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Stochastic processes and the non-perturbative structure of the QCD vacuum

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Abstract. Based on a local Gaussian evaluation of the functional integral representation, a method is developed to obtain ground state functionals. The method is applied to the gluon sector of QCD. For the leading term in the ground state functional, stochastic techniques are used to check consistency of the quantum theory, finiteness of the mass gap and the scaling relation in the continuum limit. The functional also implies strong chromomagnetic fluctuations which constrain the propagators in the fermion sector.

1 Introduction

There is a wide belief that quantum chromodynamics is the theory of strong interactions. However reliable predictions can be extracted from the theory only for a restricted class of experimentally observable quantities. For short-distance (large transverse momentum) phenomena, perturbation theory seems reliable and, for the static properties of hadrons, a few results have been obtained by lattice Monte Carlo calculations. However high energy small transverse momentum reactions, for example, are out of reach of both techniques.

Due to asymptotic freedom the (perturbative) physical picture of large transverse momentum reactions is well understood. For low energies however, even if future dedicated machines get around the small lattice limitation and are eventually able to crunch out the right numbers, some sort of understanding of the physical picture at low energies would be convenient. The physical picture is probably in the tapes containing thousands of computer-generated lattice configurations, but this is not very transparent for the common mortal.

The construction of approximations to the vacuum and other low-lying states in QCD has been attempted by many authors [1–10] who used several approaches and conjectures. An approach that has been widely followed is the variational one, wherein a trial state is conjectured, dependent on some parameters, which are then adjusted to minimize the energy. However, as Feynman [11] has pointed out, although the trial states are constructed to describe the low energy properties, the non-linear nature of the QCD couplings (and the large number of degrees of freedom) have as a consequence that the variational parameters tend to adjust themselves to the high frequency modes because they have the largest contribution to the energy. This in general leads to unrealistic values of the parameters as far as the low energy properties are concerned, which are exactly what we were trying to describe to begin with. In addition because one needs to compute high dimensional energy integrals, this essentially restricts the trial states to be Gaussian. These two limitations seriously restrict the usefulness of the variational approach in field theory*.

In other approaches a systematic perturbative expansion of the vacuum is constructed where typically the first term has the structure of the Abelian ground state. In the perturbative expansions the individual terms are not invariant under non-Abelian gauge transformations and non-perturbative features are hard to extract from the expansion. It has been proposed [6] to improve the perturbative expansion by a so-called gauge invariant completion. I.e. order by order the expansion terms are replaced by gauge-invariant expressions which agree, to that order, with perturbation theory. A difficult consistency problem occurs however, because the gauge-invariant completion of a given order contains contributions of all higher orders.

The general conclusion is that methods to approximate the QCD vacuum are needed which do not rely on the minimization of the energy and that are gauge invariant. Furthermore they should also not rely on expansions on powers of the coupling constant to ensure that the leading terms already contain non-perturbative information. An attempt in this direction is presented in Sect. 2,

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* A different point of view has recently been advocated by A. Neveu [12]

based on a local Gaussian evaluation of the functional integral representation of the ground state.

When an approximation to the QCD vacuum is derived or conjectured, the second problem is what to do with it. In principle when the ground state is known, Green's functions and, by LSZ reduction, scattering amplitudes may be computed and the theory is completely defined. However one still has to compute functional integrals on the time-zero fields which, for non-trivial (non-Gaussian) ground states is not simple.

There is however another way to extract useful information from the ground state functionals $\psi_0(\phi)$ that relies on the interpretation of the measure $\psi_0^2(\phi)\{d\phi\}$ as the invariant measure of a stochastic process

$$d\phi = -L(\phi)\,dt + d\,W(t), \tag{1.1}$$

with drift

$$-L(\phi) = \frac{1}{\psi_0(\phi)}\frac{\delta\psi_0}{\delta\phi}, \tag{1.2}$$

the generator of this process being the Hamiltonian operator. Consistency of the quantum theory defined by the ground state measure may then be rigorously formulated as a problem of closability of the associated energy form [10] or as a problem of existence of solutions of the stochastic differential equation (1.1) [13].

Furthermore the Dirichlet problem of the generator (i.e. the eigenvalue problem of the Hamiltonian) is related to the statistics of the exit time of the stochastic process from the domain where boundary conditions are imposed. In particular the Wentzell-Freidlin theory [14] and its associated large deviation estimates are especially adequate to characterize the mass gap and scaling properties of the continuum limit. The Wentzell-Freidlin technique seems in fact to be the most natural non-perturbative technique in the sense that quantities behaving like $\exp(-c/g^2)$ are the simplest ones to deal with. For details on this technique I refer to [14–17]. A summary of the main results and some applications may be found in [18].

Extensive use will be made of the ground state functional interpretation as the invariant measure of a stochastic process in the study of the properties of the ground state approximation constructed in Sect. 2.

The plan of the paper is the following. In Sect. 2 a method is developed to approximate the ground state functional which is then applied to the gluon sector of QCD. One of these approximations, which will be called QCD_0, is then shown to lead to a well-defined quantum theory, in the sense that the ground state functional is the density of a closable Dirichlet form [19].

In Sect. 3 the interpretation of the ground state functional as the invariant measure of a stochastic process and the Wentzell-Freidlin technique are used to prove that the long-range non-abelian QCD_0-dynamics has a finite mass gap and to derive its scaling properties in the continuum limit. Finally Sect. 4 deals with the structure of the strong chromomagnetic vacuum fluctuations implied by the functional of QCD_0 and its effect on the propagation of fermions. Many results in the paper hold for any compact gauge group. However, whenever specific calculations were required, I have, for simplicity, used the $SU(2)$ group.

2 Approximating the ground state functional

The starting point is the path integral representation of the ground state $\psi_0(x)$ as a sum over Euclidean paths pinned down, at time zero, to the x coordinates

$$\psi_0(x) \sim \int \mathscr{D}x(\tau)\,\delta(x(0)-x)\,e^{\int_{-\infty}^{0} L_E(x(\tau),\dot{x}(\tau))\,d\tau}, \tag{2.1}$$

with $L_E(x,\dot{x}) = -c(\dot{x})^2 - V(x)$.

