

# Modular quantum computing and quantum-like devices

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## Abstract

The two essential ideas in this paper are, on the one hand, that a considerable amount of the power of quantum computation may be obtained by adding to a classical computer a few specialized quantum modules and, on the other hand, that such modules may be constructed out of classical systems obeying quantum-like equations where a space coordinate is the evolution parameter (thus playing the role of time in the quantum algorithms).

Keywords: Quantum computation, Quantum Fourier transform, Oracles, Fiber and wave-guide optics

## 1 Introduction

### 1.1 Computation models

Classical, probabilistic and quantum computing are three computing modalities which, adopting a Turing Machine-like scheme [1] [2], may be briefly described in the following way:

Let  $M$  be a states machine with one working tape with alphabet  $\Gamma$  and an input tape with alphabet  $\Sigma$ . At each time the machine configuration  $c$  is the content of the working tape, the position of two pointers (in the input and working tapes) and the current state. Let  $\mathcal{C}(x)$ , of cardinality  $N$ , be the set of all possible configurations when the input is  $x$ .

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At each time step the machine, in a state  $q \in Q$ , reads a symbol  $\sigma \in \Sigma$  in the input tape and the current symbol  $\gamma \in \Gamma$  in the working tape, changes to a state  $q' \in Q$ , prints a symbol  $\gamma' \in \Gamma$  in the working tape and the pointers move right ( $R$ ) or left ( $L$ ) in the respective tapes. The probability of these operations is controlled by a mapping  $T$  from  $\mathcal{C}(x)$  into a space  $S$

$$T : Q \times \Sigma \times \Gamma \times Q \times \Gamma \times \{L, R\}^2 \rightarrow S$$

This mapping is called the transition function from which the transition probability between  $c_i$  and the next  $c_{i+1}$  configuration  $p(c_i, c_{i+1}) = F(T(c_i, c_{i+1}))$  may be obtained. In all cases it is assumed that the internal state of the machine is not observed except at the final time of the calculation. The mapping  $T$  defines a matrix in the space of configurations  $\mathcal{C}$ .

$$T \left( q_j, \gamma_j, p_j^{(1)}, p_j^{(2)} \mid q_i, \gamma_i, p_i^{(1)}, p_i^{(2)} \right) \doteq T(c_j, c_i)$$

The three computation models correspond to different choices of  $T$  and of  $p(c_i, c_{i+1}) = F(T(c_i, c_{i+1}))$

*Classic deterministic computation:*

$$\begin{aligned} S &= \{s : s = 0, 1\} \\ p(c_j, c_i) &= T(c_j, c_i) = s \end{aligned} \tag{1}$$

Only one element in each line of the transition matrix  $T$  is different from zero.

*Classical probabilistic computation:*

$$\begin{aligned} S &= \{s : s \in [0, 1]\} \\ p(c_j, c_i) &= T(c_j, c_i) = s \end{aligned} \tag{2}$$

with the condition

$$\sum_j T(c_j, c_i) = 1 \tag{3}$$

$T$  is a stochastic matrix preserving the  $\mathcal{L}^1$  norm in the space of configurations.

*Quantum computation:*

$$\begin{aligned} S &= \{s \in \mathbb{C} : |s|^2 = 1\} \\ p(c_j, c_i) &= |T(c_j, c_i)|^2 = |s|^2 \end{aligned} \tag{4}$$

with the condition

$$\sum_j |T(c_j, c_i)|^2 = 1 \tag{5}$$

that is,  $T$  is a unitary matrix preserving the  $\mathcal{L}^2$  norm in the space of configurations.

In all cases the transition probabilities between initial and final states are positive and normalized. The difference between the three computational models is the method used to find the transition probabilities.

Physical implementations of the computational models require physical elements for **coding**, **interaction** between the elements to perform the writing and change of states and finally an **evolution** process to represent the transition function. *Coding, interaction and evolution.* And, in each case, the evolution should be such as to satisfy the constraints (1) or (3) or (5).

Some quantum systems, when sufficiently isolated from the environment, because their coherent time-evolution is unitary, provide physical models of quantum computation. However, quantum computation is not quantum mechanics. Any other system, that provides coding, interaction and a change of states compatible with (4) (5), may also provide a model of quantum computation. In particular the state evolution of these systems should be unitary. Such systems have been called quantum-like.

In Ref.[3] it has been proposed that classical paraxial light propagation, being ruled by a Schrödinger-like equation may also provide a model of quantum computation. There is, of course, no contradiction with the physical rules of quantum mechanics because in the classical paraxial system the propagation is along a space coordinate which plays the same role as time in the quantum mechanical Schrödinger equation. As a consequence the transfer function may be implemented by the unitary propagation of information along a space coordinate. Considering the coding and interaction requirements, a good candidate for this implementation seems to be fiber or wave-guide optics.

The idea of using quantum-like systems for quantum computation and simulation of quantum effects has been later explored (see for example [4] - [12]) by several authors.

## 1.2 Modular computation

Although it has not yet been rigorously proven that  $BPP \subsetneq BQP$ , that is, that quantum circuits cannot be efficiently simulated in a bounded-error

probabilistic machine, the quantum oracle algorithms, that have been developed, provide circumstantial evidence that quantum computing is indeed more efficient than classical computing.

The power of quantum computing hinges both on the capacity to deal with superpositions of many different states (quantum parallelism) and on the enhancement of particular computational paths (quantum interference). The following three resources are responsible for the efficiency of the known quantum algorithms:

- (i) Preparation of a linear superposition of all possible basis states  $\sum_x |x\rangle$ ;
- (ii) Call to a reversible oracle operation

$$\sum_x |x\rangle |\psi\rangle \rightarrow \sum_x |x\rangle |f(x) \oplus \psi\rangle = \sum_x |x\rangle U_{f(x)} |\psi\rangle$$

the target qubit(s)  $|\psi\rangle$  being usually chosen to be eigenstates of the controlled unitary operations  $U_{f(x)}$  with eigenvalues  $e^{i\alpha(x)}$ ;

- (iii) Use of the  $e^{i\alpha(x)}$  phases (kicked back to  $|x\rangle$ ) to enhance, by interference, particular computational paths.

The oracle is the quantum subroutine that contains the information specific to each particular problem. The way the oracle is chosen to act (in particular the choice of the target qubit as an eigenstate of  $U_{f(x)}$ ) implies that the natural interference device is the quantum Fourier transform (QFT). On the other hand, the QFT, operating on the state  $|00\dots 0\rangle$ , also generates a superposition of all the basis states. This suggests that most of the power of quantum computing may be obtained by adding to a classical computer a few basic modules, namely:

- (i) A quantum Fourier transform module
- (ii) Programmable oracle modules.

In theoretical discussions the oracle is considered to be a subroutine call, invocation of which only costs unit time. However, one should not forget that it is an operation acting in all basis states and therefore, to benefit from quantum parallelism the practical requirements for its implementation are not very different from those of the quantum Fourier transform.

Quantum computing requires the coding, manipulation and detection of entangled qubits. Nuclear spins, atom states, flux units, Cooper pairs or single photon polarizations have been proposed and used to encode qubits and exhibit quantum logic operations. Qubits encoded in such fundamental matter units might indeed be the ultimate building blocks of future quantum

computers. For practical computing applications, a scalable tensor product structure is required to avoid an exponential demand for physical resources. However, this seems difficult to achieve with the prototype quantum gates that have been developed. Therefore a search for alternative implementations seems appropriate.

Section 2, improving and extending a previous proposal [3], discusses an implementation of quantum computing operations in classical systems that propagate according to a Schrödinger equation with a space coordinate playing the role of time. Here one tries to make a concrete proposal for the implementation of the theory using fiber or wave-guide optics, the qubits being robustly coded in particular modes or on their polarizations, with the result of the (unitary) operations being read off at particular locations of the optical systems. Fiber or planar wave-guide optics implementations benefit from the large amount of technological sophistication already developed for communications. Therefore, the emphasis is on the construction of quantum gates using devices and techniques currently available in this field. As the sophisticated optical elements developed so far have been done mostly for telecommunication purposes one also clarifies the implementation progress needed to make them appropriate for the quantum computation purposes. As well as the issues of coding and gate implementation, also polarization effects, signal coupling and the notions of mixing, entanglement and coherence are discussed in this setting.

Finally, Section 3 discusses how these quantum-like elements might scale-up to construct a quantum Fourier transform module as well as programmable oracles.

## **2 Quantum-like computation with fiber or wave-guide optics**

### **2.1 Unitary evolution**

In optical fibers or planar wave-guides, mode propagation may be well approximated by a Schrödinger equation with the longitudinal  $z$ -coordinate playing the role of time. Ref.[3] follows a reasoning similar to the Leontovich-Fock [13] description of paraxial beams in the parabolic approximation. Here one generalizes the derivation in [3] by explicitly including polarization effects.

From the Maxwell equations, with  $\rho = J = M = 0$ , one obtains the Helmholtz equation for a space-varying dielectric constant

$$\nabla \left( \frac{1}{\varepsilon} E \cdot \nabla \varepsilon \right) + \Delta E = \varepsilon \mu_0 \frac{\partial^2 E}{\partial t^2} \quad (6)$$

Consider now a fixed frequency transversal mode  $E(x, y, z, t) = E(x, y, z) \exp(i\omega t)$  and an index of refraction profile

$$\varepsilon(x, y, z) = n^2(x, y, z) = n_0^2(z) - V(x, y) \quad (7)$$

where  $n_0^2(z)$  is the index of refraction at the fiber axis and  $V(x, y) \ll n_0^2(z)$ . With this last condition and neglecting terms in  $(\nabla V)^2$  and  $V \nabla V$ , one obtains, for a transversal electric mode

$$\left( \frac{\partial^2}{\partial z^2} + \Delta_2 \right) E + n^2 k_0^2 E - \frac{1}{n_0^2} \nabla_2 (E \cdot \nabla_2 V) \simeq 0 \quad (8)$$

where  $k_0 = \frac{\omega}{c}$  and  $\lambda_0 = \frac{2\pi}{k_0}$  is the wavelength in vacuum.

