



ELSEVIER

5 February 1996

PHYSICS LETTERS A

Physics Letters A 211 (1996) 87–93

A fully connected network of Bernoulli units with correlation learning

J.A. Dente ^a, R. Vilela Mendes ^b

^a *Laboratório de Mecatrónica, DEEC, Instituto Superior Técnico, Av. Rovisco Pais, P 1096 Lisbon Codex, Portugal*

^b *Grupo de Física-Matemática, Universidade de Lisboa, Av. Gama Pinto 2, P 1699 Lisbon Codex, Portugal*

Received 16 February 1995; revised manuscript received 31 October 1995; accepted for publication 21 November 1995

Communicated by A.R. Bishop

Abstract

Biological evidence suggests that pattern recognition and associative memory in the mammalian nervous system operates through the establishment of spatio-temporal patterns of activity and not by the evolution towards an equilibrium point as in attractor neural networks. Information is carried by the space-time correlation of the activity intensities rather than by the details of individual neuron signals. Furthermore the fast recognition times that are achieved with relatively slow biological neurons seem to be associated to the chaotic nature of the basal nervous activity. To copy the biology hardware may not be technologically sound, but to look for inspiration in the efficient biological information processing methods is an idea that deserves consideration. Inspired by the mechanisms at work in the mammalian olfactory system we study a network where, in the absence of external inputs, the units have a dynamics of the Bernoulli shift type. When an external signal is presented, the pattern of excitation bursts depends on the learning history of the network. Association and pattern identification in the network operates by the selection, by the external stimulus, of distinct invariant measures in the chaotic system. The simplicity of the node dynamics, that is chosen, allows a reasonable analytical control of the network behavior.

1. Introduction

Freeman and collaborators [1–6] have extensively studied and modeled the neural activity in the mammalian olfactory system. Their conclusions challenge the idea that pattern recognition in the brain is accomplished as in an attractor neural network [7]. Pattern recognition in the brain is the process by which external signals arriving at the sense organs are converted into internal meaningful states. The studies of the excitation patterns in the olfactory bulb of the rabbit lead to the conclusion that, at least in this biological pattern recognition system, there is no evolution towards an equilibrium fixed point nor does it seem to be minimizing an energy function. Other interesting conclusions of these biological studies are:

– The main component of the neural activity in the olfactory system is chaotic. This is also true in other parts of the brain, periodic behavior occurring only in abnormal situations like deep anesthesia, coma, epileptic seizures or in areas of the cortex that have been isolated from the rest of the brain.

– The low-level chaos that exists in absence of an external stimulus is, in the presence of a signal, replaced by bursts lasting for about 100 ms which have different intensities in different regions of the olfactory bulb.

Olfactory pattern recognition manifests itself as a spatially coherent pattern of intensity.

– The recognition time is very fast, in the sense that the transition between different patterns occurs in times as short as 6 ms. Given the neuron characteristic response times this is clearly incompatible with the global approach to equilibrium of an attractor neural network.

– The biological measurements that have been performed do not record the action potential of individual neurons, but the local effect of the currents coming out of thousands of cells. Therefore the very existence of measurable activity bursts implies a synchronization of local assemblies of many neurons.

Freeman, Yao and Burke [4,5] model the olfactory system with a set of non-linear coupled differential equations, the coupling being adjusted by means of an input correlation learning scheme. Each variable in the coupled system is assumed to represent the dynamical state of a local assembly of many neurons. Based on numerical simulations they conjecture that olfactory pattern recognition is realized through a multilobe strange attractor. The system would be, most of the time, in a basal (low-activity) state, being excited to one of the higher lobes by the external stimulus.

