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Noncommutative spacetime and the PeV photons from Crab

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Considering the visible and the PeV photons from SN1054 to be emitted at similar times at the source, one estimates the value of a fundamental time constant τ arising in the noncommutative spacetime formulation.

Keywords: Noncommutative space time; PeV photons.

1. Introduction

Cosmic ray photons at TeV and PeV energies are believed to originate either from massive stars or from supernova remnants (SNR).[1] In SNR, PeV photons might be produced by electrons, accelerated at the pulsar wind termination shock by inverse Compton scattering. With this mechanism SNR would be cosmic pevatron accelerators, although the details of the acceleration mechanism remain largely unknown.

Recently,[2] a few PeV photons were detected, which were assigned without ambiguity to the direction of the Crab nebula. No clear evidence of photons at PeV energies from the Crab nebula had been found before.[3] This, of course, could be explained by the improvement on the experimental techniques. On the other hand, the spectral lag data in gamma ray bursts (GRBs) have suggested that a small energy dependence, on the speed of photon wavepackets, might be detectable when they propagate over cosmological distances. This has been interpreted either as a small Lorentz invariance violation (LIV)[4] or as a manifestation of spacetime non-commutativity.[5] In the second instance, Lorentz invariance is maintained, the energy dependence arising from the fact that, because time and space are non-commuting variables, the speed of massless wave packets is different from c.

A hypothesis to be explored here is that, instead of originating from the nebula pevatron accelerator, the PeV photons, recently detected, might provide from the mechanisms associated to the core collapse of the supernova.[7,8] And because this only takes a few days, the emission of the PeV photons would have occurred at around the same time as the light signal detected in A.D. 1054.[6] From this, one would then be able to estimate the parameters of spacetime non-commutativity.

The noncommutative spacetime theory, that is used here, is based on deformation theory. The transition from non-relativistic to relativistic mechanics corresponds to deformation of the unstable Galilean algebra to the stable Lorentzian one and the transition from classical to quantum mechanics to the deformation of the unstable Poisson algebra to the stable Moyal algebra (equivalent to Heisenberg's). Likewise, the Poincaré-Heisenberg algebra of relativistic quantum mechanics (Poincaré + Heisenberg + the commutators of position and momenta with the Poincaré generators) is an unstable algebra. A two-parameter stabilization of this algebra[9,10] leads to an algebra isomorphic to either $so(5,1)$, $so(4,2)$ or $so(3,3)$, depending on the signs of the parameters. Except for the signs of the parameters, the deformation is essentially unique, given that the momentum operators should be noncommuting in the presence of gravity. For a discussion of the full manifold of deformations of the Poincaré-Heisenberg algebra refer to Ref. 11.

The stabilization introduces two new small parameters ℓ and ϕ, of dimensions L and L^{-1}, associated to the commutators

$$[x_\mu, x_\nu] = -i\epsilon\ell^2 M_{\mu\nu} \tag{1}$$

and

$$[p_\mu, p_\nu] = -i\epsilon'\phi^2 M_{\mu\nu} \tag{2}$$

ϵ and ϵ' being ± 1. In addition, the Heisenberg commutator has a nontrivial right-hand side operator $\Im$ with

$$\begin{aligned}
[p^\mu, x^\nu] &= i\eta^{\mu\nu}\Im, \\
[x^\mu, \Im] &= i\epsilon\ell^2 p^\mu, \\
[p^\mu, \Im] &= -i\epsilon'\phi^2 x^\mu, \\
[M^{\mu\nu}, \Im] &= 0,
\end{aligned} \tag{3}$$

the remaining commutators being the usual ones. In the tangent space, that is, neglecting local gravity effects, one may approximate $\epsilon' \approx 0$, obtaining the inhomogeneous algebras $iso(3,2)$ for $\epsilon = +1$ or $iso(4,1)$ for $\epsilon = -1$, the operator $\Im$ playing the role of fifth component of the momentum.

In the past, other authors have suggested non-commutativity of the spacetime coordinates, mostly in connection with gravity at short distances (see a large set of references in Ref. 10), leading to linear or nonlinear extensions of the spacetime algebra. A novel feature in the approach of stabilization by deformation is the emergence of two independent fundamental length scales. Non-commutativity of momenta being associated to gravity, the fundamental length scale associated to the commutator (2) might be related to the Planck length. However, the parameter ℓ associated to the commutator (1) is an independent parameter and if, for example, $\ell \simeq 10^{-19}$–10^{-21} cm (or $\tau = \frac{1}{\ell} \simeq 3.3 \times 10^{-30} - 3.3 \times 10^{-32}$ s) the spacetime noncommutative effects might already be observable in the laboratory[5,12] or, at least, in astrophysical events and not only at Planck scales (10^{-33} cm).