Introduce the slowly varying (in  $z$ ) complex vectorial function  $\psi(x, y, z)$

$$E(x, y, z) = \psi(x, y, z) \exp\left(ik_0 \int^z n_0(\zeta) d\zeta\right) \quad (9)$$

For slow variation of the index of refraction along the fiber axis over distances of the order of one wavelength

$$\frac{\lambda_0}{n_0^2(z)} \left| \frac{dn_0(z)}{dz} \right| \ll 1$$

one may neglect second-order derivatives of  $\psi$  along  $z$  and derivatives of  $n_0(z)$  and end up with

$$i\lambda_0 \frac{\partial}{\partial z} \begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix} = \left\{ \begin{array}{l} \frac{\lambda_0^2}{4n_0(z)} \begin{pmatrix} -\Delta_2 & 0 \\ 0 & -\Delta_2 \end{pmatrix} + \frac{\pi}{n_0(z)} \begin{pmatrix} V(x, y) & 0 \\ 0 & V(x, y) \end{pmatrix} \\ + \frac{\lambda^2}{4\pi n_0^3(z)} \begin{pmatrix} \partial_x^2 V + \partial_x V \partial_x & \partial_{xy}^2 V + \partial_y V \partial_x \\ \partial_{xy}^2 V + \partial_x V \partial_y & \partial_y^2 V + \partial_y V \partial_y \end{pmatrix} \end{array} \right\} \begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix} \quad (10)$$

which is a quantumlike version of the Schrödinger-Pauli equation. The role of time in this equation is played by the spatial (longitudinal) coordinate of the light beam, the role of Planck's constant is played by the light wavelength

and the role of potential energy by the index of refraction of the medium. Thus, a beam of light, a purely classical object, obeys equations formally identical to those of quantum mechanics.

The unitary  $z$ -evolution of the electromagnetic complex amplitude is described by the evolution operator  $\hat{U}(z)$

$$\hat{U}(z, z_0)\psi(x, y, z_0) = \psi(x, y, z), \quad (11)$$

associated to the Hamiltonian

$$\hat{H}(z) = \left( \frac{\hat{p}_x^2}{2} + \frac{\hat{p}_y^2}{2} \right) \frac{1}{n_0(z)} + \Gamma(x, y, z). \quad (12)$$

with  $\hat{p}_x = -i\lambda \frac{\partial}{\partial x}$ ,  $\hat{p}_y = -i\lambda \frac{\partial}{\partial y}$  and a potential function  $\Gamma(x, y, z)$  which, for general  $V(x, y)$ , has local and nonlocal terms mixing the polarizations as follows from Eq.(10). Manipulation of the polarization will play an important role in this quantumlike computation approach. As seen from the last term in Eq.(10) it is obtained by engineering the index of refraction profile.

Other quantumlike systems are reviewed in [14] [15] [16]. They include sound-wave propagation in acoustic waveguides, charged-particle beams and light beams inside diode lasers. Full implementation of quantum algorithms might also be obtained in these systems. For each unitary operation a steady state is to be established and the result of the computation is read at the appropriate space location. The notion of preservation of *time coherence* needed to define the reliability and maximum number of operations in quantum computation is here replaced by *space coherence* of the steady state that is established in the device.

Before discussing practical implementations of the quantumlike representation, I add two speculative remarks:

(i) Abrams and Lloyd [17] have shown that were quantum mechanics nonlinear, more computational power could still be achieved. There is no evidence indicating that actual quantum mechanics is nonlinear. However, in the quantumlike scheme it is quite simple to implement a nonlinear Schrödinger equation evolving in the  $z$ -coordinate. Therefore, quantumlike nonlinear circuits might provide an adequate framework to test Abrams and Lloyd's ideas.

(ii) Brun [18] has pointed out that hard problems could in principle be solved, even by a classical computer, if it had access to a closed timelike curve. Except maybe in extreme cosmological conditions, closed timelike curves are

not readily available. However, if in a computational scheme (both classical and quantum) time is replaced by space, simulation of closed timelike curves is not unthinkable.

Before proceeding it should be pointed out that other implementation of some features of quantum algorithms by linear optical methods have been proposed by several authors (see for example [19] [20] and references therein). To obtain the entanglement needed for universal quantum computation, the proposed optics implementations use either:

- (i) Kerr nonlinearities, which are hard to achieve at the single-photon level or
- (ii) a probabilistic scheme based on the nonlinearity implicit in the selection by single-photon detectors.

What is proposed here and in Ref.[3] is a more radical proposal in the sense that, instead of setting up a time sequence of optical events as the implementation of the quantum algorithm, one uses the fact that, in optical fibers, mode propagation is well approximated by a Schrödinger equation with the  $z$ -coordinate along the fiber playing the role of time.

## 2.2 Coding

In Ref.[3], several ways to code qubits on a fiber, using either discrete or continuous variables, were already discussed. Here simpler implementations are proposed which might be robustly obtained with the materials available for optical communication applications.

Consider three types of qubit codings in two types of fibers:

(a) In single-mode (double-polarization) fibers a qubit would correspond to the two polarizations directions of the  $LP_{01}$  mode [21] (Fig.1).

(b) In single-mode fibers a qubit might also be associated to the amplitudes of a particular polarization in two distinct fibers, one of the fibers associated to  $|0\rangle$  and the other to  $|1\rangle$ .

(c) In fibers with normalized frequency allowing for  $LP_{01}$  and  $LP_{11}$  modes, a qubit may be associated to the two distinct  $LP_{11}$  modes, without distinguishing polarization states (Fig.1). Counting the polarizations one has four degrees of freedom associated to the  $LP_{11}$  mode, which allows for the coding of two qubits and the implementation of a two-qubit gate in a single fiber (see below).

These codings are the simplest ones for optical fiber implementations. Notice however that the reliability of fiber optics techniques allows for reliable

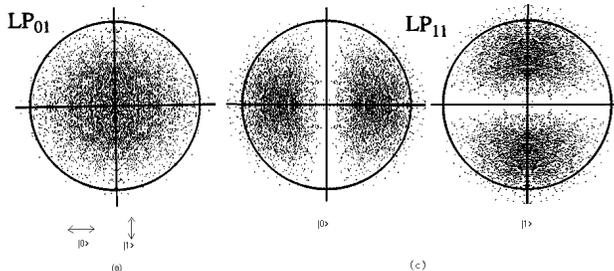


Figure 1: Two distinct qubit coding choices

manipulation and separation of many other modes. For example for a fiber with normalized frequency  $5.5201 < \frac{2\pi r_{co}}{\lambda} \sqrt{n_{co}^2 - n_{cl}^2} < 6.3802$ , modes up to  $LP_{12}$  may be excited, allowing for 40 different quantumlike degrees of freedom in a single fiber. ( $r_{co}, n_{co}, n_{cl}$  denote the core radius and the index of refraction of core and cladding)

### 2.3 One-qubit gates

Universal quantum computation requires one-qubit gates performing arbitrary unitary transformation and, at least, a two-qubit gate performing a unitary transformation in the four-dimensional tensor space which, together with the one-qubit transformations, generates the unitary group in four dimensions. For the one-qubit gates, two schemes seem appropriate:

#### (1) Polarization coding

Isotropic single-mode fibers support two degenerate polarization modes which propagate with the same constants  $\beta_i = k_0 n_i$ . However it is relatively easy to make the fibers to behave as linearly birefringent or circularly birefringent media [22] [23]. The birefringence of the fiber is conventionally characterized by a Jones matrix  $J$  [24] [25] [26] which defines the amount of transformation of the phase induced by the difference  $\Delta\beta_L = \beta_x - \beta_y$  (or  $\Delta\beta_C = \beta_R - \beta_L$  for circular polarization). Because a global phase associated to  $k_0 \int^z n_0(\zeta) d\zeta$  is already taken into account, the Jones matrix is what defines the phase rotation of  $\psi(x, y, z)$  in Eq.(9). For linear birefringence the Jones matrix relating the output and input phase of a fiber of length  $L$ , is

$$J_L(\Delta\beta_L) = \begin{pmatrix} e^{iL\Delta\beta_L/2} & 0 \\ 0 & e^{-iL\Delta\beta_L/2} \end{pmatrix} \quad (13)$$

In this expression it is assumed that the fast axis, that is, the one with the largest  $\beta$ , is in the  $x$ -direction. If the fast axis is at an angle  $\theta$  relative to the  $x$ -direction the Jones matrix would be

$$J_L(\Delta\beta_L, \theta) = \begin{pmatrix} \cos \frac{L\Delta\beta_L}{2} + i \cos 2\theta \sin \frac{L\Delta\beta_L}{2} & i \sin 2\theta \sin \frac{L\Delta\beta_L}{2} \\ i \sin 2\theta \sin \frac{L\Delta\beta_L}{2} & \cos \frac{L\Delta\beta_L}{2} - i \cos 2\theta \sin \frac{L\Delta\beta_L}{2} \end{pmatrix} \quad (14)$$

Any  $U(2)$  matrix may be decomposed into

$$U(\alpha, \theta, \beta) = \begin{pmatrix} e^{i\alpha/2} & 0 \\ 0 & e^{-i\alpha/2} \end{pmatrix} \begin{pmatrix} \cos \frac{\theta}{2} & i \sin \frac{\theta}{2} \\ i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} e^{i\beta/2} & 0 \\ 0 & e^{-i\beta/2} \end{pmatrix} \quad (15)$$

hence it follows from (13) and (14) that any  $U(2)$  transformation may be obtained on linearly birefringent fibers.

Linear birefringence is easily obtained by elliptical cores, lateral stress, bending or application of an electrical field. For fixed one qubit gates the most robust method is probably the use of cooling induced stress [27]. Circular birefringence is obtained by geometrical twisting (spun fibers) or axial magnetic fields (Faraday rotation). Adjustment of the intensity of these properties by the variation of applied electromagnetic fields is a potentially useful feature for the construction of programmable modules.

Engineering the birefringency properties is a very flexible way to obtain one qubit gates using single mode double-polarization fibers. For example, in the cases above one has assumed that the fast axis is fixed along the fiber segment. If instead one has a continuously rotating fast axis, a more complex Jones matrix is obtained

$$J_L(\Delta\beta_L, \xi) = \begin{pmatrix} \cos \frac{\delta}{2} + i \frac{L\Delta\beta_L}{\delta} \sin \frac{\delta}{2} & \frac{L\xi}{\delta} \sin \frac{\delta}{2} \\ -\frac{L\xi}{\delta} \sin \frac{\delta}{2} & \cos \frac{\delta}{2} - i \frac{L\Delta\beta_L}{\delta} \sin \frac{\delta}{2} \end{pmatrix}$$

with  $\delta = \sqrt{(L\Delta\beta_L)^2 + 4(L\xi)^2}$  and  $\xi = \frac{d\theta}{dz}$  the constant rate of rotation of the fast axis along the  $z$ -coordinate.