To compute or even prove the existence of chaotic measures in coupled differential equation systems is an awesome task. Therefore, even if it may be biologically accurate, the analytical model of these authors is difficult to deal with and unsuitable for wide application in technological pattern recognition tasks, although one such application has indeed been attempted by the authors [6]. However, the idea that efficient pattern recognition may be achieved by a chaotic system, which selects distinct invariant measures according to the class of external stimuli, is quite interesting and deserves further exploration. Being more interested in technological applications of biologically inspired ideas than in accurate biological modeling, we have attempted to develop a model which behaves roughly as an olfactory system (in Freeman's sense) and, at the same time, is easier to describe and control by analytical means. To play the role of the local chaotic assembly of neurons we have chosen a Bernoulli unit with a well defined measure. The connection between the units is realized by linear synapses with an input correlation learning law and the external inputs also have adjustable gains, changing as in a biological potentiation mechanism. This last feature, which as far as we understand is not contained in the models of Refs. [5,6], turns out to be useful to enhance the novelty-filter qualities of the system.

Inspired by the biological evidence described above, we are here mostly concerned with the role of the networks as information processing devices. Therefore the modulation, through learning, of the connection strengths and the selection of distinct invariant measures play a major role. We must, however, mention that fully connected networks of chaotic elements are interesting dynamical systems in a wider context. They have been discussed in connection with mean field models of coupled map lattices, fluid turbulence models, chemical reaction systems and oscillatory formal neurons [8–12].

2. A network of Bernoulli units

The network we study is the fully connected system shown in Fig. 1. The output of the nodes is denoted by y_i and the x_i are the external inputs. W_{ij} with $i, j \in \{1, 2, \dots, N\}$ are the connection strengths and W_{i0} the input gains. Both W_{ij} and W_{i0} are real numbers in the interval $[0, 1]$. The input patterns are zero-one sequences ($x_i \in \{0, 1\}$). The learning laws for the connection strengths and the input gains are the following:

Let $S_{ij}(t) = W_{ij}(t) + \eta x_i(t) x_j(t)$. Then

$$\begin{aligned} W_{ij}(t+1) &= S_{ij}(t), & \text{if } \sum_{k \neq i} S_{ik}(t) \leq C, \\ &= C \frac{S_{ij}(t)}{\sum_{k \neq i} S_{ik}(t)}, & \text{if } \sum_{k \neq i} S_{ik}(t) > C, \quad i \neq j, \end{aligned} \quad (2.1)$$

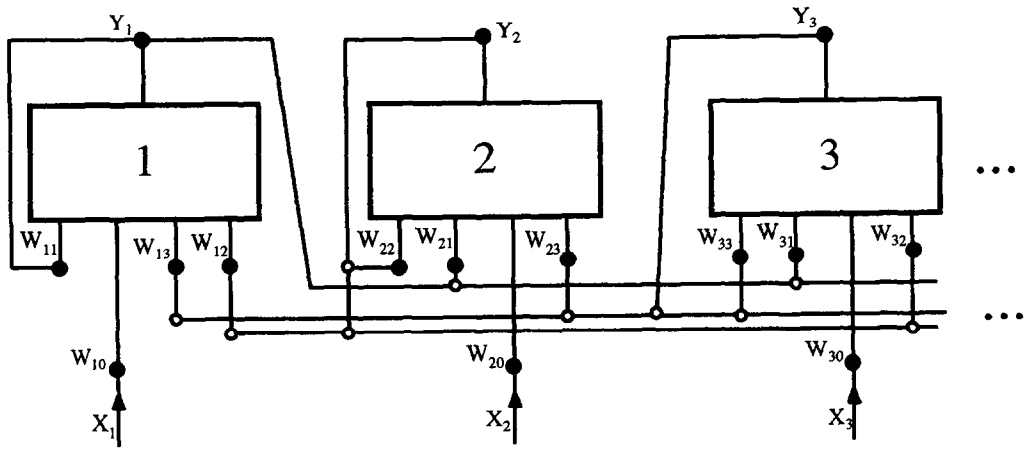


Fig. 1. The network.

$$W_{ii}(t+1) = 1 - \sum_{j \neq i} W_{ij}(t+1), \tag{2.2}$$

$$W_{i0}(t+1) = \alpha \frac{N_i(1)_t}{N_i(1)_t + N_i(0)_t}. \tag{2.3}$$

According to Eq. (2.1), when an input pattern has a one in both the i and j positions, the correlation of the units i and j becomes stronger. $C < 1$ is a constant related to the node dynamics, which the sum of the off-diagonal connections is not allowed to exceed. η is a small parameter that controls the learning speed. Finally the diagonal element W_{ii} is chosen in such a way that the sum of all connections entering each unit adds to one.