It is more convenient to work with paths from $\tau=-\infty$ to $\tau=+\infty$ and this leads to

$$|\psi_0(x)|^2 = \frac{1}{N}\int \mathscr{D}x(\tau)\,\delta(x(0)-x)\,e^{\int_{-\infty}^{\infty} L_E(x(\tau),\dot{x}(\tau))\,d\tau}, \tag{2.2}$$

where the normalization factor N is the same functional integral without the delta function. We are going to make the change of variables

$$x(\tau) = x + z(\tau).$$

The delta function may be included in the exponential by

$$\delta(z^i(0)) = \left(\frac{1}{2\pi}\right)\int dy^i \exp\{i\int d\tau\, y^i z^i(\tau)\,\delta(z)\}.$$

Now add a term $z(\tau)\cdot J(\tau)$ to the Euclidean Lagrangian and separate the terms quadratic or less than quadratic in $z(\tau)$ from higher order terms

$$\begin{aligned} &L_E(x(\tau),\dot{x}(\tau)) + z(\tau)\cdot J(\tau) \\ &= -V(x) - z(\tau)\cdot\left(-c\frac{\partial^2}{\partial\tau^2} + S(x)\right)z(\tau) \\ &\quad -(\Gamma(x)-J(\tau))\cdot z(\tau) + G(z(\tau)), \end{aligned} \tag{2.3}$$

where $G(z(\tau))$ denotes the terms of order higher than two. Representing $G(z(\tau))$ by $G\left(\frac{\partial}{\partial J(\tau)}\right)$ and computing the Gaussian integral the following representation is obtained for $|\psi_0(x)|^2$

$$|\psi_0(x)|^2 = \left.\frac{e^{\int d\tau\, G\left(\frac{\delta}{\delta J(\tau)}\right)}\, e^{\frac{1}{4}\int d\tau (\Gamma(x)-J(\tau))\frac{1}{-c\frac{\partial^2}{\partial\tau^2}+S(x)}(\Gamma(x)-J(\tau))}\, e^{2\sqrt{c}L\sqrt{S}L}}{e^{\int d\tau\, G\left(\frac{\delta}{\delta J(\tau)}\right)}\, e^{\frac{1}{4}\int d\tau (\Gamma(x)-J(\tau))\frac{1}{-c\frac{\partial^2}{\partial\tau^2}+S(x)}(\Gamma(x)-J(\tau))}}\right|_{J=0}, \tag{2.4}$$

with

$$L = -\frac{i}{4\sqrt{c}}\int d\tau\,\frac{1}{\sqrt{S(x)}}\,e^{-|\tau|\sqrt{\frac{S(x)}{c}}}(\Gamma(x)-J(\tau)), \tag{2.5a}$$

for $S(x)>0$. L reduces to

$$L_0=-\frac{i}{2}\frac{1}{S(x)}\Gamma(x), \tag{2.5b}$$

in the limit $J\to 0$.

From (2.4), expanding $\exp\left(\int d\tau G\left(\frac{\partial}{\partial J(\tau)}\right)\right)$, one may construct successive approximations to the ground state. Of particular interest is the leading term

$$|\psi_0(x)|^2_{(0)}=\exp\left\{-\frac{\sqrt{c}}{2}\Gamma(x)\frac{1}{S(x)}\sqrt{S(x)}\frac{1}{S(x)}\Gamma(x)\right\}. \tag{2.6}$$

This differs essentially from a perturbative estimate in that, at each point x of the wave function, a different expansion point is chosen, which is x itself. In the functional integral representation of the ground state the wave function is the integrated effect of paths coming from the infinite past to the point x. Because near x the difference $z(\tau)-x$ is small, the leading term will contain accurate information coming from the paths in the neighbourhood of x, and will be inaccurate only concerning non-harmonic contributions to the paths far away from x.

For non-Abelian gauge fields, $\mathscr{A}_i^\alpha(\tau,x)$ is defined to be the space-time Euclidean vector potential, $A_i^\alpha(x)=\mathscr{A}_i^\alpha(0,x)$ the time-zero field, $\mathscr{B}_i^\alpha(\tau,x)=\varepsilon_{ijk}\left(\partial_j\mathscr{A}_k^\alpha-\frac{g}{2}f_{\alpha\beta\gamma}\mathscr{A}_j^\beta\mathscr{A}_k^\gamma\right)$ the chromomagnetic field and $B_i^\alpha(x)=\varepsilon_{ijk}\left(\partial_j A_k^\alpha-\frac{g}{2}f_{\alpha\beta\gamma}A_j^\beta A_k^\gamma\right)$ the time-zero chromomagnetic field.

Because one is dealing with approximations to the ground state measure, it is convenient to use the Schrödinger formulation for quantum fields [20]. The temporal gauge $\mathscr{A}_a^0=0$ is chosen, the time-zero fields are the canonical variables, the chromoelectric fields the conjugate momenta and Gauss' law is imposed as a constraint, i.e. the wave functionals must be gauge invariant. Make the following change of variables

$$\mathscr{A}_i^\alpha(\tau,x)=A_i^\alpha(x)+\phi_i^\alpha(\tau,x).$$

The Euclidean Lagrangian is

$$L_E=\frac{1}{2}\left(\frac{\partial}{\partial\tau}\phi_i^\alpha\right)^2-\frac{1}{2}(\mathscr{B}_i^\alpha)^2.$$

To construct the ground state approximation according to (2.6), notice that

$$\mathscr{B}_i^\alpha(\tau,x)=B_i^\alpha(x)+\varepsilon_{ijk}D_j(A)_{\alpha\beta}\phi_k^\beta(\tau,x) -\frac{g}{2}\varepsilon_{ijk}f_{\alpha\beta\gamma}\phi_j^\beta(\tau,x)\phi_k^\gamma(\tau,x), \tag{2.7}$$

with

$$D_j(A)_{\alpha\beta}=\partial_j\delta_{\alpha\beta}-gf_{\alpha\gamma\beta}A_j^\gamma(x). \tag{2.8}$$