Also, for a simple circularly birefringent fiber the Jones matrix is

$$J_L(\Delta\beta_C) = \begin{pmatrix} \cos \frac{L\Delta\beta_C}{2} & \sin \frac{L\Delta\beta_C}{2} \\ \sin \frac{L\Delta\beta_C}{2} & \cos \frac{L\Delta\beta_C}{2} \end{pmatrix}$$

and for a fiber that is both linearly and circularly birefringent (for example a linearly birefringent spun fiber or a linearly birefringent one with an axial

magnetic field) the Jones matrix is

$$J(\Delta\beta_C, \alpha) = \begin{pmatrix} \cos \frac{L\Delta\beta_C}{2} - i\frac{1-\alpha^2}{1+\alpha^2} \sin \frac{L\Delta\beta_C}{2} & \frac{2\alpha}{1+\alpha^2} \sin \frac{L\Delta\beta_C}{2} \\ -\frac{2\alpha}{1+\alpha^2} \sin \frac{L\Delta\beta_C}{2} & \cos \frac{L\Delta\beta_C}{2} + i\frac{1-\alpha^2}{1+\alpha^2} \sin \frac{L\Delta\beta_C}{2} \end{pmatrix}$$

with  $\alpha = \frac{2\gamma}{n_x^2 - n_y^2 + \sqrt{(n_x^2 - n_y^2)^2 + 4\gamma^2}}$  and  $\gamma$  being the nondiagonal term in the relative

dielectric constant tensor  $\begin{pmatrix} n_x^2 & i\gamma & 0 \\ -i\gamma & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{pmatrix}$ .

Linear and circular birefringence allow for the implementation of any  $U(2)$  transformation in the polarization-encoded qubits. Preparation and measurement of the polarization-encoded qubits is obtained by polarizing fibers and polarizing beam-splitters.

## (2) $LP_{11}$ coding

For a fiber with a parabolic index profile, the  $LP_{11}$  modes may be approximated by the first harmonic excitations along the  $x$  and  $y$  directions. Denoting by  $a^\dagger$  and  $b^\dagger$  the corresponding creation operators, one has the following correspondence

$$|0\rangle \leftrightarrow LP_{01}; (a^\dagger |0\rangle, b^\dagger |0\rangle) \leftrightarrow LP_{11} \quad (16)$$

The  $SU(2)$  group, operating irreducibly in the 2-dimensional space  $(a^\dagger |0\rangle, b^\dagger |0\rangle)$ , is the following subgroup of the Weyl-symplectic group in 2-dimensions

$$\begin{aligned} J_+ &= a^\dagger b \\ J_- &= b^\dagger a \\ J_3 &= \frac{1}{2} (a^\dagger a - b^\dagger b) \end{aligned} \quad (17)$$

As explained in Ref.[3] and as follows from Eq.(10) in Sect. 2.1, changing the index profile along  $x$  and  $y$  as well as the coefficient of the Laplacian one has access to all generators of the two-dimensional Weyl-symplectic group and in particular to those of the  $SU(2)$  subgroup. Therefore, by engineering the index profile, all unitary rotations may be implemented on the  $LP_{11}$ -encoded qubits.

Requiring a precise adjustment of the index profile, an unitary manipulation of the  $LP_{11}$ -encoded qubits is more complex than the corresponding operation on polarization-encoded qubits. Therefore this encoding might be only recommended for control qubits.

In the quantumlike scheme one deals not with single photon events, but with steady-state beams. Therefore conversion between the two encodings is relatively easy using standard optical techniques.

## 2.4 Two-qubit gates

To obtain universal computation, in addition to one-qubit gates performing arbitrary unitary transformations, one needs at least one entangling gate. This is a gate that, together with one-qubit gates, generates all  $U(4)$  transformations. The CNOT, CS (controlled sign) or CP (controlled phase) gates are such gates, but there are many others (Appendix A).

### 2.4.1 A two-qubit controlled gate using $LP_{11}$ coding

Here one shows how to obtain a controlled (entangling) gate using the two qubit codings discussed before. On a fiber carrying  $LP_{11}$  modes, the  $LP_{11}$  mode has four degrees of freedom, two of them associated to the two possible orientations of the mode (see Fig.1) and the other two to the polarization. Let the two orientations of the  $LP_{11}$  mode code the control qubit and the polarization code the target qubit. For later convenience the codes for the  $|1\rangle$  and  $|0\rangle$  qubits will be  $V, H$  (vertical, horizontal) for the polarizations (target) and  $a, b$  for the positions (control) of the  $LP_{11}$  modes. If the fiber is constructed in such a way that the  $|1\rangle$  sectors in the  $LP_{11}$  mode are linearly birefringent and the  $|0\rangle$  sectors are isotropic (see Fig.2), a phase gate is obtained corresponding to the matrix

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix} \quad (18)$$

in the basis ( $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ ), the first entry being the control qubit and the second the target qubit.  $\theta$  is the additional phase that the  $|1\rangle_t$  target qubit obtains in the  $|1\rangle_c$  sector of the control qubit. In all cases there is a global phase that should be taken into account arising from the  $z$ -propagation in the gate. With different choices of the birefringence distribution other entangling  $U(4)$  matrices may be obtained.

Suppose that at the input of the gate the beam is a superposition of the  $LP_{11}$  modes polarized on the  $x, y$  plane ( $\alpha_1 |0\rangle_t + \alpha_2 |1\rangle_t$ ) and that it is the

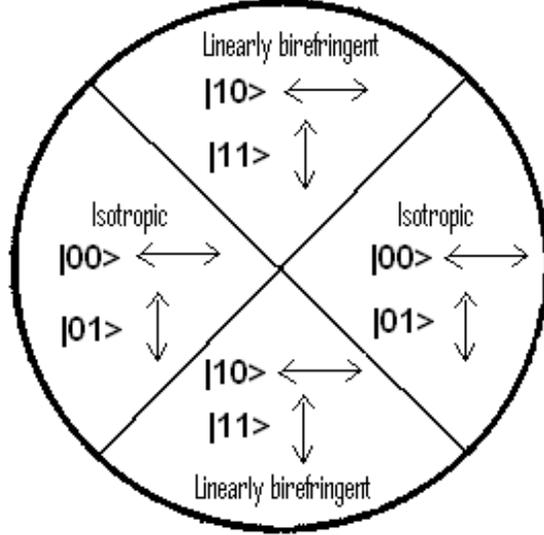


Figure 2: Coding of a controlled gate, using  $LP_{11}$  modes for the control qubit and polarization for the target qubit

position (control) mode  $a$  that is active. Then in the sector  $a$  of the fiber the output is

$$|1\rangle_c \otimes (\alpha_1 |0\rangle_t + \alpha_2 e^{i\theta} |1\rangle_t) = \alpha_1 |10\rangle + \alpha_2 e^{i\theta} |11\rangle, \quad (19)$$

whereas in the  $b$  sector the target qubit is unchanged. That is, the degrees of freedom of the beam are entangled.

The nature of this entanglement<sup>1</sup> is what has been called *local entanglement* in the sense that it refers to the degrees of freedom carried by the same physical entity. For a more general control qubit  $(\beta_1 |0\rangle_c + \beta_2 |1\rangle_c)$  one has

$$\beta_1 |0\rangle_c \otimes (\alpha_1 |0\rangle_t + \alpha_2 |1\rangle_t) + \beta_2 |1\rangle_c \otimes (\alpha_1 |0\rangle_t + \alpha_2 e^{i\theta} |1\rangle_t) \quad (20)$$

which would be faithfully implemented in the  $LP_{11}$  gate. The target qubit changes but only in the sector  $a$  of the gate.

The usual statement that entangling two-qubit gates requires a nonlinear effect, actually refers to the tensor product in (20), which here is obtained by

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<sup>1</sup>Some authors have claimed that the notion of entanglement should include other features in addition to non-separability. Here entanglement is simply used in the sense of non-separability.

local entanglement. The local entanglement, here associated to the sharing of degrees of freedom by the same physical system, is, after all, not so very different from the *nonlocal entanglement* in quantum mechanics. In quantum mechanics two photons may become entangled if they have interacted in the past, in general because they were produced by a common source<sup>2</sup>. They then share a common wavefunction and, in this sense they are also parts of the same physical system. They only become independent entities if the wavefunction decoheres, and then entanglement is gone. So local and nonlocal entanglement are not so very different as it might seem. On this optical entanglement of the beam degrees of freedom there is another parallel with quantum mechanics. In quantum mechanics the more noteworthy feature of entanglement is the fact that correlation between the photons remains if at a later time they are well separated in space. Here the role of time is played by the longitudinal  $z$ -coordinate of the fiber and the entanglement that occurs in the gate may be observed at a later  $z$ . This, of course, if noise or the fiber imperfections do not destroy space coherence. Like in quantum mechanics. In short, entanglement requires interaction and remembrance of the interaction effects along the propagation path.

In some quantum computing applications, for example in quantum Fourier transform (QFT) as will be seen later, the full entangled output of the phase gates is not used. Instead, in each line of the output of the QFT one would want to find

$$\beta_1 (\alpha_1 |0\rangle_t + \alpha_2 |1\rangle_t) + \beta_2 (\alpha_1 |0\rangle_t + \alpha_2 e^{i\theta} |1\rangle_t),$$

that is, a partial trace over the control qubit is effectively done.

If instead of linearly birefringency the  $|1\bullet\rangle$  region is circularly birefringent, also entangling gates may be constructed. Here the two-bit gate is based on the four degrees of freedom of the  $LP_{11}$  modes of a circular fiber. A similar construction might be done using the  $TE, TM - 12$  modes of a rectangular fiber. Modern fiber optics technology is also able to handle multimode fibers which would provide entangling gates for many more qubits.