In the input gain learning law, $N_i(1)_t$ (or $N_i(0)_t$) is the number of times that a one (or a zero) has appeared at the input i , up to time t . Eq. (2.3) means that if an input is excited many times, during the learning phase, it becomes more sensitive.

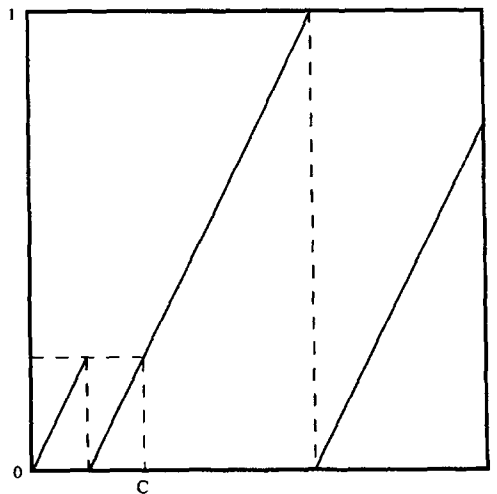


Fig. 2. The node dynamics function.

The node dynamics is

$$y_i(t + 1) = f\left(\sum_j W_{ij}(t) y_j(t) + W_{i0} x_i(t)\right), \tag{2.4}$$

f being the function depicted in Fig. 2.

The learning process starts with $W_{ii} = 1$ and $W_{ij} = 0$ for $i \neq j$. Each unit has then an independent absolutely continuous invariant measure which is the Lebesgue measure in $[0, C]$ and zero outside. When the W_{ij} ($j \neq i$) become different from zero but the inputs x_i are still zero, all variables y_i stay in the interval $[0, C]$ because of the convex linear combination of inputs imposed by the normalization of the W_{ij} . When some inputs x_i are $\neq 0$, there is a finite probability for an irregular burst in the interval $[C, 1]$, of some of the node variables, with reinjection into $[0, C]$ whenever the iterate falls on the interval $[\frac{1}{2} + \frac{1}{2}C, \frac{1}{2} + C]$.

The bursts in the interval $[C, 1]$, in response to some of the input patterns, is the recognition mechanism of the network. The basal chaotic dynamics insuring an uniform covering of the interval $[0, C]$, the timing of the onset of the bursts depends only on the correlation probability and on the clock time of the discrete dynamics. We understand therefore why a chaos-based network may have a recognition time faster than an attractor network.

3. Learning, invariant measures and simulations

Both the connection strengths and the nature of the bursts, for a given set of W and an applied input pattern, may be estimated in probability.

In Eq. (2.1), either the node i is not correlated to any other node and then all off-diagonal elements W_{ij} are zero or, as soon as the input patterns begin to correlate the node i with any other node, the off-diagonal elements start to grow and only the second case in Eq. (2.1) needs to be considered. Let the learning gain η be small. Then, in first order in η , we have

$$W_{ij}(t + 1) = W_{ij}(t) + \eta x_i(t) x_j(t) - \frac{\eta}{C} W_{ij}(t) \sum_{k \neq i} x_i(t) x_k(t).$$

For N learning steps, in first order in η

$$W_{ij}(t + N) = W_{ij}(t) + \eta \sum_{n=0}^{N-1} x_i(t+n) x_j(t+n) - \frac{\eta}{C} W_{ij}(t) \sum_{k \neq i} \sum_{n=0}^{N-1} x_i(t+n) x_k(t+n).$$

