{dV_i}{dt} = \frac{e^2}{m_D} \sum_{j \neq i} \frac{\varepsilon_{ij}}{r_{ij}^2} \frac{r_{ij}}{\|r_{ij}\|} \ , \tag{2.1a}$$

$$\frac{dY_j}{dt} = W_i \ , \qquad \frac{dW_i}{dt} = \frac{e^2}{m_e} \sum_{j \neq i} \frac{\varepsilon'_{ij}}{r_{ij}^2} \frac{r_{ij}}{\|r_{ij}\|} \ , \tag{2.1b}$$

where ε_{ij} and ε'_{ij} are $+$ or $-$ signs according to the charges of the interacting particles.

Let a be a natural length scale and define the following scaled variables

$$X_i = a\, x_i \ , \qquad V_i = \mu_\nu \nu_i \ , \tag{2.2a}$$

$$r_{ij} = a\, \rho_{ij} \ , \qquad t_i = \mu_\tau \tau_i \ , \tag{2.2b}$$

$$Y_i = a\, y_i \ , \qquad W_i = \mu_\nu \omega_i \ , \tag{2.2c}$$

with

$$\mu_\nu = \frac{a}{\mu_\tau} \ , \qquad \mu_\tau = \frac{\sqrt{m_D a^3}}{e} \ . \tag{2.2d}$$

Then the equations of motion become

$$\frac{dx_j}{d\tau} = \nu_i \ , \qquad \frac{d\nu_j}{d\tau} = \sum_{j \neq i} \frac{\varepsilon_{ij}}{\rho_{ij}^2} \frac{\rho_{ij}}{\|\rho_{ij}\|} \ , \tag{2.3a}$$

$$\frac{dy_i}{d\tau} = \omega_i \ , \qquad \frac{d\omega_i}{d\tau} = \frac{1}{\gamma} \sum_{j \neq i} \frac{\varepsilon'_{ij}}{\rho_{ij}^2} \frac{\rho_{ij}}{\|\rho_{ij}\|} \ , \tag{2.3b}$$

where $\gamma = m_e/m_D$ is the ratio of the masses. Whenever needed the singularity of the Coulomb potential is regularized by introducing a cut-off corresponding to the classical radius of the particles.

3. Three-Body Dominance of Near Collisions

We now study the system defined by the scaled Eq. (2.3) for a very large number of particles. We will assume that the total asymptotic energy of the system (i.e. the energy when all velocities vanish and the particles are at infinite relative distances) is zero. Furthermore we assume the existence of an invariant measure μ for the dynamical system with the following characteristics:

(a) Quasi-local energy balance.[23]

Conservation of energy requires the measure to contain a $\delta(E)$ where E is the total energy of all the particles in the system. By quasi-local balance we mean that there is a constant K such that for any region of volume $\mathbb{V}$ in configuration space the energy associated to the particles in this volume is less than $K\mathbb{V}$. Physically, it corresponds to the situation where energy exchanges are local in character, the energy change of a particle being due mainly to interactions and collisions with nearby particles. In particular the quasi-local energy balance hypothesis will exclude contributions to the phase space integral where, for example, a local decrease of energy is compensated by an increase of energy in a far away region. This quasi-local balance situation is to be expected under conditions of Debye shielding which have been proven to occur in very general conditions in a Coulomb gas.[21]

(b) Uniform density in phase space as long as condition (a) is fulfilled.

(c) The measure μ is indecomposable, i.e., motion in its support is ergodic.

Formally we may write the following explicit expression for the measure μ:

$$d\mu = N^{-1}\delta(E)\delta^3(P)\prod_{\substack{i\\ \{V_i\}}}\theta\left(KV_i - \|E_i\|\right)\prod_{i\in\{D\}} d^3x_i d^3\nu_i \prod_{j\in\{e\}} d^3y_j d^3\omega_j\ , \qquad (3.1)$$

where

$$E = \frac{1}{2}\sum_i \|\nu_i\|^2 + \frac{\gamma}{2}\sum_j \|\omega_j\|^2 + \sum_{\substack{i<j\\ i,j\in\{D\}}} \frac{1}{\|x_i - x_j\|} + \sum_{\substack{i<j\\ i,j\in\{e\}}} \frac{1}{\|y_i - y_j\|} - \sum_{\substack{i\in\{D\}\\ j\in\{e\}}} \frac{1}{\|x_i - y_j\|}\ , \qquad (3.2a)$$

$$P = \sum_i \nu_i + \gamma\sum_j \omega_j\ , \qquad (3.2b)$$

and the first product in (3.1) runs over all partitions of configuration space into regions of volume $\mathbb{V}_i, E_i$ being the energy function, as in (3.2) but involving only the particles contained in the volume $\mathbb{V}_i$. N is a normalization factor.