Instead of a single fiber carrying  $LP_{11}$  modes, one may use two fibers (or light wave guides on a chip) one for the control position code  $a$  and the other for the code  $b$ . Each one of the light guides might carry the full polarization information or the  $a$ -fiber might only contain the vertical ( $V$ ) component

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<sup>2</sup>They may also be entangled by *entanglement swapping* which involves measurement, a nonlinear operation.

and the  $b$ -fiber the horizontal ( $H$ ) component. For future reference all these equivalent possibilities will be denoted as a  $G$ -gate.

Depending on its position on the quantum circuits, qubits may play the role of target or control qubits. Therefore to each qubit one associates two synchronous wave guides, to carry both position and polarization information. While one of the lines carries optically the full polarization, the other might well be electrical, with the interaction of polarization ( $V, H$ ) and position ( $a, b$ ) modes carried out by optical or electro-optical means. Notice also that conversion of polarization to position and vice versa is easily obtained by polarizing beam splitters and polarization preserving fibers. The main challenge in this dual coding scheme is to preserve linearity in the gate. In a controlled phase gate only the  $b$ -line needs to enter the gate, the polarization coming from the target line being established in this line which is then passed through the appropriate retarder.

#### 2.4.2 Two-qubit gates with polarization coding

A different alternative for the construction of two-qubit gates would be to use only one type of coding, for example polarization coding. In this case the tensor product of control and target qubits is not achieved by the coupling position-polarization, but it requires an interaction between the two polarized beams, which only occurs through interaction with an optical active medium. Fig.3 sketches the required mechanism. After being split by a polarizing beam splitter (PBS) the  $V$  component of the target beam is further split by another unit (controlled beam splitter, CBS) that is controlled by the  $V$  component of the control beam. One of the branches is then passed through a phase retarder ( $\theta$ ) to implement the controlled phase 2-qubit gate. This implements the operation in Eq.(21). The essential element is the controlled beam splitter (CBS) which can be achieved by a dynamical holography mechanism. A grating, dynamically created on a material by interaction of the control and a reference beam, splits the target beam. Optically and electro-optically controlled beam splitters have been discussed and constructed before (see for example [28] - [32] and references in [33],[34]). However they operate mostly in an ON-OFF regime and here, as seen in Eq.21, one needs linear operation. In Appendix B, the basic theory of one such device is discussed as well as the requirements and challenges faced to obtain linear operation.

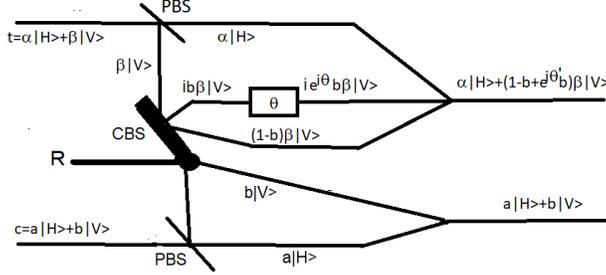


Figure 3: Optical two qubit phase gate with polarization coding. PBS = polarizing beam splitter; CBS = controlled beam splitter; R = reference beam.  $\theta' = \theta + \frac{\pi}{2}$

$$\left. \begin{array}{l} \alpha_1 |H\rangle_t + \alpha_2 |V\rangle_t \\ \beta_1 |H\rangle_c + \beta_2 |V\rangle_c \end{array} \right\} \rightarrow \beta_1 \alpha_1 |H_c H_t\rangle \oplus \beta_1 \alpha_2 |H_c V_t\rangle \oplus \beta_2 \alpha_1 |V_c H_t\rangle \oplus \beta_2 \alpha_2 e^{i\theta} |V_c V_t\rangle \quad (21)$$

## 2.5 On the physical implementation of the gates

As a general remark on the optical implementation of the operations of quantum-like computing, it should be pointed out that one is in a more favorable position than in the usual one-photon quantum computing implementation. Here one deals with light beams and therefore nonlinear effects are much easier to obtain. Furthermore one deals not with a transient temporal phenomenon, but with the establishment, in an optical network, of a steady state phenomenon. The initial state at the input of the quantum-like circuit must be established by a coherent source which also acts as a reference beam at other points of the circuit. The role of time being played by a particular space coordinate, all the interference and gate operations are performed until a steady state configuration is established in the network, the final result of the calculation being read-off at some well defined coordinate.

This also means that, as long as all superposition and interference phenomena are implemented by optical waves, some intermediate gate operations might be performed by electro-optical means. For example in a controlled phase gate the amplitude and phase of the vertical polarizations of control

and target beams may be measured by heterodyning with the reference beam and then, with the result of the gate operation computed by electronic means, the same reference beam might by the appropriate retarders generate the optical output beams. Also at intermediate points of the network the signals may even be split, examined or amplified as long as the phase is preserved or the phase change is duly taken into account. Of course all-optical operation of the gates and of the whole circuit is desirable and a goal to be achieved.

There is, in these intermediate measurements, no conflict with the no cloning theorem of quantum information. In the usual proof of the no cloning theorem, one assumes that an unitary operator  $U$  exists such that  $U|\psi0\rangle = |\psi\psi\rangle$  for all  $\psi$  and then, by applying  $U$  to  $\gamma = \alpha\phi_1 + \beta\phi_2$  obtain  $U|\gamma0\rangle = \alpha|\phi_1\phi_1\rangle + \beta|\phi_2\phi_2\rangle \neq |\gamma\gamma\rangle$ , a contradiction. No cloning means that, given an unknown quantum state, no measurement can find out what was exactly its wave function before the measurement. By contrast given a beam of light one can split it in a polarization basis by a polarizing beam splitter and then by heterodyning it with a coherent reference beam find the amplitude and phase of each one of the components. Given that knowledge, and because the phase is defined module  $2\pi$ , the beam may then be synchronously reproduced.

In conclusion: the possibility to measure and then reproduce the quantum-like signal, means that it will not be appropriate for cryptography purposes. However, because it may have interference, parallelism, (local) entanglement and unitary propagation along a (computing) coordinate, it may be used for computation purposes.

## 2.6 Nonlinear gates

In the previous subsections the emphasis has been on linear gates, because they are the ones most useful for computation purposes. However quantum technology is not only quantum computing and nonlinear quantum (or quantum-like) effects are also of interest. The electric field associated to a single photon is very weak. This poses a major problem for all-optical quantum operations using single photons, because significant, medium-mediated, nonlinear interactions would be required between two photons. A very strong cooperative effect of atoms would be required to perform interaction of single-photon signals. The Kerr effect at the one photon level might be enhanced by choosing frequencies near resonances of the material, but then appreciable loss effects would be expected.

In the optical quantum-like approach the signals, being coded not with

single photons but with light beams, nonlinear effects are much easier to obtain. In particular, a great development has already been achieved with nonlinear effects for switching purposes in classical all-optical networks. Directional couplers are used as optical switches, as power dividers or combiners, multiplexers, demultiplexers and intensity modulators. On-off logic gates based on the Kerr effect have also been proposed by several authors.

First studied by Jensen [35] the nonlinear directional coupler is a robust device exploring the Kerr effect. In spite of its nonlinear nature, by exploring the role of constants of motion, an analytic solution may be obtained for the input-output transfer function of the device [36]. Therefore a precise quantitative control of the transfer function is obtained. For the reader convenience, the main equations and parameters of the coupler are summarized in the Appendix C. Denoting by  $\vec{E}^{(1)}(0)$ ,  $\vec{E}^{(2)}(0)$ ,  $\vec{E}^{(1)}(L)$ ,  $\vec{E}^{(2)}(L)$  the transversal electric fields at the input and output of the two ports of a coupler (1 and 2) of length  $L$ , one has a transfer function

$$\begin{aligned}
 E_j^{(1)}(L) &= \frac{1}{2} \left\{ \begin{aligned} &\left( e^{iL\beta^{-(+)}} M^{(+)} + e^{iL\beta^{-(-)}} M^{(-)} \right)_{jk} E_k^{(1)}(0) \\ &+ \left( e^{iL\beta^{-(+)}} M^{(+)} - e^{iL\beta^{-(-)}} M^{(-)} \right)_{jk} E_k^{(2)}(0) \end{aligned} \right\} \\
 E_j^{(2)}(L) &= \frac{1}{2} \left\{ \begin{aligned} &\left( e^{iL\beta^{-(+)}} M^{(+)} - e^{iL\beta^{-(-)}} M^{(-)} \right)_{jk} E_k^{(1)}(0) \\ &+ \left( e^{iL\beta^{-(+)}} M^{(+)} + e^{iL\beta^{-(-)}} M^{(-)} \right)_{jk} E_k^{(2)}(0) \end{aligned} \right\}
 \end{aligned} \tag{22}$$

where the matrices  $M^{(+)}$ ,  $M^{(-)}$  and the propagation factors  $\beta^{(+)}$ ,  $\beta^{(-)}$  associated to the symmetric and asymmetric modes are completely specified by the material parameters of the coupler (Eqs. 57, 58). Through the constants of motion they have a nonlinear dependence on the coupler medium and on the intensity of the beams. Of course in the linear case  $M^{(+)}$  and  $M^{(-)}$  are unit matrices.

For practical purposes one should notice that propagating through the coupler each beam suffers changes of phase and polarization rotations due both to itself and to the signal in the other beam, this latter action being the one that is more relevant for the computational effect of the device. Many different nonlinear operations may be obtained by the appropriate choice of the parameters.

### 3 Quantum modules

#### 3.1 Quantumlike Fourier transform with 2-qubit optical gates

A very important element in the quantum algorithms is the quantum Fourier transform (QFT). For  $n$  qubits and  $N = 2^n$  the QFT is

$$y_k = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} x_l e^{i2\pi lk/N}, \quad (23)$$

the  $N$  number sets  $\{y\}$  and  $\{x\}$  being coded by the  $n$  qubits as follows

$$x = (j_1, j_2, \dots, j_n) = j_1 2^{n-1} + j_2 2^{n-2} + \dots + j_n 2^0 \quad (24)$$

The QFT may be looked at as an unitary transformation in the computational basis of  $n$  qubits, implementing the transformation [37]

$$\begin{aligned} & |j_1 j_2 \dots j_n\rangle \\ \rightarrow & \frac{1}{2^{n/2}} \left\{ \left( |0\rangle + e^{i2\pi \frac{j_1}{2}} |1\rangle \right) \left( |0\rangle + e^{i2\pi \left( \frac{j_1-1}{2} + \frac{j_2}{4} \right)} |1\rangle \right) \dots \left( |0\rangle + e^{i2\pi \left( \frac{j_1}{2} + \frac{j_2}{4} + \dots + \frac{j_n}{2^n} \right)} |1\rangle \right) \right\} \end{aligned} \quad (25)$$

This decomposition of the QFT leads directly to the quantum circuit (for 4 qubits) in Fig.4 where  $H$  and  $R_k$  are the Hadamard and the controlled phase gates

$$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}; R_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\pi/2^k} \end{pmatrix} \quad (26)$$

This circuit has  $n(n+1)/2$  gates which, exploring the non-conflicting simultaneous application of the gates, may be implemented in  $O(2n)$  steps. There are however more efficient wirings [38] - [40].