Separating only the contributions of the local linear approximation to $\mathscr{B}_i^\alpha(\tau,x)$ one obtains for the S, Γ and G functions of (2.3)

$$S_0(A(x))_{nn'}^{\beta\beta'}=\tfrac{1}{2}\varepsilon_{nmk}D_m(A)^{\beta\alpha}\varepsilon_{km'n'}D_{m'}(A)^{\alpha\beta'}, \tag{2.9a}$$

$$\Gamma_0(A(x))_n^\beta=\varepsilon_{nmk}D_m(A)^{\beta\alpha}B_k^\alpha, \tag{2.9b}$$

$$G_0(\mathscr{A})=\frac{1}{2}\varepsilon_{kmn}\varepsilon_{km'n'}f_{\alpha\beta'\gamma'}\left\{-gD_m(A)^{\alpha\beta}\mathscr{A}_n^\beta\mathscr{A}_{m'}^{\beta'}\mathscr{A}_{n'}^{\gamma'}+\frac{g^2}{4}f_{\alpha\beta\gamma}\mathscr{A}_m^\beta\mathscr{A}_n^\gamma\mathscr{A}_{m'}^{\beta'}\mathscr{A}_{n'}^{\gamma'}\right\} +\frac{g}{2}\varepsilon_{nkn'}f_{\alpha\beta\beta'}B_k^\alpha(A)\mathscr{A}_n^\beta\mathscr{A}_{n'}^{\beta'}. \tag{2.9c}$$

If instead, one isolates all quadratic terms in L_E, including the cross terms between $-\frac{g}{2}\varepsilon_{ijk}f_{\alpha\beta\gamma}\phi_j^\beta(\tau,x)\phi_k^\gamma(\tau,x)$ and $B_i^\alpha(x)$ the result would be

$$S_1(A(x))_{nn'}^{\beta\beta'}=\frac{1}{2}\varepsilon_{nmk}D_m(A)^{\beta\alpha}\varepsilon_{km'n'}D_{m'}(A)^{\alpha\beta'} +\frac{g}{2}\varepsilon_{nkn'}f_{\alpha\beta\beta'}B_k^\alpha(A), \tag{2.10a}$$

$$\Gamma_1(A(x))_n^\beta=\varepsilon_{nmk}D_m(A)^{\beta\alpha}B_k^\alpha, \tag{2.10b}$$

$$G_1(\mathscr{A})=\frac{1}{2}\varepsilon_{kmn}\varepsilon_{km'n'}f_{\alpha\beta'\gamma'}\left\{-gD_m(A)^{\alpha\beta}\mathscr{A}_n^\beta\mathscr{A}_{m'}^{\beta'}\mathscr{A}_{n'}^{\gamma'}+\frac{g^2}{4}f_{\alpha\beta\gamma}\mathscr{A}_m^\beta\mathscr{A}_m^\gamma\mathscr{A}_{m'}^{\beta'}\mathscr{A}_{n'}^{\gamma'}\right\}, \tag{2.10c}$$

The ground state approximations obtained using (2.9) or (2.10) in (2.6) are qualitatively similar. However only the use of (2.9) is fully consistent because $S_1'(A(x))$ is not a positive operator and positivity of the Gaussian kernel plays a role in the derivation of (2.6). Therefore I will concentrate on the functional obtained from (2.9), namely

$$\psi_0(A)_{(0)}=\exp\left\{-\frac{1}{2}\int d^3x\, B_k^\alpha(A(x)) \cdot\left(\frac{1}{\sqrt{R(A(x))\,R(A(x))}}\right)_{kk'}^{\alpha\alpha'}B_{k'}^{\alpha'}(A(x))\right\}, \tag{2.11}$$

where the following operator has been defined

$$R(A)_{nn'}^{\alpha\alpha'}=\varepsilon_{nmn'}D_m(A)^{\alpha\alpha'}. \tag{2.12}$$

(2.11) is similar to the gauge invariant ansatz discussed in [10]. However unlike the ansatz of [10], which was simply a non-Abelian generalization of the Abelian ground state, the functional (2.11) is the leading term in an exact representation of the (gluon sector) QCD vacuum. It displays qualitative features of physical relevance. Namely, as discussed in [10], the long-distance behaviour appears associated to maximally disordered field configurations, whereas for high-momentum (short distances) the kernel approaches the free theory. The exponent of the exponential is negative semi-definite. Therefore the maximum is reached for the $B_k^\alpha(x)=0$ configurations, the functional being peaked at all homotopically non-equivalent classical vacua.

The quantum theory corresponding to the ground state approximation defined by (2.11) will be called

QCD_0. The remainder of this paper is dedicated to the study of some properties of this quantum theory. The first question is to show that the ground state functional (2.11) indeed defines a quantum theory. This will be done in two steps. First a lattice version of the stochastic process associated to (2.11) is constructed and a result of Fukushima [20] used to show that it defines a closed Dirichlet form, hence a self-adjoint generator and a well-defined quantum theory. The second step which relates to the scaling in the continuum limit will be carried out in Sect. 3 after the behaviour of the mass gap is discussed.

In the construction of quantum theories using ground-state functionals [10, 13, 21, 22] $\psi_0=\exp\{\sigma(A)\}$, these are always considered either as densities of a form

$$\mathscr{E}(u,v)=\int \frac{\delta}{\delta A}u\cdot\frac{\delta}{\delta A}v\,\mathrm{e}^{2\sigma(A)}\{\mathrm{d}A\},$$

or as generating a stochastic process

$$\mathrm{d}A=-L(A)\,\mathrm{d}t+\mathrm{d}W(t),$$

with drift $-L=\frac{\delta\sigma}{\delta A}$. Once closability of the form or existence of solutions to the stochastic differential equation and an ergodic invariant measure are proven, one is sure to have a positive self-adjoint Hamiltonian operator (the generator of the process). The measure then allows the reconstruction of a Hilbert space and one has a well-defined quantum theory.