Our basic assumption is the existence of the measure μ for the system (2.3). In the following we will derive exact results relating to properties of the motion of the system in the support of such a measure.

Definition: We will say that there is a ε-collision of two particles of type D, when there is a pair of i, j such that $\|x_i - x_j\| \le \varepsilon$ and $(x_i - x_j)\cdot(v_j - v_i) > 0$. (i.e., when the particles reach a distance equal or smaller than ε and move towards each other).

We may now prove the following results:

 # For ergodic motion with zero asymptotic energy and quasi-local energy balance, the rate of ε-collisions, when $\varepsilon \to 0$, is dominated by three-body near collisions.

Proof: Define the following time-dependent distribution

$$\delta_+(\Delta(t)) = \begin{cases} \delta(t-\tau) \ \text{ if } \ \Delta < 0 \ \text{ for } \ t = \tau - \xi \ \text{ and } \ \Delta > 0 \ \ t = \tau + \xi, \\ \qquad\qquad\qquad \xi \ \text{ being a positive infinitesimal} \\ 0 \ \text{ otherwise.} \end{cases} \qquad (3.3)$$

The average number n_ε of ε–collisions per unit time is therefore

$$n_\varepsilon = \lim_{T\to\infty} \frac{1}{T} \sum_{i<j} \int_0^T \delta_+ \left(\varepsilon - \|x_i(t) - x_j(t)\|\right) dt \ . \tag{3.4}$$

Under our hypothesis, the ergodic theorem

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T f(t)dt = \int f(x, y, \nu, \omega) d\mu$$

may now be applied to replace the time average in (3.4) by a phase space average, μ being the measure defined in (3.1). Taking into account the change of variables, in the delta-function of (3.3), from τ to x and ν one obtains

$$n_\varepsilon = \int d\mu \sum_{k<1} \delta\left(\|x_k - x_l\| - \varepsilon\right) \|\nu_k - \nu_l\| \, \theta\left((x_k - x_l)\cdot(\nu_l - \nu_k)\right) \ . \tag{3.5}$$

Consider now the limit of small ε. The delta function in (3.5) requires any k, l pair of D-particles that contributes to the integral to have $\|x_k - x_l\| = \varepsilon$. From the E_i energy function (Eq. (3.2a)), associated to the volume V_i that contains the k, l pair, one sees that no matter how large (but finite) K is, quasi-local energy balance will be violated in the $\varepsilon \to 0$ limit unless there are near to the k, l pair one or more particles of the e-type. An obvious phase space density consideration implies then that the leading contribution comes from events involving two D-particles and one nearby e-particle □

4. Rate Estimates

Given the fact, proven in the previous Section, that, under ergodicity and quasi-local energy balance conditions, the rate n_ε of near collisions in a Coulomb system is dominated by three-body events, we now evaluate n_ε in the small ε limit.

Let the density of D and e-particles be ρ_D and ρ_e, and assume $\rho_e \geq \rho_D$. On the average a sphere of radius

$$a = \left(\frac{3}{2\pi\rho_D}\right)^{\frac{1}{3}} \tag{4.1}$$

will contain 2 particles of D-type and $N_e = (2\rho_e)/(\rho_D)$ particles of e-type. Taking a to be the natural length scale, as defined in Sec. 2, the rate of n_ε near collisions per scaled unit volume is, according to the discussion in Sec. 3, obtained from the following 3-body integral

$$\begin{aligned} n_\varepsilon = \frac{3N^{-1}N_e}{4\pi} \int & d^3x_1 d^3x_2 d^3\nu_1 d^3\nu_2 d^3y d^3\omega \ \delta\left(\|x_1 - x_2\| - \varepsilon\right) \|\nu_1 - \nu_2\| \\ & \times \theta\left((x_1 - x_2)\cdot(\nu_2 - \nu_1)\right) \delta\left(T + \frac{1}{\|x_1 - x_2\|} - \frac{1}{|x_1 - y\|} - \frac{1}{\|x_2 - y\|}\right) \\ & \times \delta^3(\nu_1 + \nu_2 + \gamma\omega) \end{aligned} \tag{4.2}$$

where

$$N = \int d^3x_1 d^3x_2 d^3\nu_1 d^3\nu_2 d^3y d^3\omega \; \delta\left(T + \frac{1}{\|x_1 - x_2\|} - \frac{1}{\|x_1 - y\|} - \frac{1}{\|x_2 - y\|}\right)$$
$$\times \delta^3(\nu_1 + \nu_2 + \gamma\omega) \tag{4.3}$$

and

$$T = \frac{1}{2}\left\{\|\nu_1\|^2 + \|\nu_2\|^2\right\} + \frac{\gamma}{2}\|\omega\|^2 \tag{4.4}$$

and we have considered also quasi-local momentum balance.