Griffiths and Niu [41] have proposed a semiclassical approach to the quantum Fourier transform. It is semiclassical in the sense that it requires a measurements of the output qubits to obtain a signal to control the gates. In the time evolution approach to quantum computing this scheme would only be applicable when the QFT is the final step in the quantum circuit. However in the quantum-like approach because, as discussed before, measured beams may be fully restored, the Griffiths and Niu configuration may be used at any point in the circuit.

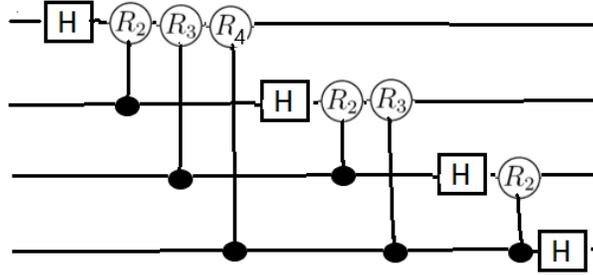


Figure 4: A  $O(2n)$  quantum Fourier transform circuit for 4 qubits

When using a single coding in the optical gates, for example polarization coding, the QFT circuits for quantum-like computation would be identical to the classical ones. However, when the  $LP_{11}$  coding scheme (with one or two wave guides) is used, the configuration might be slightly different. Fig.5 displays one such implementation. In each input, except the first, the input qubits are duplicated, assigned both to the polarization modes of single mode ( $LP_{01}$ ) fiber and to position  $LP_{11}$  modes. The  $H$ -modules are Hadamard gates implemented by one-qubit gates with  $LP_{01}$  modes polarization. Both the polarization ( $V, H$ ) and the position ( $a, b$ ) information are fed to the gate. There the  $a, b$  information and the polarization ( $V, H$ ) are used to generate a polarized  $LP_{11}$  signal which is fed to a partially birefringent fiber, which implements a two-qubit phase gate, as described in Section 2.4.1. Notice that whereas the polarization information is naturally carried in a  $LP_{01}$  mode (the fine lines in Fig.5) the position information (the thick lines) for the  $LP_{11}$  mode may be carried to the gate electronically or by a  $LP_{11}$  fiber, whatever is more convenient. At the end of the polarized  $LP_{11}$  fiber in the gate, the output polarization is obtained by merging the  $a$  and  $b$  modes into a  $LP_{01}$  polarized mode. All gates are identical, differing only on the length of the birefringent  $LP_{11}$  fiber section.

### 3.2 Optical gateless quantum-like Fourier transforms

The form of the quantum Fourier transform (Eq.23) is formally identical to the classical discrete Fourier transform (DFT). In this sense, what the QFT does is a DFT on the amplitudes of the quantum state. On the other hand

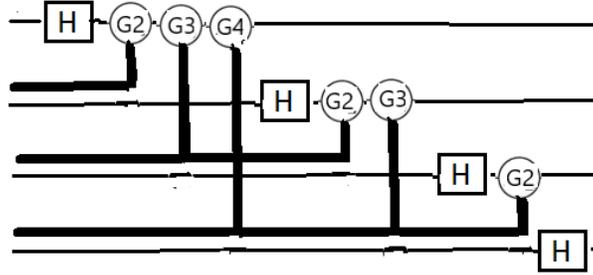


Figure 5: Quantum-like Fourier transform using  $LP_{11}$  coding

it is known that the Fourier transform may be obtained from a light front, representing the function, by observation of the far field (or the focused far field) at several angles. This led to several optical proposals for the DFT by, for example, passing a coherent light through a zero or  $\pi$  phase mask and observing the far field in the focus plane of a lens. These purely optical approaches that have also been proposed [42] - [45] for the QFT with single photons may, even more easily, be adapted to the light beam quantum-like approach.

### 3.3 Oracles

Oracles [46] [47] are functions

$$f : (0, 1)^m \rightarrow (0, 1)^n$$

which, typically, are needed both for the preparation of the input signal to the quantum circuit and for queries about the final state. In terms of a polarization coding of beams in the quantum-like approach, these are functions

$$f : (H, V)^m \rightarrow (H, V)^n$$

Such functions may be implemented by linear couplers, beam splitters, interferometers, phase rotaters and the two-qubit gates discussed before. It is desirable to use electro-optical control in these units to have programmable flexibility of the oracles.

## 4 Conclusions

1) In this paper (and in [3]) by identifying a Schrödinger-like evolution along a space coordinate of a classical system, we have concluded that quantum computation might be carried out both by quantum systems evolving in time and by a classical wave system evolving along a space coordinate. This steals the primacy of quantum systems to execute quantum computing operations. Even more, one might say that quantum computing is more general than quantum mechanics or simply that in quantum mechanics Nature is doing quantum computing along the time direction.

2) There is, of course, a difference in these two modalities of quantum computing due to the particular nature of our observer status in the universe for which, to look at a timeline (at a particular space) has properties distinct from looking at a spaceline (at a particular time). When looking at a timeline, after the operation the same time is no longer there, in contrast with the timely permanence of a spaceline. As a result if a measurement is made with a projection filter in the space evolution, the same results are obtained as in quantum mechanics, but on the other hand there are alternative ways to observe which give complete access to the value of the wave function.

3) The optical implementations of the one and two qubit gates in this paper have been kept as simple as possible, using only  $LP_{01}$  and  $LP_{11}$  modes. However with the growing sophistication on handling multimode fibers it is conceivable that, using this optical quantum-like approach, it will be possible to obtain high degrees of circuit compactness and parallelism. Of particular interest for the development of interesting quantum-like devices are the recent technological advances in space light modulators (SLM) [48] [49].

4) The current and potential applications of quantum technology are not restricted to quantum computing, other promising uses are in fields of control and communications. Whereas it seems that in quantum computing the linear gates are the most useful, nonlinear gates are expected to be potentially useful in other applications. This was the main motivation to discuss at some length in section 2 and in the appendix C the analytical aspects of the nonlinear circuits.

5) As stated before, there are, in addition to light waves, other systems which display quantum-like behavior when its evolution along a space coordinate is observed. Not all of them will be as appropriate as light to perform computations, in particular because of the need to maintain coherence in the evolution. Nevertheless a case that might deserve some attention is the case

of spin waves [50] [51].

## 5 Appendix A. Entangling two-qubit gates

It is known [52] that arbitrary one-qubit gates together with a two-qubit CNOT are capable of universal quantum computation. It follows that, more generally, any two-qubit gate capable of generating, together with the one-qubit gates, the full  $U(4)$  group would also be universal. Such two-qubit gates have been called *entangling* (or imprimitive) gates, because they map decomposable states into indecomposable ones. A gate that is not entangling is called *primitive* [53].

Let  $e_{ij}$  be a  $4 \times 4$  matrix with elements

$$(e_{ij})_{mn} = \delta_{im}\delta_{jn} \quad (27)$$

Then, the 16 Lie algebra generators of  $U(4)$  are

$$\begin{aligned} I_{ij} &= i(e_{ij} - e_{ji}) \\ J_{ij} &= e_{ij} + e_{ji} \quad i \neq j \\ e_{ii} & \end{aligned} \quad (28)$$

They are related to the Lie algebra generators of  $U(2) \otimes U(2)$  by

$$\sigma_\mu \otimes \sigma_\nu = \begin{pmatrix} \sum_i e_{ii} & J_{12} + J_{34} & -I_{12} - I_{34} & e_{11} - e_{22} + e_{33} - e_{44} \\ J_{13} + J_{24} & J_{14} + J_{23} & -I_{14} + I_{23} & J_{13} - J_{24} \\ -I_{13} - I_{24} & -I_{14} - I_{23} & J_{14} - J_{23} & -I_{13} + I_{24} \\ e_{11} + e_{22} - e_{33} - e_{44} & J_{12} - J_{34} & -I_{12} + I_{34} & e_{11} - e_{22} - e_{33} + e_{44} \end{pmatrix} \quad (29)$$

where  $\sigma_\mu = \{\sigma_0 \equiv \mathbf{1}, \sigma_1, \sigma_2, \sigma_3\}$  are the identity  $2 \times 2$  matrix and the Pauli matrices.

The elements in the first line and the first column of the matrix in (29), namely  $\mathbf{1} \otimes \sigma_\nu$  and  $\sigma_\mu \otimes \mathbf{1}$ , are the algebraic elements associated to one-qubit operations. The remaining 9 elements in (29) are of the form  $\sigma_i \otimes \sigma_j$  ( $i, j = 1, 2, 3$ ). From the commutators

$$\begin{aligned} [\mathbf{1} \otimes \sigma_i, \sigma_a \otimes \sigma_b] &= \sigma_a \otimes [\sigma_i, \sigma_b] \\ [\sigma_i \otimes \mathbf{1}, \sigma_a \otimes \sigma_b] &= [\sigma_i, \sigma_a] \otimes \sigma_b \end{aligned} \quad (30)$$

it follows that, given any one of the 9 elements  $\sigma_i \otimes \sigma_j$  it is possible to generate the full  $U(4)$  algebra by commutation with the (one-qubit) generators  $\mathbf{1} \otimes \sigma_\nu$

and  $\sigma_\mu \otimes \mathbf{1}$ . These 9 elements are therefore a basis for the imprimitive (entangling) elements of the algebra. Linear combinations of these elements as well as linear combinations with one-qubit transformations are also entangling.

## 6 Appendix B. An optically controlled beam splitter

Many controllable beam splitters have been proposed in the past. They use either mechanical displacement of metasurfaces [29], electro-optical modulators and a Mach-Zehnder interferometer [30], optical bistability by surface plasmons [31], etc.