A lattice regularization is used. Define the variables $\theta^\alpha(x,x+\hat{\imath})$ corresponding to a rescaling of the fields $A_i^\alpha(x)$ and make the replacements

$$gaA_i^\alpha(x)\to\theta^\alpha(x,x+\hat{\imath})=\theta_i^\alpha(x), \tag{2.13}$$

$$ga^2B_i^\alpha(x)\to\beta_i^\alpha(x)=\tfrac{1}{4}\gamma_{ijk}(\theta^\alpha(x+\hat{\jmath},x+\hat{\jmath}+\hat{k}) -\tfrac{1}{2}f_{\alpha\beta\gamma}\theta^\beta(x,x+\hat{\jmath})\theta^\gamma(x,x+\hat{k})), \tag{2.14}$$

where $\gamma_{ijk}=(\operatorname{sign} i)(\operatorname{sign} j)(\operatorname{sign} k)\varepsilon_{|i||j||k|}$ $(i,j,k\in\{1,2,3\})$, $\hat{\imath}$ denotes the unit lattice vector along the i-direction, x is a point in a three-dimensional lattice and a is the lattice spacing. The covariant derivative of a quantity $v^\alpha(x)$ transforming under the adjoint representation of the internal symmetry group becomes

$$\begin{aligned}(D_i)_{\alpha\beta}v^\beta(x)&\to\frac{1}{a}\{\tfrac{1}{2}(v^\alpha(x+\hat{\imath})-v^\alpha(x-\hat{\imath}))\\&\quad-f_{\alpha\gamma\beta}\theta^\gamma(x,x+\hat{\imath})v^\beta(x)\}\\&=\frac{1}{a}(\mathscr{D}_i)_{\alpha\beta}v^\beta(x).\end{aligned} \tag{2.15}$$

With these definitions the lattice version of (2.11) is

$$\begin{aligned}\psi_0(\theta)_{(0)}=\exp\bigg\{&-\frac{1}{2\pi g^2}\sum_x\int_0^\infty \mathrm{d}\lambda\,\lambda^{-\frac{1}{2}}\beta_k^\alpha(x)\\&\cdot\left(\frac{1}{\lambda+\mathscr{R}_i\cdot\mathscr{R}_i}\right)_{\alpha\gamma}\beta_k^\gamma(x)\bigg\},\end{aligned} \tag{2.16}$$

where the $\mathscr{R}$-operator is now

$$\mathscr{R}(\theta)_{nn'}^{\alpha\alpha'}=\varepsilon_{nmn'}\mathscr{D}_m(\theta)^{\alpha\alpha'},$$

and a standard integral representation for fractional powers of positive operators [23] has been used

$$\frac{1}{\sqrt{\mathscr{R}_i\mathscr{R}_i}}=\frac{1}{\pi}\int_0^\infty\lambda^{-\frac{1}{2}}\frac{1}{\lambda+\mathscr{R}_i\mathscr{R}_i}\,\mathrm{d}\lambda.$$

We may now state that:

Theorem 1. *For a finite lattice with periodic boundary conditions the ground state measure* $\psi_0^2\,d\theta$, *with* ψ_0 *as in* (2.16), *characterizes a well-defined quantum theory.*

The proof is the same as for the functional of [10] to which we refer for the details. The operator $\mathscr{R}_i\mathscr{R}_i$ plays now the same role as the operator $-\mathscr{D}_i\mathscr{D}_i$ in [10]. In a finite lattice with periodic boundary conditions $\mathscr{R}_i\mathscr{R}_i$ is a positive operator and the density of the ground state measure is bounded. Furthermore the singular set has zero measure and therefore the density satisfies the conditions of Theorem 1 in [19]. The corresponding energy form is closable, there is associated to it a diffusion process and, in the Hilbert space of square-integrable field configurations, its generator defines a positive self-adjoint Hamiltonian operator, i.e. a well-defined quantum theory is associated to $\psi_0^2\,\mathrm{d}\theta$.

To discuss the continuum limit of this lattice quantum theory one needs to control the transformation law of physical quantities when the lattice spacing a tends to zero. This is approached through the characterization of the behaviour of the mass gap (the first excited level of the Hamiltonian) in the next section.

3 The mass gap

The purpose of this section is to show that the long-range non-abelian dynamics of the QCD_0 ground state measure (on the lattice) has a finite mass gap at small but non-zero coupling and to derive its scaling behaviour when the lattice spacing a tends to zero.

The existence of a finite mass gap in QCD is related to the question of confinement. The statement of confinement contains two parts:

(i) All observables are colour neutral
(ii) All physical states are colour singlets

The first part is a trivial consequence of the existence of an exact non-Abelian gauge symmetry [24]. The second part may or may not be of "kinematical" origin, in the sense that it does not depend on the particular dynamical details of the theory [25]. However, whether or not the search for dynamical proofs of confinement is useful, it is certainly necessary to look for its manifestations. In particular, when constructing approximations to QCD, it is essential to check whether they are consistent with some of the consequences of confinement. An important consequence is the existence of a mass gap in the gluon sector. If massless gluons cannot appear in the asymptotic fields, then there should exist a finite gap in the energy spectrum above the ground state of the Yang-Mills Hamiltonian and the first excited state is a massive excitation (glueball). Otherwise if the gap is arbitrarily small it is hard to see why a massless gluon cannot propagate to infinity. This latter situation occurs in QED and so some relevant difference must be found between the mass gap in Abelian and non-Abelian gauge theories. An appropriate setting to discuss this question is the Wentzell-Freidlin large deviation theory.