Denoting by $p_{ij}(1)$ the probability for the occurrence, in the input patterns, of a one both in the i and the j positions, the above equation has the stationary solution

$$W_{ij} = C p_{ij}(1) \left(\sum_{k \neq i} p_{ik}(1) \right)^{-1}. \tag{3.1}$$

Now we establish an equation for the burst probabilities. Consider the case where C is much smaller than one, that is, the basal chaos is of low intensity. In this case, because of the normalization chosen for W_{ii} , the dynamics inside the interval $[0, C]$ is dominated by $y \rightarrow 2y \pmod{C}$ and in the interval $[C, 1]$ by $y \rightarrow 2y \pmod{1}$. Hence, to a good approximation, we may assume uniform probability measures for the motion inside each one of the intervals. Denoting the interval $[0, C]$ as the state 1 and the interval $[C, 1]$ as the state 2, the dynamics of each node is a two-state Markov process with transition probabilities between the states corresponding to the probabilities of falling in some subintervals of the intervals $[0, C]$ and $[C, 1]$. Namely, the

probability $p(2 \rightarrow 1)$ equals the probability of falling in the reinjection interval $[\frac{1}{2} + \frac{1}{2}C, \frac{1}{2} + C]$ and the probability $p(1 \rightarrow 2)$ that of falling near the point C at a distance smaller than the off-diagonal excitation,

$$p_i(2 \rightarrow 1)_t = \frac{C}{2(1 - C)}, \tag{3.2a}$$

$$p_i(2 \rightarrow 2)_t = 1 - p_i(2 \rightarrow 1)_t, \tag{3.2b}$$

$$p_i(1 \rightarrow 2)_t = \left\{ \frac{1}{W_{ii}C} \left(W_{i0}x_i(t) - C(1 - W_{ii}) + \sum_{j \neq i} W_{ij}y_j(t) \right) \right\}^\#, \tag{3.2c}$$

$$p_i(1 \rightarrow 1)_t = 1 - p_i(1 \rightarrow 2)_t, \tag{3.2d}$$

where we have used the notation $f^\# = (f \vee 0) \wedge 1$ for functions truncated to the range $[0, 1]$. That is, $f^\# = 0$ if $f < 0$, $f^\# = 1$ if $f > 1$ and $f^\# = f$ if $1 \geq f \geq 0$.

The sum in the right-hand side of Eq. (3.2c) is approximated in probability by

$$\sum_{j \neq i} W_{ij} \left\{ \frac{1}{2} p_j(2)_t + \frac{1}{2} C [1 - p_j(2)_t] \right\}, \tag{3.3}$$

where $p_j(2)_t$ denotes the probability of finding the node j in the state 2 at time t . The probability estimate (3.3), for the outputs y_j , assumes statistical independence of the units. This hypothesis fails when there are synchronization effects, which are to be expected mainly when a small group of units is strongly correlated.

With the probability estimate for the y_j and the detailed balance principle it is now possible to write a self-consistent equation for the probability $p_i(2)_t$ to find an arbitrary node i in the state 2 at time t ,

$$p_i(2)_t = \left\{ \frac{1}{W_{ii}C} \left(W_{i0}x_i(t)_{ij} - C(1 - W_{ii}) + \sum_{j \neq i} W_{ij} \left\{ \frac{1}{2} p_j(2)_t + \frac{1}{2} C [1 - p_j(2)_t] \right\} \right) \right\}^\# \left[\left\{ \frac{1}{W_{ii}C} \left(W_{i0}x_i(t) - C(1 - W_{ii}) + \sum_{j \neq i} W_{ij} \left\{ \frac{1}{2} p_j(2)_t + \frac{1}{2} C [1 - p_j(2)_t] \right\} \right) \right\}^\# + \frac{C}{2(1 - C)} \right]^{-1}. \tag{3.4}$$