Optically controlled beam splitters have been discussed. For example [28] uses a grating made of polymer slices alternated with layers of aligned nematic liquid crystal. When the liquid crystal is aligned the input light beam is split into a transmitted and a refracted component, however when another pump beam is turned on, the liquid crystal suffers a nematic to isotropic phase transition, the refractive index contrast vanishes and the structure becomes transparent to the incoming light. Because fine-tuning of the index contrast seems difficult, this interesting device is mostly suited for an ON-OFF operation mode. The same applies to electro-optic operated liquid crystal devices [32].

The ON-OFF behavior of the controlled beam splitters is appropriate for digital communication purposes, but for analog or quantum-like computing applications a smoother, linear or quasi-linear, dependence on the control signal is desirable. Quantum-like applications are even more demanding because information on the phase of the control signal should be taken into account.

The propagation of a transversal electric field in a nonlinear media is described by the equation

$$\Delta \mathbb{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \mathbb{E}}{\partial t^2} = \mu_0 \frac{\partial^2 P_L}{\partial t^2} + \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \quad (31)$$

For the nonlinear contribution to the refraction index one considers either a Kerr or a photorefractive medium. Let

$$\mathbb{E} = \vec{E}(x, z) e^{i\omega t} \quad (32)$$

$(x, z)$  being the coordinates of the propagation plane of field. Consider a fixed thick sinusoidal grating along the  $x$  coordinate (Fig.6), and the propagation of a light wave on this grating, that is

$$\Delta \vec{E}(x, z) + \mu_0 \varepsilon_0 \omega^2 \vec{E}(x, z) = -\mu_0 \varepsilon_0 \chi^{(1)} \omega^2 \vec{E}(x, z) - \mu_0 \varepsilon_0 \chi^{(NL)} \cos(Qx) \omega^2 \vec{E}(x, z) \quad (33)$$

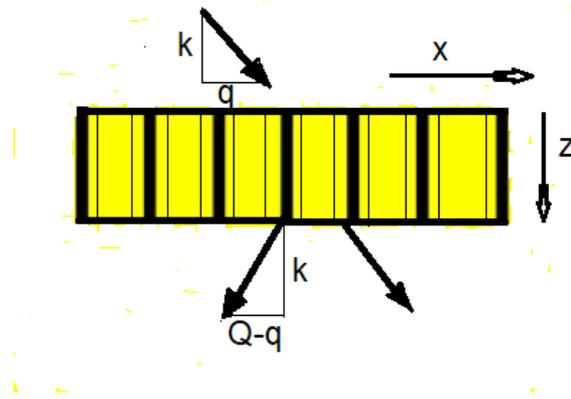


Figure 6: A thick grating

Passing to the Fourier transform on the  $x$  coordinate and writing

$$\vec{E}(q, z) = \vec{\psi}(q, z) e^{ikz} \quad (34)$$

with  $\vec{\psi}(q, z)$  having a slow variation on  $z$ , one obtains

$$(-k^2 - q^2) \vec{\psi}(q, z) + 2ik \partial_z \vec{\psi}(q, z) + \alpha \vec{\psi}(q, z) + \frac{\beta}{2} (\vec{\psi}(Q + q, z) + \vec{\psi}(Q - q, z)) \simeq 0 \quad (35)$$

with

$$\begin{aligned} \alpha &= \mu_0 \varepsilon_0 \chi^{(1)} \omega^2 \\ \beta &= \mu_0 \varepsilon_0 \chi^{(NL)} \omega^2 \end{aligned} \quad (36)$$

being the linear and nonlinear refractive coefficients. In Eq.(35) one has neglected the term  $\partial_z^2 \vec{\psi}$ .

With  $q$  and  $Q > 0$ , constructive interference of the diffractive components in the thick grating slab requires

$$\begin{aligned} k^2 + q^2 &= \alpha \\ k^2 + (Q - q)^2 &= \alpha \end{aligned} \quad (37)$$

When only the 0th and the 1st diffraction orders are non-evanescent,  $Q = 2q$  and one has the following equations for the transmitted and diffracted components

$$\begin{aligned} 2ik \frac{\partial \vec{\psi}(q, z)}{\partial z} &= -\frac{\beta}{2} \vec{\psi}(Q - q, z) \\ 2ik \frac{\partial \vec{\psi}(Q - q, z)}{\partial z} &= -\frac{\beta}{2} \vec{\psi}(q, z) \end{aligned} \quad (38)$$

with solution

$$\begin{aligned} \vec{\psi}(q, z) &= \vec{\psi}(q, 0) \cos\left(\frac{\beta}{4k}z\right) \\ \vec{\psi}(Q - q, z) &= i \vec{\psi}(q, 0) \sin\left(\frac{\beta}{4k}z\right) \end{aligned} \quad (39)$$

In conclusion: the amount of splitting of the beam by the thick grating is controlled by the nonlinear contribution to the refractive index.

Now, to have the splitting of the beam controlled by another light beam, the grating should not be fixed but created by the intensity of the other beam. Because one wants to have the tuning to be also a function of the phase, consider 3 light beams (target, control and reference,  $t, c, R$ )

$$\begin{aligned} \vec{E}_t &= |\vec{E}_t| \vec{e}_t e^{i(\omega t - \vec{k}_t \cdot \vec{x} + \theta_t)} \\ \vec{E}_c &= |\vec{E}_c| \vec{e}_c e^{i(\omega t - \vec{k}_c \cdot \vec{x} + \theta_c)} \\ \vec{E}_R &= |\vec{E}_R| \vec{e}_R e^{i(\omega t - \vec{k}_R \cdot \vec{x} + \theta_R)} \end{aligned} \quad (40)$$

The intensity of the sum of the three signals is

$$\begin{aligned} \left| \vec{E}_t + \vec{E}_c + \vec{E}_R \right|^2 &= |\vec{E}_t|^2 + |\vec{E}_c|^2 + |\vec{E}_R|^2 \\ &+ \vec{e}_t \cdot \vec{e}_c |\vec{E}_t| |\vec{E}_c| \left( e^{i(\theta_c - \theta_t + (\vec{k}_t - \vec{k}_c) \cdot \vec{x})} + c.c. \right) \\ &+ \vec{e}_t \cdot \vec{e}_R |\vec{E}_t| |\vec{E}_R| \left( e^{i(\theta_R - \theta_t + (\vec{k}_R - \vec{k}_t) \cdot \vec{x})} + c.c. \right) \\ &+ \vec{e}_c \cdot \vec{e}_R |\vec{E}_c| |\vec{E}_R| \left( e^{i(\theta_R - \theta_c + (\vec{k}_R - \vec{k}_c) \cdot \vec{x})} + c.c. \right) \end{aligned} \quad (41)$$

With linearly polarized signals it is always possible to have

$$\begin{aligned}\vec{\epsilon}_t \cdot \vec{\epsilon}_c &= \vec{\epsilon}_t \cdot \vec{\epsilon}_R = 0 \\ \vec{\epsilon}_c \cdot \vec{\epsilon}_R &\neq 0\end{aligned}\quad (42)$$

Then, only the last of the mixed terms is nonvanishing and,

$$\left| \vec{E}_t + \vec{E}_c + \vec{E}_R \right|^2 = \left| \vec{E}_t \right|^2 + \left| \vec{E}_c \right|^2 + \left| \vec{E}_R \right|^2 + \vec{\epsilon}_c \cdot \vec{\epsilon}_R \left| \vec{E}_R \right| \left| \vec{E}_c \right| \left( e^{i(\theta_c - \theta_R)} e^{i(\vec{k}_R - \vec{k}_c) \cdot \vec{x}} + c.c. \right) \quad (43)$$

By the Kerr effect or on a photorefractive material, one may use this intensity to create a grating along the  $\vec{k}_R - \vec{k}_c$  direction. This holographic-like pattern carries the information on the intensity and phase of the target signal, which with the choice (42) is not contaminated by the interaction with the target signal nor by the interaction of the reference beam with the target. The intensity of the reference beam, in general larger than the one of the other signals defines the amplitude of the grating effect.

For Kerr materials

$$P_{NL} = \chi^{(3)} \left| \vec{E}_t + \vec{E}_c + \vec{E}_R \right|^2 \left( \vec{E}_t + \vec{E}_c + \vec{E}_R \right) \quad (44)$$

therefore the  $\beta$  factor in Eq.(39) is proportional to  $\vec{\epsilon}_c \cdot \vec{\epsilon}_R \left| \vec{E}_c \right| e^{i\theta_c}$ , that is, the splitting of the target beam would be directly controlled by the control beam. For small  $\beta$  this action is approximately linear on the amplitude of the control, however deviations from linearity occur for large  $\beta$ . The situation might be improved by manipulation of the  $\vec{\epsilon}_c \cdot \vec{\epsilon}_R$  term, that is, making the control beam pass through a medium that rotates the polarization as a function of the intensity. Alternatively one might act on the intensity of the control beam by electro-optical means to obtain  $\beta = \sin^{-1} \left( \alpha \left| \vec{E}_c \right| \right)$ .

For photorefractive materials the change of the refractive index is proportional to the space derivative of the intensity (see for example [33] ch. 21.4) and the mechanism is quite similar.

## 7 Appendix C. Nonlinear directional couplers

Directional couplers are useful devices currently used in fiber optics communications. Because of the interaction between the two input fibers, power fed

into one fiber is transferred to the other. The amount of power transfer is controlled by the coupling constant, the interaction length or the phase mismatch between the inputs. If, in addition the material in the coupler region has strong nonlinearity properties, the power transfer will also depend on the intensities of the signals [35] [54]. A large number of interesting effects take place in nonlinear directional couplers [55] [56] [57] [58] with, in particular, the possibility of performing all classical logic operations by purely optical means [59].

Here one summarizes how, by exploring the constants of motion of the coupler equation, explicit analytical solutions are obtained for both the linear and nonlinear couplers, as used in Sect.2 for two-qubit gates. Further details may be found in Ref. [36].