In the lattice formulation the exact Hamiltonian associated to the functional (2.16), i.e. the Hamiltonian for which it is the exact ground state, is

$$H=\frac{g^2}{2a}\sum_{x,i,\alpha}\left\{-\frac{\partial}{\partial\theta_i^\alpha(x)}+L_i^\alpha(x)\right\}\left\{\frac{\partial}{\partial\theta_i^\alpha(x)}+L_i^\alpha(x)\right\}, \tag{3.1}$$

where

$$-L_i^\alpha(x)=\frac{1}{\psi_0}\frac{\partial\psi_0}{\partial\theta_i^\alpha(x)}. \tag{3.2}$$

Making the unitary transformation $H\to H'=\psi_0^{-1}H\psi_0$ one obtains

$$-aH'=\frac{g^2}{2}\sum_{x,i,\alpha}\frac{\partial}{\partial\theta_i^\alpha(x)}\frac{\partial}{\partial\theta_i^\alpha(x)}+\sum_{x,i,\alpha}b_i^\alpha(x)\frac{\partial}{\partial\theta_i^\alpha(x)}, \tag{3.3}$$

with

$$b_i^\alpha(x)=-g^2L_i^\alpha(x)=g^2\frac{\partial}{\partial\theta_i^\alpha(x)}(\ln\psi_0). \tag{3.4}$$

The operator in (3.3) has the standard form of a second-order elliptic operator which is the generator of a diffusion process

$$\mathrm{d}\theta_i^\alpha(x)=b_i^\alpha(x)\,\mathrm{d}t+g\,\mathrm{d}W_i^\alpha(t), \tag{3.5}$$

with drift $b_i^\alpha(x)$ and diffusion coefficient g. This provides a characterization of the eigenvalue problem of the Hamiltonian through stochastic techniques. Consider the eigenvalue problem

$$-aH'u=\lambda u \tag{3.6}$$

in a domain D with boundary condition $u=0$ on ∂D. From the theory of positive operators there is, at least, one positive eigenvalue, the eigenfunction space of the smallest positive eigenvalue λ_1 being one-dimensional. λ_1 is called the principal eigenvalue. In the lattice theory it would be $\lambda_1=am$, m being called the mass gap. There is a probabilistic characterization of the principal eigenvalue λ_1 in terms of the first exit time τ from D of the process defined by (3.5), namely

$$\lambda_1=\sup\left\{\lambda\geqq 0;\ \sup_{x\in D}E_x e^{\lambda\tau}<\infty\right\}. \tag{3.7}$$

E_x denotes the statistical expectation when the starting point of the process is x. Equation (3.7) may be used for example as a reliable tool for the numerical evaluation of the mass gap [26]. Of more interest here is the fact that near the continuum limit one is in the weak coupling regimen. For small g it follows from (3.5) that one is in the "weak noise" limit. Then analytical results can be obtained for the mass gap from the Wentzell-Freidlin estimates [14]

$$\exp\left\{\frac{-I(s,x,\partial D)-h}{2g^2}\right\}<P_x\{\tau<s\}<\exp\left\{\frac{-I(s,x,\partial D)+h}{2g^2}\right\}, \tag{3.8a}$$

for any $h>0$ and sufficiently small g. $P_x\{\tau<s\}$ is the probability for the first exit time τ to be smaller than s when the process starts from x. The functional $I(s,x,\partial D)$ is

$$I(s,x,\partial D)=\inf_{\chi\in\psi} I_{0,s}(\chi), \tag{3.8b}$$

$$\psi_s=\left\{\chi\in C_{0,s}\colon \chi(0)=x;\ \min_{0\leq s'\leq s}\mathrm{d}(\chi(s'),\partial D)=0\right\}, \tag{3.8c}$$

with

$$I_{T_1,T_2}(\chi)=\frac{1}{2}\int_{T_1}^{T_2}\left(\frac{\mathrm{d}\chi}{\mathrm{d}s}-b(\chi(s))\right)^2\mathrm{d}s, \tag{3.8d}$$

i.e. $I(s,x,\partial D)$ is the infimum of $I_{T_1,T_2}(\chi)$ over all continuous paths that, starting from x, hit the boundary ∂D of the domain D in time less than or equal to s.

From (3.7–8) it is clear that the $g\to 0$ behaviour of the mass gap is controlled by the nature of the deterministic dynamical system

$$\frac{\mathrm{d}\theta_i^\alpha(x)}{\mathrm{d}t}=b_i^\alpha(x). \tag{3.9}$$

The case of most interest here is when the following two conditions are verified:

(a) There are a certain number of ω-limit sets K_i of (3.9) inside the domain D, with all points in each set K_i being equivalent for the functional I, i.e. $I(s,x,y)=0$ if both $x,y\in K_i$.
(b) $b\cdot v>0$ on ∂D, v being the inward normal to ∂D.

Let $V_i=\inf I(s,x,y)$ for $x\in K_i$ and $y\in\partial D$ and

$V^*=\max(V_1,\ldots,V_r)$,

$V_*=\min(V_1,\ldots,V_r)$.

Then

$$\lim_{g\to 0}(-g^2\ln\lambda_1(g))\leq V^*;\quad \lim_{g\to 0}(-g^2\ln\lambda_1(g))\geq V_*.$$

In particular if there is only one V_i

$$\lambda_1(g)=am(g)\asymp\exp\left(-\frac{V}{g^2}\right), \tag{3.10}$$

the symbol $\asymp$ meaning logarithmic equivalence in the sense of large deviation theory.

For systems of the type (3.5), when the drift is the gradient of a function $(\ln\psi_0)$, the minimizing paths are those that exactly follow the flow or exactly oppose it. Therefore if, for example, there is only one attractive fixed point in D, V is given (up to a factor) by the smallest difference between $\ln\psi_0$ at the fixed point and at the boundary ∂D. If this difference is $\neq 0$ then, at small but finite g, λ_1 will be different from zero and approaches zero with g according to (3.10).

The situation described above does not apply to the vacuum of Abelian gauge fields which, using Fourier-transformed transverse fields, may be written

$$\psi_0[A]=\exp\left\{-\tfrac{1}{2}\int \mathrm{d}^3k\,A_i^T(-k)|k|A_i^T(k)\right\}. \tag{3.11}$$

Fix the boundary ∂D of the eigenvalue problem at some distance from the origin $A_i=0$, for example $D=\{A:|A|<M\}$. Now, for any M and any ε arbitrarily small, there is always a sufficiently small k such that on a path along k one has $I(s,\{A=0\},\partial D)<\varepsilon$, i.e. the boundary cannot be separated from the point $\{A=0\}$ by the functional I. Notice that for transverse fields the points in the boundary, that we are considering, are not gauge equivalent to $\{A=0\}$. Therefore the conditions (a) and (b) above are not satisfied. This is consistent with the fact that Abelian gauge fields have no positive mass gap.