These equations give the rate n_ε of ε-collisions per unit τ and unit volume in scaled variables. To obtain the rates in physical variables the relations (2.2) and (4.1) should be used.

Changing variables from $(x_1, x_2, y, \nu_1, \nu_2, \omega)$ to $(\rho_1, \theta_1, \phi_1, \rho_2, r, \phi_2, y, T, \Omega_\xi, \eta, \omega)$ where

$$\begin{aligned} x_1 - y &= (\rho_1, \theta_1, \phi_1) \,, \quad \xi = \frac{1}{2}(\nu_1 + \nu_2) = (\|\varepsilon\|, \Omega_\xi) \,, \\ x_2 - y &= (\rho_2, \theta_2, \phi_2) \,, \quad \eta = \nu_1 - \nu_2 \,, \\ r &= \|x_1 - x_2\| \,, \end{aligned}$$

and performing the velocity integrations one obtains:

$$n_\varepsilon = N^{-1} N_e \frac{1024}{15} \pi^4 \frac{1}{\gamma^3} \left(1 + \frac{2}{\gamma}\right)^{-\frac{3}{2}} \int_0^{T_{\max}} dT T^{\frac{5}{2}} I_{\delta_\varepsilon}(T, k) \tag{4.5a}$$

$$N = \frac{128}{3} \pi^6 \frac{1}{\gamma^3} \left(1 + \frac{2}{\gamma}\right)^{-\frac{3}{2}} \int_0^{T_{\max}} dT T^2 I_1(T, k) \tag{4.5b}$$

where

$$I_{\delta_\varepsilon}(T, k) = \int_k^1 \rho_1 d\rho_1 \int_k^1 \rho_2 d\rho_2 \int_{\|\rho_1 - \rho_2\|}^{\rho_1 + \rho_2} r dr \delta(r - \varepsilon) \delta\left(T + \frac{1}{r} - \frac{1}{\rho_1} - \frac{1}{\rho_2}\right) \tag{4.6a}$$

$$I_1(T, k) = \int_k^1 \rho_1 d\rho_1 \int_k^1 \rho_2 d\rho_2 \int_{\|\rho_1 - \rho_2\|}^{\rho_1 + \rho_2} r dr \delta\left(T + \frac{1}{r} + \frac{1}{\rho_1} - \frac{1}{\rho_2}\right) . \tag{4.6b}$$

In the above expressions the singularity of the Coulomb interaction has been dealt with by imposing that the minimum distance between an e-particle and a D-particle is k and between two D particles is $2k$. Therefore $T_{\max} = \frac{3}{2k}$. It turns out however that the phase space integrals in (4.5a–b) are finite in the $k \to 0$ limit and, for simplicity, these values are used.

From (4.5a–b) follows the result that *under the stated conditions of ergodicity, quasi-local balance and non-relativistic motion the rate of ε-collisions of D-particles does not depend on the mass of the e-particle negative scatterer.* Later we will see how this statement is changed by relativistic corrections.

We now compute

$$\begin{aligned} J_1^{(2)}(0) &= \int_0^\infty dT\, T^2 I_1(T,0) \\ &= \int_0^1 \rho_1 d\rho_1 \int_0^1 \rho_2 d\rho_2 \int_0^\infty dT\, T^2 \left(\frac{1}{\rho_1}+\frac{1}{\rho_2}-T\right)^{-3} \\ &\quad \times \theta\left(\rho_1+\rho_2-\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}-T\right)^{-1}\right)\theta\left(\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}-T\right)^{-1}-\|\rho_1-\rho_2\|\right) \\ &= \int_0^1 d\rho_1 \int_0^1 d\rho_2 \left\{\frac{(\rho_1+\rho_2)^2}{2\rho_1\rho_2}[(\rho_1+\rho_2)^2-\|\rho_1-\rho_2\|^2]\right. \\ &\quad \left. -2(\rho_1+\rho_2)[\rho_1+\rho_2-\|\rho_1-\rho_2\|\,]+\rho_1\rho_2 \ln\frac{\rho_1+\rho_2}{\|\rho_1-\rho_2\|}\right\} \\ &= \frac{7}{6}\,; \end{aligned} \tag{4.7}$$