Consider two linear optical fibers coming together into a coupler of non-linear material. The equation for the electric field is

$$\Delta E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_L}{\partial t^2} + \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}, \quad (45)$$

$P_L(r, t) = \varepsilon_0 \chi^{(1)} E(r, t)$  being the linear polarization of the medium,  $P_{NL}(r, t) = \varepsilon_0 \chi^{(3)} |E(r, t)|^2 E(r, t)$  the nonlinear polarization in the instantaneous non-linear response approximation and transversal dependence of  $\chi^{(1)}$  and  $\chi^{(3)}$  have been considered negligible.

Separating fast and slow (time) variations

$$\begin{aligned} E(r, t) &= \frac{1}{2} \{ \mathcal{E}(r, t) e^{-i\omega_0 t} + c.c. \} \\ P_{NL}(r, t) &= \frac{1}{2} \{ \mathcal{P}_{NL}(r, t) e^{-i\omega_0 t} + c.c. \} \end{aligned} \quad (46)$$

one obtains for the  $e^{-i\omega_0 t}$  part of a transversal mode

$$P_{NL_{1,2}}(r, t) = \frac{3\varepsilon_0}{8} \chi^{(3)} \left\{ e^{-i\omega_0 t} \left[ \left( |\mathcal{E}_{1,2}|^2 + \frac{2}{3} |\mathcal{E}_{2,1}|^2 \right) \mathcal{E}_{1,2} + \frac{1}{3} \mathcal{E}_{2,1} \mathcal{E}_{2,1} \mathcal{E}_{1,2}^* \right] + c.c. \right\} \quad (47)$$

the labels 1 and 2 denoting two orthogonal polarizations.

The dependence on transversal coordinates  $(x, y)$  is separated by considering

$$E_k(r, t) = g \Psi_k^{(i)}(x, y, z) e^{i\beta_i z} e^{-i\omega_0 t} \quad (48)$$

with  $\Psi_k^{(i)}(x, y, z)$  being an eigenmode of the coupler with slow variation along  $z$

$$\Delta_2 \Psi_k^{(i)} + \left( \frac{\omega_0^2}{c^2} (1 + \chi^{(1)}) - \beta^{(i)2} \right) \Psi_k^{(i)} = 0 \quad (49)$$

( $i$ ) denotes the mode index,  $k$  the polarization and  $\Delta_2 = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$ .

Neglecting  $\frac{\partial^2 \Psi^{(i)}}{\partial z^2}$  one obtains

$$2i\beta^{(i)} \frac{\partial \Psi_{1,2}^{(i)}}{\partial z} = -\frac{3\omega_0^2}{4c^2} \chi^{(3)} \left\{ \left( \left| \Psi_{1,2}^{(i)} \right|^2 + \frac{2}{3} \left| \Psi_{2,1}^{(i)} \right|^2 \right) \Psi_{1,2}^{(i)} + \frac{1}{3} \Psi_{2,1}^{(i)} \Psi_{2,1}^{(i)} \Psi_{1,2}^{(i)*} \right\} \quad (50)$$

In the directional couplers the propagating beams are made to overlap along one of the transversal coordinates ( $x$ ). Typically, in the overlap region of the directional coupler, the eigenmodes are symmetric (+) and antisymmetric (-) functions on  $x$ , the amplitudes in each fiber at the input and output of the coupler being recovered by

$$\begin{aligned} \Psi_k^{(1)} &= \frac{1}{2} \left( \Psi_k^{(+)} + \Psi_k^{(-)} \right) \\ \Psi_k^{(2)} &= \frac{1}{2} \left( \Psi_k^{(+)} - \Psi_k^{(-)} \right) \end{aligned} \quad (51)$$

An explicit analytic solution, also for the nonlinear coupler equation (50), may be obtained by noticing that it has two constants of motion

$$\begin{aligned} \frac{\partial}{\partial z} \left\{ \left| \Psi_1^{(i)} \right|^2 + \left| \Psi_2^{(i)} \right|^2 \right\} &= 0 \\ \frac{\partial}{\partial z} \left\{ \Psi_1^{(i)*} \Psi_2^{(i)} - \Psi_1^{(i)} \Psi_2^{(i)*} \right\} &= 0 \end{aligned} \quad (52)$$

Therefore, defining

$$\begin{aligned} \left| \Psi_1^{(i)} \right|^2 + \left| \Psi_2^{(i)} \right|^2 &= \alpha^{(i)} \\ \Psi_1^{(i)*} \Psi_2^{(i)} - \Psi_1^{(i)} \Psi_2^{(i)*} &= i\gamma^{(i)} \end{aligned} \quad (53)$$

one obtains for the electrical field of the eigenmodes

$$\begin{aligned} i \frac{\partial E_1^{(i)}}{\partial z} &= -\beta^{-(i)} E_1^{(i)} - ik E_2^{-(i)} \\ i \frac{\partial E_2^{(i)}}{\partial z} &= -\beta^{-(i)} E_2^{(i)} + ik E_1^{-(i)} \end{aligned} \quad (54)$$

with

$$\begin{aligned} \beta^{-(i)} &= \beta^{(i)} + \frac{3\omega_0^2}{8c^2} \frac{\chi^{(3)}}{\beta^{(i)}} \alpha^{(i)} \\ k^{-(i)} &= \frac{\omega_0^2}{8c^2} \frac{\chi^{(3)}}{\beta^{(i)}} \gamma^{(i)} \end{aligned} \quad (55)$$

Notice that, through  $\alpha^{(i)}$  and  $\gamma^{(i)}$ ,  $\beta^{-(i)}$  and  $k^{-(i)}$  depend on the material properties, on the geometry of the mode and also on its intensity. One may now obtain, for each eigenmode, the input-output relation of the nonlinear coupler

$$\begin{aligned} E_1^{(i)}(z) &= e^{i\beta^{-(i)}z} \left\{ E_1^{(i)}(0) \cos\left(\frac{-(i)}{k}z\right) - E_2^{(i)}(0) \sin\left(\frac{-(i)}{k}z\right) \right\} \\ E_2^{(i)}(z) &= e^{i\beta^{-(i)}z} \left\{ E_1^{(i)}(0) \sin\left(\frac{-(i)}{k}z\right) + E_2^{(i)}(0) \cos\left(\frac{-(i)}{k}z\right) \right\} \end{aligned} \quad (56)$$

the nonlinearity being embedded into  $\beta^{-(i)}$  and  $k^{-(i)}$

$$\begin{aligned} \beta^{-(i)} &= \beta^{(i)} + \frac{3\omega_0^2 \chi^{(3)}}{8c^2 \beta^{(i)}} \left( \left| E_1^{(i)}(0) \right|^2 + \left| E_2^{(i)}(0) \right|^2 \right) \\ k^{-(i)} &= \frac{\omega_0^2 \chi^{(3)}}{4c^2 \beta^{(i)}} \text{Im} \left( E_1^{(i)*}(0) E_2^{(i)}(0) \right) \end{aligned} \quad (57)$$

To obtain the corresponding input-output relations in the two fibers one defines a matrix

$$M^{(\pm)}(z) = \begin{pmatrix} \cos\left(\frac{-(\pm)}{k}z\right) & -\sin\left(\frac{-(\pm)}{k}z\right) \\ \sin\left(\frac{-(\pm)}{k}z\right) & \cos\left(\frac{-(\pm)}{k}z\right) \end{pmatrix} \quad (58)$$

Eq.(56) is rewritten

$$E^{(\pm)}(z) = e^{iz\beta^{-(\pm)}} M^{(\pm)}(z) E^{(\pm)}(0) \quad (59)$$

$z$  being the interaction length of the directional coupler. Using (51) the fields

at the output of the coupler are related to the input fields by

$$\begin{aligned}
E_j^{(1)}(z) &= \frac{1}{2} \left\{ \begin{aligned} &\left( e^{iz\beta^{-(+)}} M^{(+)} + e^{iz\beta^{-(-)}} M^{(-)} \right)_{jk} E_k^{(1)}(0) \\ &+ \left( e^{iz\beta^{-(+)}} M^{(+)} - e^{iz\beta^{-(-)}} M^{(-)} \right)_{jk} E_k^{(2)}(0) \end{aligned} \right\} \\
E_j^{(2)}(z) &= \frac{1}{2} \left\{ \begin{aligned} &\left( e^{iz\beta^{-(+)}} M^{(+)} - e^{iz\beta^{-(-)}} M^{(-)} \right)_{jk} E_k^{(1)}(0) \\ &+ \left( e^{iz\beta^{-(+)}} M^{(+)} + e^{iz\beta^{-(-)}} M^{(-)} \right)_{jk} E_k^{(2)}(0) \end{aligned} \right\}
\end{aligned} \tag{60}$$

For the linear coupler case the  $M^{(\pm)}(z)$  matrices are the unit matrices and the coupling arises only from the difference in the propagation constants  $\beta^{-(+)}$ ,  $\beta^{-(-)}$  of symmetric and antisymmetric modes. However in both cases, linear and nonlinear, explicit analytical expressions are obtained for the coupling as a function of the input intensities and the material properties. In  $\beta^{-(i)}$  the nonlinear effect is a function of the energy of the incoming signals and  $k^{-(i)}$  has a geometrical interpretation as

$$k^{-(i)} = \frac{\omega_0^2 \chi^{(3)}}{8c^2 \beta^{(i)}} |\Psi^{(i)*} \times \Psi^{(i)}|$$

Here it was assumed that the frequency of the two incoming signals to the coupler is the same. If they have different frequencies  $\omega_1$  and  $\omega_2$  the corresponding constants of motion, as a function of the associated fields  $\Psi^1, \Psi^2$ , would be

$$|\Psi^1|^2; |\Psi^2|^2; \frac{\beta_1}{\omega_1^2} \Psi^{1*} \times \Psi^1 + \frac{\beta_2}{\omega_2^2} \Psi^{2*} \times \Psi^2$$

However, in this case, these constants of motion do not seem to be sufficient to obtain an explicit analytical solution.

## References

- [1] D. Deutsch; *Quantum theory, the Church-Turing principle and the universal quantum computer*, Proc. R. Soc. London A400 (1985) 97-117.