For non-Abelian gauge fields, with dynamics defined by the QCD_0 functional, it is not so simple to take into account all types of possible field configurations and, by analogy with the abelian case, one considers only the case of (non-trivial) gauge potentials with minimal variation in space, which in this case corresponds to choose non-commuting constant potentials. This is one of the choices that leads to constant chromomagnetic fields, the other being the choice of commuting potentials with linear space dependence [27, 28]. For definiteness consider the gauge group to be $SU(2)$. Consider the matrix $M_{ij}=\sum_\alpha \theta_i^\alpha\theta_j^\alpha$, $\theta_i^\alpha=\theta^\alpha(x,x+\hat{i})$ being a constant lattice gauge potential. M, being a symmetric matrix, may be diagonalized by a space rotation. In the new coordinate axis the three $SU(2)$-vectors $\theta_1^\alpha\,\theta_2^\alpha$ and θ_3^α are orthogonal. Without loss of generality, the internal $SU(2)$-space coordinates may be chosen such that

$$\theta_1^\alpha=(a_1,0,0);\quad \theta_2^\alpha=(0,a_2,0);\quad \theta_3^\alpha=(0,0,a_3), \tag{3.12a}$$

to which correspond the chromomagnetic lattice fields

$$\beta_1^\alpha=(-a_2a_3,0,0);\quad \beta_2^\alpha=(0,-a_3a_1,0);\quad \beta_3^\alpha=(0,0,-a_1a_2). \tag{3.12b}$$

We have to compute now the exponent of (2.16) for this field configuration (Recall that, up to a constant, the exponent of (2.16) is the potential of the deterministic motion (3.9)). The result is

$$\begin{aligned}\sigma(a_1,a_2,a_3)=-\frac{N}{2\pi g^2}\int_0^\infty d\lambda\lambda^{-\frac{1}{2}}\{&(a_1a_2a_3)^2(a_1^2+a_2^2+a_3^2)\\ &+\lambda\{a_1^4(a_2^2+a_3^2)+a_2^4(a_1^2+a_3^2)\\ &+a_3^4(a_1^2+a_2^2)\}+\lambda^2(a_2^2a_3^2+a_1^2a_3^2+a_1^2a_2^2)\}\\ &\cdot\{4(a_1a_2a_3)^2+\lambda(\lambda+a_1^2+a_2^2+a_3^2)^2\}^{-1}.\end{aligned} \tag{3.13}$$

N is the number of sites on the lattice.

For the dynamics in this subspace, one decides whether the mass gap is finite or not by looking for neutral paths from the point $a_1=a_2=a_3=0$ to the boundary of the Dirichlet problem. The integrand in (3.13) is positive for generic a_1,a_2,a_3 and is zero only if two among these variables vanish. Therefore in the $\mathbb{R}^3$ space of a_1,a_2,a_3 the integral is strictly different from zero except along the axis. Thus it would seem that there is a neutral path along the axis. However all points along each one of the axis are gauge equivalent to the origin and should be identified with it. Therefore when the boundary is placed at some finite distance from the origin (which means at a finite distance from the axis) there are no neutral paths and, according to the discussion above, the mass gap is positive.

The fact that the small noise estimates give a specific form for the dependence of the mass gap on g, at small g, provides a way to discuss the scaling of am and $g=g(a)$ when the lattice spacing a goes to zero. The result is that "*the mass gap scales as* $am\sim\exp\{-c/g^2\}$ *when* $a\to0$, $g(a)\to0$". The statement is again a consequence of the discussion above. Notice that the occurrence of the power -2 in the g-dependence of the exponent is a consequence of the fact that the drift $-L$ of the lattice process is independent of g, a necessary condition discussed in [18]. The specific form $\exp\{-c/g^2\}$ follows then from the fact that the ω-limit set of the deterministic system (3.9) is attractive in the domain of the Dirichlet problem. This is fulfilled because $g^2\ln\{\psi_0\}$ is negative definite with maxima in the $\beta=0$ field configurations. From $am\sim\exp\{-c/g^2\}$ one sees that for the physical mass gap m to remain fixed when $a\to0$, one should require $g^2(a)\sim\left|\frac{c}{\log a}\right|$. Therefore, when $a\to0$, $g(a)\to0$ consistent with the use of the small noise estimates.

Estimates using large deviation theory are only accurate up to logarithmic equivalence. In particular they are not sensitive to power factors in front of the exponentials. In Appendix A we display this fact, in a simple example, by comparing exact asymptotic expressions with those obtained by the Wentzell-Freidlin technique. The example is also used to explain why the technique is not appropriate to estimate the wave functional of the lowest excited state. A better way to estimate this wave functional at small values of the coordinates and weak coupling would be to solve the equation

$$-\sum_{x,i,\alpha} b_i^\alpha(x)\frac{\partial}{\partial\theta_i^\alpha(x)}u\{\theta\}=\lambda u\{\theta\}.$$

4 Vacuum background fluctuations and the fermion sector

The ground state structure derived in Sect. 2 is the leading term of an exact (nonperturbative) representation of the QCD vacuum. We will now see that it provides an unambiguous description of a chromomagnetic vacuum structure.

The chromomagnetic structure of the QCD vacuum has been the object of many conjectures and phenomenological models, ranging from the search for low energy configurations [29–31] to the QCD sum rules [32] and the parametrization of hadronic properties as functions of the vacuum correlators of a fluctuating vacuum [33, 34]. The discussions of background chromomagnetic configurations in the QCD vacuum faced the problem of showing both that they minimize the energy and that they are stable. No such problem arises in QCD_0. Because the ground state measure itself defines the theory and it has been shown to lead to a closable energy form, the theory (and the QCD_0 vacuum) are necessarily stable. The picture that emerges is that the magnetic structure of the vacuum is not to be found in some stable background field configuration, but in the nature of the fluctuations of the ground state process. No stability problem of the background fields arises because they are simply fluctuations of the vacuum and what is stable is the vacuum itself as a stochastic process associated to a closed Dirichlet form.