$$\begin{aligned} J_{\delta_\varepsilon}^{(\frac{5}{2})}(0) &= \int_0^\infty dT\, T^{\frac{5}{2}} I_{\delta_\varepsilon}(T,0) \\ &= \varepsilon \int_0^1 \rho_1 d\rho_1 \int_0^1 \rho_2 d\rho_2 \left(\frac{1}{\rho_1}+\frac{1}{\rho_2}-\frac{1}{\varepsilon}\right)^{\frac{5}{2}} \theta(\rho_1+\rho_2-\varepsilon)\theta(\varepsilon-\|\rho_1-\rho_2\|) \\ &\quad \times \theta\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}-\frac{1}{\varepsilon}\right) \\ &= \frac{\varepsilon^{\frac{5}{2}}}{4}\left\{\int_1^4 dx \int_0^1 dy + \int_4^{2+\sqrt{5}} dx \int_{\sqrt{x^2-4x}}^1 dy\right\}(x^2-y^2)\left(\frac{4x}{x^2-y^2}-1\right)^{\frac{5}{2}} \\ &= \alpha(0)\varepsilon^{\frac{5}{2}} = 7.2\,\varepsilon^{\frac{5}{2}}\,, \end{aligned} \tag{4.8}$$

where, in the last integral, the change of variables

$$x = \frac{1}{\varepsilon}(\rho_1+\rho_2)\,, \quad y = \frac{1}{\varepsilon}(\rho_1-\rho_2)$$

has been used. Substituting (4.7) and (4.8) in (4.5a–b) one obtains the rate of ε-collisions per unit volume (at $k=0$ and small ε)

$$n_\varepsilon = N_e \varepsilon^{\frac{5}{2}} \frac{9.9}{\pi^2}\,. \tag{4.9}$$

Notice that the purpose of taking into account the integration over the full phase space and correct flux factor $\|\nu_1-\nu_2\|$, is to reduce the power of ε to $5/2$ (rather than 6 as would be obtained in configuration space). The actual rate of ε-collisions under ergodic conditions is therefore expected to be much larger than the lower bound obtained in Ref. 20 using a reduced phase space and flux uniformity assumptions.

n_ε in Eq. (4.9) is the rate of ε-collisions per unit volume and unit τ in scaled units. Reducing to physical units, using (4.1) and (2.2) one obtains

$$n_\varepsilon^{(\text{phys})} = \frac{19.8}{\pi^2} \frac{\rho_e}{\rho_D} \varepsilon_{\text{phys}}^{\frac{5}{2}} \left(\frac{2\pi\rho_D}{3}\right)^{\frac{7}{3}} \frac{e}{\sqrt{m_D}} \tag{4.10}$$

with $\varepsilon_{\text{phys}} = \varepsilon \times a^3$. N_e in (4.9) and ρ_e, ρ_D in (4.10) should be interpreted as the densities of e and D particles participate in the ergodic motion at a certain moment, which are not necessarily the total material densities.

To understand the physical meaning of this result consider a density of 2 D-particles in a sphere of radius 2 angstrom and $\varepsilon = 10$ Fermi with $m_D = 3.34358 \times 10^{-24}$ gram, the deuteron mass. One obtains $\frac{4\pi}{3} n_\varepsilon = N_e\ 7.4 \times 10^{-11}$ per unit τ, i.e. $(4\pi/3)(n_\varepsilon/\mu_\tau) = N_e\ 6.9 \times 10^3$ per second as the average number of times 2 D-particles, at this density and in ergodic motion, would approach each other within 10 Fermi.

For $\varepsilon = 5$ Fermi the corresponding results are $N_e\ 1.3 \times 10^{11}$ per unit τ and $N_e\ 1.2 \times 10^3$ per second.