- [2] E. Bernstein and U. Vazirani; *Quantum complexity theory*, SIAM J. Computing 26 (1997) 1411-1473.
- [3] M. A. Man'ko, V. I. Man'ko and R. Vilela Mendes; *Quantum computation by quantum-like systems*, Phys. Lett. A 288 (2001) 132-138.
- [4] S. Chávez-Cerda, H. M. Moya-Cessa and J. R. Moya-Cessa; *Quantum-like entanglement in classical optics*, Optics and Photonics News 18 (2007) 38.
- [5] O. Crasser, H. Mack and W. P. Schleich; *Could Fresnel Optics be Quantum Mechanics in Phase Space?*, Fluctuation and Noise Letters 4 (2004) L43-L51.
- [6] S. Chávez-Cerda, J. R. Moya-Cessa and H. M. Moya-Cessa; *Quantumlike systems in classical optics: applications of quantum optical methods*, J. Opt. Soc. Am. B 24 (2007) 404-407.
- [7] J. Fu, Z. Si, S. Tang and J. Deng; *Classical simulation of quantum entanglement using optical transverse modes in multimode waveguides*, Phys. Rev. A 70 (2004) 042313.
- [8] A. Aiello et al.; *Quantum-like nonseparable structures in optical beams*, New J. Phys. 17 (2015) 043024.
- [9] N. Nefedov; *Quantum-like computations using coupled nano-scale oscillators*, in NanoNet 2009, Proc. Int. ICST Conf., pp. 64-69, Springer, Berlin 2009.
- [10] K. Nishimura, M. Takeuchi and T. Kuga, *Experimental simulation of a decohering Schrödinger's cat state in wave optics*, J. of the Optical Society of America B 35 (2018) 337-345.
- [11] M.A. Garcia-March, N.L. Harshman, H. da Silva, T. Fogarty, Th. Busch, M. Lewenstein and A. Ferrando; *Graded-index optical fiber emulator of an interacting three-atom system: Classical non-separability and illumination control of particle statistics*, arXiv:1902.01748.
- [12] A. R. Urzúa, F. Soto-Eguibar, V. M. Arrizón and H. M. Moya-Cessa; *Light propagation in inhomogeneous media, coupled quantum harmonic oscillators and phase transitions*, arXiv:1905.06897.

- [13] M.A. Leontovich and V.A. Fock; *Solution of the problem of electromagnetic wave propagation along the Earth's surface by the method of parabolic equation*, J. Phys. USSR, 10 (1946) 13-23.
- [14] R. Fedele and P. K. Shukla (Eds.); *Quantumlike models and coherent effects*, World Scientific, Singapore 1995.
- [15] S. De Martino, S. De Nicola, S. De Siena, R. Fedele and G. Miele (Eds.); *New perspectives in Physics of Mesoscopic Systems: Quantumlike descriptions and Macroscopical Coherence Phenomena*, World Scientific, Singapore 1997.
- [16] M. A. Man'ko; *Beam optics and signal analysis in a quantumlike approach*, J. Russian Laser Research 22 (2001) 48-60.
- [17] D. S. Abrams and S. Lloyd; *Nonlinear Quantum Mechanics implies polynomial-time solution for NP-complete and # P problems*, Phys. Rev. Lett. 81 (1998) 3992-3995.
- [18] T. A. Brun; *Computers with closed timelike curves can solve hard problems efficiently*, Foundations of Physics Lett. 16 (2003) 245-253.
- [19] T. C. Ralph; *Quantum optical systems for the implementation of quantum information processing*, Reports on Progress in Optics 69 (2006) 853-898.
- [20] J. L. O'Brien; *Optical quantum computing*, Science 318 (2007) 1567-1570.
- [21] D. Gloge; *Weakly guiding fibers*, Appl. Optics 10 (1971) 2252-2258.
- [22] C. R. Pollock and M. Lipson; *Integrated photonics*, Springer N. Y. 2003.
- [23] S. O. Kasap; *Optoelectronics and photonics, principles and practices*, Pearson, Essex 2013.
- [24] R. C. Jones; *A new calculus for the treatment of optical systems I. Description and discussion of the calculus*, J. Opt. Soc. Am. 31 (1941) 488-493.

- [25] H. Hurwitz Jr. and R. C. Jones; *A new calculus for the treatment of optical systems II. Proof of three general equivalence theorems*, J. Opt. Soc. Am. 31 (1941) 493-499.
- [26] Chin-Lin Chen; *Foundations for guided-wave optics*, Wiley-Interscience, Hoboken NJ 2007.
- [27] W. Eickhoff; *Stress-induced single polarization single mode fiber*, Opt. Lett. 12 (1982) 629-631.
- [28] L. De Sio, A. Tedesco, N. Tabirian and C. Umeton; *Optically controlled holographic beam splitter*, Appl. Phys. Lett. 97 (2010) 183507.
- [29] C. Wang et al.; *Tunable beam splitter using bilayer geometric metasurfaces in the visible spectrum*, Optics Express 28, 19 (2020) 28673.
- [30] X. Ma et al.; *A high-speed tunable beam splitter for feed-forward photonic quantum information processing*, Optics Express 19, 23 (2011) 22723.
- [31] G. Song et al.; *Tunable multi-function broadband splitter with optical bistability based on surface plasmon*, Optik 124 (2013) 4721-4724.
- [32] D. C. Zografopoulos, R. Beccherelli and E. E. Kriezis; *Beam-splitter switches based on zenithal bistable liquid-crystal gratings*, Phys. Rev. E 90 (2014) 042503.
- [33] B. E. A. Saleh and M. C. Teich; *Fundamentals of Photonics*, 3rd Edition, John Wiley & Sons, Hoboken NJ, 2019.
- [34] X. Li, Z. Shao, M. Zhu and J. Yang; *Fundamentals of Optical Computing Technology*, Springer Nature, Singapore 2018.
- [35] S. M. Jensen; *The nonlinear coherent coupler*, IEEE J. of Quantum Electronics 18 (1982) 1580-1583.
- [36] R. Vilela Mendes; *The nonlinear directional coupler: an analytic solution*, Optics Communications 232 (2004) 425-427.
- [37] M. A. Nielsen and I. L. Chuang; *Quantum Computation and Quantum Information*, Cambridge U. P., Cambridge 2010.

- [38] R. Cleve and J. Watrous; *Fast parallel circuits for the quantum Fourier transform*, Proceedings 41st Annual Symposium on Foundations of Computer Science, pp. 526-536, Redondo Beach, CA, USA, 2000.
- [39] Z. Zilic and K. Radecka; *The role of super-fast transforms in speeding up quantum computations*, pp. 129-135, Proceedings 32nd IEEE International Symposium on Multivalued Logic 2002, pp. 129-135, 2002.
- [40] C. Moore, D. N. Rockmore and A. Russell; *Generic quantum Fourier transforms*, ACM Trans. Algorithms 2 (2006) 707-723.
- [41] R. B. Griffiths and C.-S. Niu; *Semiclassical Fourier transform for quantum computation*, Phys. Rev. Lett. 76 (1996) 3228-3231.
- [42] A. Tomita; *Quantum Information Processing with Fiber Optics: Quantum Fourier Transform of 1024 Qubits*, Optics and Spectroscopy 99 (2005) 204-210.
- [43] Y. S. Nam and R. Blüme; *Optical simulator of the quantum Fourier transform*, EPL 114 (2016) 20004.
- [44] R. C. Young, P. M. Birch and C. R. Chatwin; *Considerations for the extension of coherent optical processors into the quantum computing regime*, Proceedings of SPIE, 9845, pp. 1-9, 2016.
- [45] A. J. Macfaden, G. S. D. Gordon and T. D. Wilkinson; *An optical Fourier transform coprocessor with direct phase determination*, Scientific Reports 7 (2016) 13667.
- [46] E. Kashefi, A. Kent, V. Vedral and K. Banaszek; *Comparison of quantum oracles*, Phys. Rev. A 65 (2002) 050304(R).
- [47] A. Gilyén, S. Arunachalam and N. Wiebe; *Optimizing quantum optimization algorithms via faster quantum gradient computation*, pp. 1425-1444, Proc. of 30th Annual ACM-SIAM Symposium on Discrete Algorithms 2016.
- [48] C. Pinho et al.; *Spatial Light Modulation as a Flexible Platform for Optical Systems*, in *Telecommunication Systems – Principles and Applications of Wireless-Optical Technologie*, DOI: <http://dx.doi.org/10.5772/intechopen.88216>.

- [49] J. Park et al.; *All-solid-state spatial light modulator with independent phase and amplitude control for three-dimensional LiDAR applications*, Nat. Nanotechnol. (2020) <https://doi.org/10.1038/s41565-020-00787-y>
- [50] G. Csaba, A.Pappa and W. Poroda; *Perspectives of using spin waves for computing and signal processing*, PhysicsLetters A381(2017)1471-1476.
- [51] A. V. Chumak, V. I. Vasyuchka, A. A. Serga and B. Hillebrands; *Magnon spintronics*, Nature Physics 11 (2015) 453-461.
- [52] A. Barenco, C. Bennett, R. Cleve, D. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. Smolin and H. Weinfurter; *Elementary gates for quantum computation*, Phys. Rev. A 52 (1995) 3457-3467.
- [53] J.-L. Brylinski and R. Brylinski; *Universal quantum gates*, in Mathematics of Quantum Computation, R. Brylinski and G. Chen, (eds.), Chapman & Hall/CRC, Boca Raton, Florida, 2002, 101–116.
- [54] Y. Silberberg and G. I. Stegeman; *Nonlinear coupling of waveguide modes*, Appl. Phys. Lett. 50 (1987), 801-803.
- [55] G. I. Stegeman, C. T. Seaton, A. C. Walker and C. N. Ironside; *Nonlinear directional couplers with integrating nonlinearities*, Optics Communications 61 (1987) 277-281.
- [56] G. I. Stegeman, E. Caglioti, S. Trillo and S. Wabnitz; *Parameter trade-offs in nonlinear directional couplers: Two level saturable nonlinear media*, Optics Communications 63 (1987) 281-284.
- [57] A. M. Kenis, I. Vorobeichik, M. Orenstein and N. Moiseyev; *Non-*evanescent* adiabatic directional coupler*, IEEE Journal of Quantum Electronics 37 (2001) 1321-1328.
- [58] G. J. Liu, B. M. Liang, Q. Li and G. L. Jin; *Coupled mode analysis of the nonlinear switching in the couplers with variable coupling coefficient*, Optics Communications 223 (2003) 195-200.
- [59] Y. Wang and J. Liu; *All-fiber logical devices based on the nonlinear directional coupler*, IEEE Photonics Technology Letters 11 (1999) 72-74.