Consider again the exponent (3.13) of the ground state measure computed for the constant fields (3.12). For large volume (large N) and weak coupling ($g(a)\to 0$), the measure is dominated by those configurations that lead to values of the exponent near zero. The exponent (3.13) is never positive and is zero only when two of the variables vanish. In one of the coordinate planes, $a_1=0$ for example, it reduces to

$$\sigma(0, a_2, a_3) = -\frac{N}{2g^2}\frac{a_2^2 a_3^2}{\sqrt{a_2^2+a_3^2}}. \tag{4.1}$$

This simple function is plotted in Fig. 1. It vanishes along the axis and grows monotonically away from them. However comparing (4.1) and (3.12b) one concludes that $\sigma(0, a_2, a_3)$ may be made arbitrarily small while, at the same time, β_1^α is arbitrarily large. It suffices to consider for any $|\beta_1^\alpha| = M$, $a_3 = M/\delta$ and $a_2 = \delta$. Then, for small δ, σ is proportional to δM and may be made as small as we like without changing β_1^α. This means that there are vacuum fluctuations with arbitrarily large chromomagnetic fields and that all magnitudes are equally probable. Notice that these are non-trivial fluctuations because, although they correspond to points very near the axis in Fig. 1, they are not in the gauge orbit of the origin.

The dynamics of these fluctuations must in fact be the dominant contribution of the constant fields to the continuum theory because for large volume and small $g(a)$ only values near the axis will not be suppressed. Because the points along the axis are gauge equivalent to the origin, and the main contributions are all concentrated near the axis, one could think of getting rid of the gauge degeneracy and, by a change of coordinates, concentrate

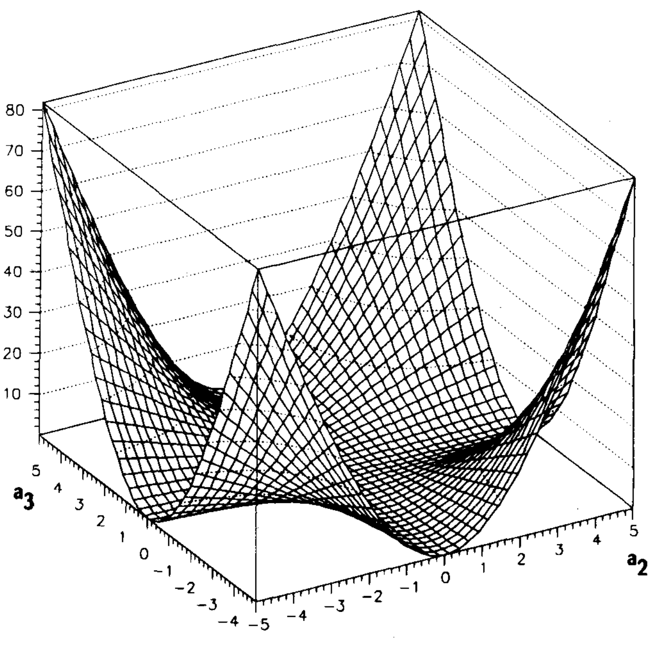

Fig. 1. The function $\sigma(0, a_2, a_3)$, (4.1)

the measure at the origin. This transformation would however be very singular because, the integral of $\exp\{2\sigma(a_2, a_3)\}$ in a ball of radius r behaving like $r^{1/2}$, the transformed quantity, if concentrated at the origin, would not be a measure. Hence it seems more convenient to work with the gauge degenerate form (3.13).

Consider the continuum version of (3.13). For the situation described above, that is, near the 3-axis in the $a_1=0$ plane, the vacuum chromomagnetic field is

$$B_1^\alpha = g(-a_2 a_3, 0, 0); \quad B_2^\alpha = B_3^\alpha = 0, \tag{4.2}$$

and the chromoelectric field is obtained by operating with $\frac{1}{i}\frac{\delta}{\delta \underset{\sim}{A}(x)}$ on ψ_0

$$E_1^\alpha = 0, \tag{4.3a}$$

$$E_2^\alpha \sim (0, i g a_2 |a_3|, 0), \tag{4.3b}$$

$$E_3^\alpha \sim \left(0, 0, i\frac{g}{2}a_2^2 \operatorname{sign}(a_3)\right), \tag{4.3c}$$

(for $a_1=0$, a_3 large and a_2 small). The vacuum chromoelectric field is imaginary consistent with the fact that ground state fluctuations are zero-energy fluctuations.

Neglecting the effect of the fermions on the gluon background, the fermion propagator is obtained from

$$\frac{1}{N}\int |\psi_0(A)|^2 \langle 0, A | T(\psi(x)\bar{\psi}(0)) | 0, A\rangle \mathrm{d}\{A\}, \tag{4.4}$$

where $|0, A\rangle$ is the fermion vacuum in the A-background and N a normalization constant chosen such that $\frac{1}{N}\int |\psi_0(A)|^2 \mathrm{d}\{A\} = 1$.

The background fields (4.2–3), computed from the ground state functional, may equivalently be described by a vector potential

$$A_0^\alpha = \left(\frac{1}{\sqrt{2}} a_2 \operatorname{sign}(a_3), 0, 0\right), \tag{4.5a}$$

$$A_1^\alpha = (0, 0, 0), \tag{4.5b}$$

$$A_2^\alpha = (0, 0, i a_3 \sqrt{2}), \tag{4.5c}$$

$$A_3^\alpha = \left(0, -\frac{i}{a_2\sqrt{2}}, 0\right). \tag{4.5d}$$

Then the fermion propagator (4.4) may be obtained by integrating

$$S(p) = \frac{1}{\left[\gamma^\mu\left(p_\mu - g A_\mu^a \frac{\sigma_a}{2}\right) - m\right]}$$

over $\mathrm{d}\{A\}$ with the density $|\psi_0(A)|^2$. The leading contributions are from configurations near the axis where $|\psi_0(A)|^2 \sim 1$. There the matrix elements of $S(p)$ are of order $1/a_3$ and $(1/a_3)^2$ and the integral $\frac{1}{\Lambda}\int^\Lambda \mathrm{d}a_3$ implies that the background-average propagator vanishes. I.e. single fermions do not propagate in this background.