These are amazingly large rates of instances of close proximity for two low energy positively charged particles, which shows that motion under ergodic conditions is quite different from a situation of static equilibrium. Some improvements are however needed in these results:

First we have considered particles of zero radius ($k = 0$). When ε is very small the phase space reduction resulting from the finiteness of k should be taken into account. For small k the value of the normalization integral J_1 changes very little, however for $J_{\delta_\varepsilon}^{(\frac{5}{2})}(0)$ the integration limits become

$$\left\{ \int_1^{1+2\frac{k}{\varepsilon}} dx \int_0^{x-2\frac{k}{\varepsilon}} dy + \int_{1+2\frac{k}{\varepsilon}}^4 dx \int_0^1 dy + \int_4^{2+\sqrt{5}} dx \int_{\sqrt{x^2-4x}}^1 dy \right\}$$

and

$$J_{\delta_\varepsilon}^{(\frac{5}{2})}(k) = \alpha\left(\frac{k}{\varepsilon}\right)\varepsilon^{\frac{5}{2}}\,.$$

One obtains for example

$\frac{k}{\varepsilon}$	0.001	0.01	0.05	0.1	0.2	0.3	0.4
$\alpha\left(\frac{k}{\varepsilon}\right)$	7.0	6.6	5.6	4.9	3.8	3.0	2.3

A reduction of n_ε for small ε is indeed observed, but it does not essentially change the order of magnitude of the estimates above.

Second, as pointed out, the result (4.9–10) does not depend on the mass of the e-particle. This is however changed by relativistic corrections. For large values of the density, large T values will contribute to the phase space integrals and relativistic corrections should be taken into account. As a simple estimate of the effect of relativistic corrections one considers the D-particles still as non-relativistic and study the limiting case $m_e = 0$ for the e-particle. Then in the measures (4.2) and (4.3) one replaces $d^3\omega$ by $d^3\pi/E_e$ (with $E_e = \|\pi\|$), T by

$$T = \frac{1}{2}\|\nu_1\|^2 + \frac{1}{2}\|\nu_2\|^2 + \|\pi\|$$

and the momentum delta function becomes $\delta^3(\nu_1 + \nu_2 + \pi)$.

Changing variables as before from $(x_1, x_2, y, \nu_1, \nu_2, \pi)$ to $(\rho_1, \theta_1, \phi_1, \rho_2, r, \phi_2, y, T, \Omega_\xi, \eta, \pi)$ one obtains

$$n_\varepsilon = N^{-1}N_e\, 64\pi^4 \int_0^{T_{\max}} dT\left\{T^2 + \frac{8}{3} - \frac{8}{3}(1+T)^{\frac{3}{2}} + 4T\right\} I_{\delta_\varepsilon}(T,k)\ ,$$

$$N = \frac{512}{3}\pi^5 \int_0^{T_{\max}} dT\left\{\frac{2}{3}T^{\frac{3}{2}} + T^{\frac{1}{2}} - (1+T)\arcsin\sqrt{\frac{T}{1+T}}\right\} I_1(T,k)\ .$$

Computing

$$J_1^{(\frac{3}{2})}(0) = \int_0^\infty dT\, T^{\frac{3}{2}} I_1(T,0) = 0.6\ ,$$

$$J_1^{(\frac{1}{2})}(0) = \int_0^\infty dT\, T^{\frac{1}{2}} I_1(T,0) = 0.28\ ,$$

$$J_1^{(s)}(0) = \int_0^\infty dT(1+T)\arcsin\sqrt{\frac{T}{1+T}}\ I_1(T,0) = 0.59\ ,$$

and

$$J_{\delta_\varepsilon}^{(2)}(0) = \int_0^\infty dT\, T^2\, I_{\delta_\varepsilon}(T,0) = 3.66\varepsilon^3\ ,$$

$$J_{\delta_\varepsilon}^{(\frac{3}{2})}(0) = \int_0^\infty dT(1+T)^{\frac{3}{2}} I_{\delta_\varepsilon}(T,0) \simeq 2.66\varepsilon^{\frac{7}{2}}\ ,$$

$$J_{\delta_\varepsilon}^{(1)}(0) = \int_0^\infty dT\, T\, I_{\delta_\varepsilon}(T,0) = 2.5\varepsilon^4\ ,$$

$$J_{\delta_\varepsilon}^{(0)}(0) = \int_0^\infty dT\, I_{\delta_\varepsilon}(T,0) = 5.32\varepsilon^5\ ,$$

one obtains (in the $k = 0$ and small ε limit)

$$n_\varepsilon = \frac{N_e}{\pi}\left\{15.2\varepsilon^3 - 29.5\varepsilon^{\frac{7}{2}} + 41.6\varepsilon^4 + 59.1\varepsilon^5\right\} \tag{4.11}$$

in scaled variables. For small ε the lowest power dominates and one obtains in physical units

$$n_\varepsilon^{(\text{phys})} \simeq \frac{30.5}{\pi}\frac{\rho_e}{\rho_D}\varepsilon_{\text{phys}}^3\left(\frac{2\pi\rho_D}{3}\right)^{\frac{15}{6}} \frac{e}{\sqrt{m_D}}\ . \tag{4.12}$$

Computing now the rates for the same values as before one obtains:

For $\varepsilon = 10$ Fermi: $(4\pi/3)\ n_\varepsilon = N_e\ 2.5\ \times 10^{-2}$ per unit τ, i.e., $(4\pi/3)(n_\varepsilon/\mu_\tau) = N_e\ 2.35\ \times 10^2$ per second.