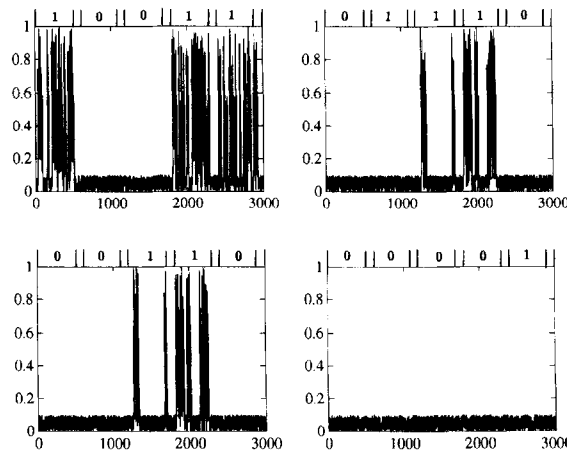


Fig. 3. Response of a four-node network after being exposed to the patterns 1000 and 0110 ($C = 0.1, \alpha = 0.01$).

For each input pattern $x_i(t)$, one obtains an estimate for $p_i(2)_t$ solving Eq. (3.4), by iteration. We find that the solution that is obtained is qualitatively similar to the numerically determined invariant measures, although it tends to overestimate the burst excitation probabilities when they are small. This may be understood from the synchronization effects between groups of units. When one unit is not excited (not in state 2) the others tend also not to be excited, hence (3.3) overestimates the sum $\sum_{j \neq i} W_{ij} y_j(t)$.

We now illustrate how the network behaves as an associator and pattern recognizer. Consider, for display simplicity, a network of four nodes that is exposed during many iterations to the patterns 1000 and 0110 where the first pattern appears twice as much as the second. After this learning period we have exposed the network to all several distinct zero-one input patterns for 500 time steps each and observed the network reaction. During the