Appendix A. The Dirichlet problem and the Wentzell-Freidlin method Comparison with exact solutions

Consider the eigenvalue problem

$$\left\{-\frac{\varepsilon^2}{2}\frac{\partial^2}{\partial x^2}+x\frac{\partial}{\partial x}\right\}u\{x\}=\lambda\, u\{x\} \tag{A.1}$$

in the domain $D=\{x:|x|\leq M\}$ with the boundary condition $u(\pm M)=0$, ε being a small parameter. If λ is not an integer two independent solutions of (A.1) are

$$u_\pm(x)=\exp\left(\frac{x^2}{2\varepsilon^2}\right)D_\lambda\left(\pm\frac{\sqrt{2}}{\varepsilon}x\right),$$

where $D_\lambda(x)$ is a parabolic cylinder function. A symmetric solution is

$$u(x)=\exp\left(\frac{x^2}{2\varepsilon^2}\right)\left\{D_\lambda\left(\frac{\sqrt{2}}{\varepsilon}x\right)+D_\lambda\left(-\frac{\sqrt{2}}{\varepsilon}x\right)\right\}. \tag{A.2}$$

When ε is small the argument of D_λ in the boundary of the interval $[-M, M]$ is very large and one may use the asymptotic expansion of the parabolic cylinder functions to obtain

$$u(M)\cong\left(\frac{\sqrt{2}M}{\varepsilon}\right)^\lambda\left\{1+\frac{\lambda(1-\lambda)}{4\left(\frac{M}{\varepsilon}\right)^2}+\ldots\right\}$$
$$-\frac{\pi\lambda}{\Gamma(1-\lambda)}\sqrt{\frac{2}{\pi}}\,e^{\frac{M^2}{\varepsilon^2}}\left(\frac{\sqrt{2}M}{\varepsilon}\right)^{-\lambda-1}$$
$$\cdot\left\{1+\frac{(\lambda+1)(\lambda+2)}{4\left(\frac{M}{\varepsilon}\right)^2}+\ldots\right\}. \tag{A.3}$$

Enforcing the boundary condition $u(\pm M)=0$ one obtains for small ε

$$\lambda_1\cong\frac{M}{\varepsilon\sqrt{\pi}}\,e^{-\frac{M^2}{\varepsilon^2}}, \tag{A.4}$$

for the smallest non-zero eigenvalue.

The stochastic process associated with this Dirichlet problem is

$$dx_t=-x\,dt+\varepsilon\, dW_t. \tag{A.5}$$

Estimating the lowest positive eigenvalue by the Wentzell-Freidlin technique (see 3.8–3.10) one obtains

$$\lambda_1\asymp e^{-\frac{M^2}{\varepsilon^2}}, \tag{A.6}$$

i.e. one obtains the exponential but not the power factor $\frac{M}{\varepsilon\sqrt{\pi}}$. This is typical of the large deviation estimates which are only accurate in the logarithmic equivalence sense.

The solutions $u(x)$ of the more general problem

$$Ku(x)+c(x)u(x)=f(x),\quad \text{with}\quad u(x)|_{x\in\partial D}=\psi(x), \tag{A.7}$$

where K is an elliptic operator, have the stochastic representation

$$u(x)=-E_x\int_0^\tau f(x_t)\,e^{\int_0^t c(x_s)ds}\,dt+E_x\psi(x_\tau)\,e^{\int_0^\tau c(x_s)ds}, \tag{A.8}$$

where τ is the first hitting time of the boundary ∂D and E_x denotes the expectation value for the process starting from x. For the particular case of (A.1), (A.8) reduces to

$$u(x)=E_x\psi(x_\tau)e^{\lambda\tau}, \tag{A.9}$$

which according to (3.7) is only valid for $\lambda<\lambda_1$. One might think that a limiting procedure $\lambda\to\lambda_1\,\psi(x_\tau)\to 0$ and large deviation techniques would lead to an estimate of the wave function corresponding to the lowest non-zero eigenvalue. However, as explained below, because large deviation methods reproduce accurately only the exponential factors, this is not a very useful technique for the wave functionals.

For the case of a process with one deterministic global attractor as in (A.1) and according to the Wentzell-Freidlin estimates (3.8), the process at small ε is well approximated by a Markov chain where:

- Transition probability from x to the boundary $\partial D=p_x\sim e^{-\frac{V(\partial D)-V(x)}{\varepsilon^2}}$
- Transition probability from x to the neighbourhood of the attractor $=1-p_x$
- Transition probability from the attractor to $\partial D=p_0\sim e^{-\frac{V(\partial D)-V(0)}{\varepsilon^2}}$
- Transition probability from the attractor to itself $=1-p_0$
- Transition probability from the boundary to the boundary $=1$

For (A.1), $V(x)=x^2$. Then

$$E_x(e^{\lambda\tau})=e^\lambda p_x+(1-p_x)p_0\,\{e^{2\lambda}+(1-p_0)e^{3\lambda}+\ldots\}$$
$$=e^\lambda p_x+\frac{(1-p_x)p_0e^{2\lambda}}{1-(1-p_0)e^\lambda}.$$

$E_x(e^{\lambda\tau})$ is finite only for $(1-p_0)e^\lambda<1$ which, according to (3.7), leads exactly to the estimate (3.10) and (A.6) for λ_1. For the wave function $u(x)$ corresponding to λ_1 we may consider a boundary condition $1-(1-p_0)e^\lambda$ which vanishes when $\lambda\to\lambda_1$. Then

$$u_1(x)\sim c_1\,e^{\frac{V(\partial D)-V(x)}{\varepsilon^2}}+c_2.$$

Comparing with (A.3) one sees that the exponential behaviour is reproduced but that the power behaviour entering in c_1 and c_2 is not reachable by this technique.

In some cases [15, 17] it has been shown that when $\varepsilon\to 0$ the solutions of (A.7) with $K=\frac{\varepsilon^2}{2}\Delta+b\cdot\nabla$ approach the solutions of the corresponding equation with $K=b\cdot\nabla$ and the same boundary conditions. For the particular case of (A.1) this leads to $u\sim x^\lambda$ which is accurate at small x.

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