For $\varepsilon = 5$ Fermi: $(4\pi/3)\ n_\varepsilon = N_e\ 3.16\ \times 10^{-13}$ per unit τ, i.e., $(4\pi/3)(n_\varepsilon/\mu_\tau) = N_e\ 29.4$ per second.

The effect of considering an extremely relativistic e-particle is to reduce the rates as compared to (4.9) but the value is still fairly large.

5. Concluding Remarks

1. Although suggesting that complex dynamics in high density Coulomb systems may lead to interesting effects, the results of the previous sections should be considered as somewhat preliminary. The nature of the phase space measure that is obtained from random initial conditions and in particular the role of quasi-local constraints on the energy and angular momenta is now being checked by direct numerical simulations. Consideration of quantum effects is of course also needed. Although the quantum theory of non-integrable systems is still being developed, an analysis similar to the one in Sec. 4 using the appropriate Wigner functions might be feasible.
2. Even if the theoretical results indicate that under ergodic motion of the type considered here the rate of near collisions is indeed large, the question remains of course whether such conditions might be created experimentally for the deuterons absorbed in a solid lattice.

   When the absorbed deuterons tend to stay in deep potential wells in the interstitial lattice sites the system will, in the steady state, be too ordered to display the kind of effects being discussed. If the effect is observed at all in such systems it would only be in conditions of equilibrium rupture. For example when charging or discharging the lattice or when conditions of localized disorder are induced by defects or dislocations. Notice also that it is typical of complex systems that have a preferred stationary state to display occasional intermittency, i.e. bursts of chaos followed by long quiescent periods.

   Continuous fusion effects are only expected to occur above the "ergodicity threshold", i.e., when a dynamical measure of the type of (3.1) is reached from most initial conditions. Below a few ideas (maybe unfeasible) on how one could possibly attain this threshold are listed:

   a) To favour deuteron delocalization, by insuring that octahedral and tetrahedral interstitial sites are occupied in sufficiently large numbers; to choose an appropriate working temperature or to dope the lattice with atoms that compete with the deuterons for the interstitial sites. This last technique will however have the shortcoming of reducing the maximum volume of absorbed deuterium.

   b) To excite both the absorbed deuterium and (or) the lattice wave functions with external fields. In general a steady (or pulsed) system of crossed electric and magnetic fields is expected to induce three-dimensional chaos in charged systems.

In conclusion, if the effects I have been discussing are of any practical use, then hybrid fusion might well become the art of inducing complex dynamical behavior in Coulomb systems.

3. Finally, if the conditions for ergodicity are realized, some consequences are to be expected at the level of the nuclear processes. They will be mostly of the three-body DeD type and, in contrast with bound state mechanism, their rate is not critically dependent on the negative scatterer mass. For fusions that occur through three-body scattering processes, there is a mechanism[24] that naturally reduces neutron production, the $t+p$ channel being the preferred one. Therefore neutron detection may not be the best nuclear signature to look for. A signature for three-body scattering fusion would be the detection of t, p pairs at the invariant mass of the 20.1 (0^+) excited $^4He^*$ state.

References and Footnotes

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16. Here I am thinking mostly about palladium where it is known that one may accumulate a number of deuterons in excess of the number of metal atoms, the deuterons being relatively free to move in the lattice. In titanium a composite TiD_2 is formed; furthermore electrolytic penetration was found to be small.[10]
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21. See for example the review by A. L. Rebenko, *Russian Math. Surveys* **43** (1988) 65.
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23. This is the only local (few-body) constraint that will be imposed on the phase space measure. For other quantum numbers like angular momentum, it will of course be conserved for the whole many-body system, but no additional restrictions will be

imposed on the angular momentum of few-body subsystems. It is debatable whether, under shielding conditions, a quasi-local bound on angular momentum should also be imposed. The need and validity of quasi-local constraints on the ergodic measure is now being checked by numerical simulations.
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