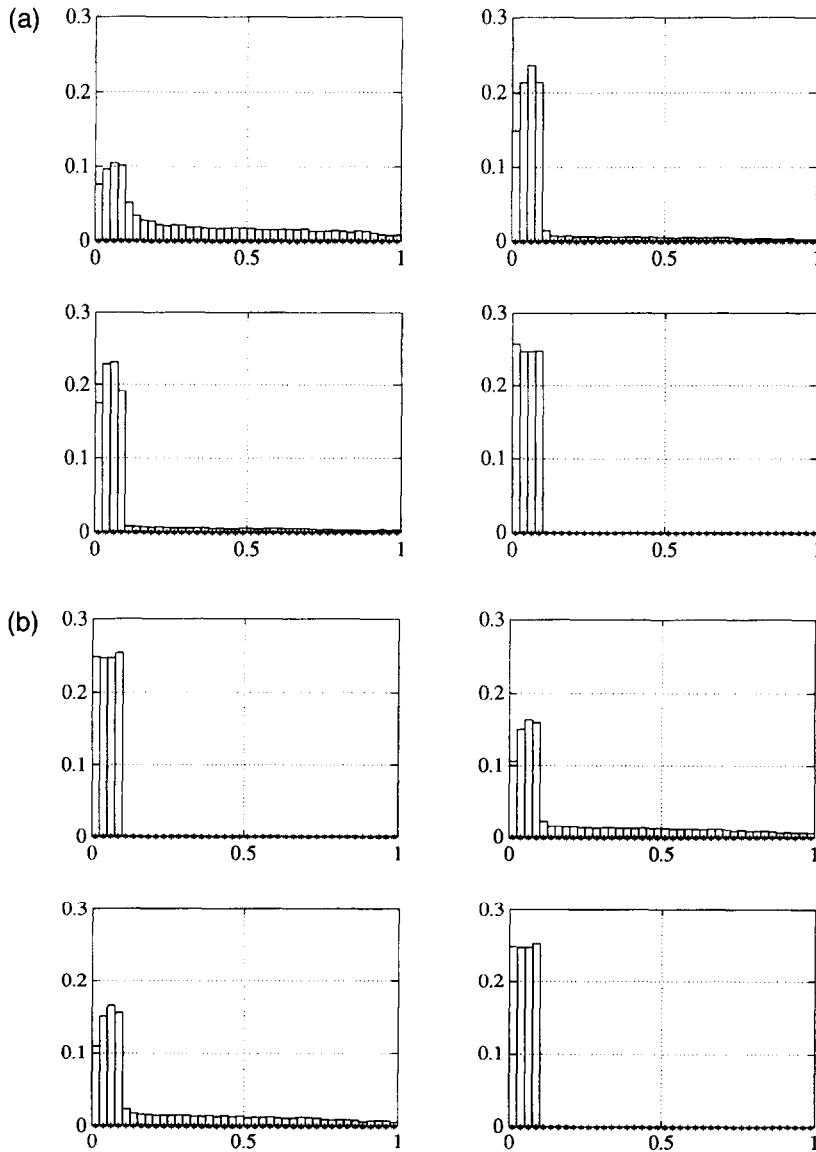


Fig. 4. Invariant measures of the same network for the input patterns (a) 1100 and (b) 0.110.

recall experiment no further adjustment of the W_{ij} is made. The result is shown in Fig. 3. The conclusions from this and other simulations is that, according to nature of the learning patterns, the network acts, for the recall input patterns, as a mixture of memory, associator and novelty filter. For example in Fig. 3 we see that after having learned the sequences 1000 and 0110, the network reproduces these patterns as a memory. The pattern 1001 is associated to the pattern 1000 and the pattern 1110 associated to a mixture of the two learned patterns. By contrast the pattern 0100 is not recognized by the network which acts then as a novelty filter. Fig. 4 shows the invariant measures of the system (expressed in probability per bin) when the input patterns are 1100 and 0110.

In conclusion, (i) the network based on Bernoulli units, with node dynamics as in Fig. 2 and correlation learning described by Eqs. (2.1)–(2.3) is a chaos-based pattern recognizer with the capability of operating on distinct invariant measures which are selected by the input patterns. (ii) The response time of the selection is controlled by the magnitude of the invariant measures and the clock time of the basal chaotic dynamics. (iii) As a pattern recognizer the network is a mixture of memory, associator and novelty filter. This, however, is sensitive to the learning algorithm that is chosen and, for the algorithm that is discussed here, it is sensitive also to the value of the parameters C and α .

References

- [1] C.A. Skarda and W.J. Freeman, *Brain Behav. Sci.* 10 (1987) 161.
- [2] W.J. Freeman and B. Baird, *Behav. Neurosci.* 101 (1987) 393.
- [3] W.J. Freeman, *Biol. Cyb.* 56 (1987) 139.
- [4] W.J. Freeman, Y. Yao and B. Burke, *Neural Networks* 1 (1988) 277.
- [5] Y. Yao and W.J. Freeman, *Neural Networks* 3 (1990) 153.
- [6] Y. Yao, W.J. Freeman, B. Burke and Q. Yang, *Neural Networks* 4 (1991) 103.
- [7] D.J. Amit, *Modeling brain function. The world of attractor neural networks* (Cambridge Univ. Press, Cambridge, 1989).
- [8] K. Kaneko, *Physica D* 54 (1991) 5; 75 (1994) 55; 77 (1994) 456.
- [9] K. Wiesenfeld and P. Hadley, *Phys. Rev. Lett.* 62 (1989) 1335.
- [10] D. Golomb, D. Hansel, B. Shraiman and H. Sompolinsky, *Phys. Rev. A* 45 (1992) 3516.
- [11] A.S. Pikovsky and J. Kurths, *Phys. Rev. Lett.* 72 (1994) 1644.
- [12] H. Nozawa, *Physica D* 75 (1994) 179.