2. Wave Packet Speed in Noncommutative Spacetime

One of the consequences of the deformed Poincaré-Heisenberg algebra is that, space and time being noncommutative coordinates, they cannot be simultaneously diagonalized and speed can only be defined in terms of expectation values, that is

$$v_\psi^i = \frac{1}{\langle \psi_t, \psi_t \rangle} \frac{d}{dt} \langle \psi_t, x^i \psi_t \rangle, \tag{4}$$

where ψ is a state with a small dispersion of momentum around a central value p. At time zero

$$\psi_0 = \int |k^0 \vec{k} \alpha\rangle f_p(k) d^3 k \tag{5}$$

with $k^0 = \sqrt{|\vec{k}|^2 + m^2}$, α standing for the quantum numbers associated to the little group of k and $f_p(k)$ a normalized function peaked at $k = p$. This implies, for example, that photon wave packets will have velocities different from c, without that implying violation of Lorentz invariance, which is still preserved in the deformed Poincaré-Heisenberg algebra. In the $\epsilon' \approx 0$ limit, the nature of these deviations from c has been studied before,[5,13] the result being

$$v_\psi = \frac{p}{p^0} \frac{1 - \epsilon \ell^2 \left(\frac{p^0}{\Im}\right)^2}{1 + \epsilon \ell^2 \left(\frac{p^0}{\Im}\right)^2}. \tag{6}$$

An explicit representation $\Im = (1 + \ell^2 p_0^2)^{1/2}$ may be obtained and, in leading ℓ^2 order, one has

$$v_\psi \simeq \frac{p}{p^0} \left(1 - 2\epsilon \ell^2 \left(\frac{p^0}{\Im}\right)^2\right). \tag{7}$$

Notice that the correction is negative or positive depending on the sign of ϵ. For example, a massless particle wave packet would be found to travel slower or faster than c according to whether $\epsilon = +1$ (quantized time) or $\epsilon = -1$ (quantized space). This deviation from c, for the velocity of the massless particle wave packet, implies no violation of relativity. Lorentz invariance is still an exact symmetry in the deformed algebra and the velocity corrections do not arise from modifications of the dispersion relation for elementary states but from the noncommutativity of time and space.

3. The PeV Photons from Crab

In July 4, 1054, as recorded by the Chinese astronomers or at about that time,[6] visible light from the SN1054 supernova was observed at earth. Consider as a working hypothesis that, as compared to the propagation time from the Crab nebula to earth, both the visible and the PeV photons[2] were emitted at similar times. A weak gravity approximation, that is, the $\epsilon' \approx 0$ limit in the deformed algebra, is a reasonable approximation along the worldline of the photons. Hence, using Eq. (7) and the arrival time delays Δt listed in the table, one may now estimate the value of the constant ℓ.

The time lag of arrival to earth of two photons of energies $p_{0(1)}$ and $p_{0(2)}$ (energies measured at earth) would be

$$\Delta t = \frac{\epsilon \ell^2}{H_0} I_2(z) \left(p_{0(1)}^2 - p_{0(2)}^2\right) + a \tag{8}$$

with

$$I_2(z) = \int_0^z dz' \frac{\Omega_{m,0}(1+z')^2(z'^2+2z'+4)+4(1+z')\Omega_{\Lambda,0}}{(\Omega_{m,0}(1+z')^3+\Omega_{\Lambda,0})^{3/2}} \tag{9}$$

z being the redshift of the source (for a derivation see Ref. 5). a is the intrinsic lag corresponding to different emission times, which will be neglected here, and use the values $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$, $H_0 = 70$ Km s^{-1} Mpc^{-1}.

For very small redshifts, as is the case for SN1054

$$I_2(z) \simeq 4z. \tag{10}$$

Using the time lags listed in the table, for the highest energy photons detected from the direction of the Crab,[2] and $p_{0(2)} \simeq 3$ eV (visible light), one obtains the corresponding ℓ values and $\epsilon = +1$

PeV	Arrival time	Δt (years)	$\ell\,(10^{-21}$ cm)
1.12	2021-01-04, 16:45:06	966.5	3.39
0.88	2020-01-11, 17:59:18	965.52	4.32
0.57	2020-05-22, 03:54:56	965.83	6.67
0.46	2020-11-05, 21:23:28	966.33	8.27
0.40	2020-04-30, 09:57:54	965.82	9.5

and averaging

$$\ell = (6 \pm 3) \times 10^{-21} \text{cm} \tag{11}$$

or

$$\tau = (2 \pm 1) \times 10^{-31} \text{ s.} \tag{12}$$

Actually, ϵ being $+1$, it is time that has a discrete spectrum and it makes more sense to refer to a fundamental time τ, rather than to a fundamental length ℓ.

Previous estimates of ℓ using data on the spectral lags of a few gamma ray bursts (GRBs)[5] or a similar analysis on the data used in the context of the search for Lorentz invariance violation (LIV)[4] implied an ℓ in the range 10^{-19}–10^{-21} cm. The spectral lag results suffer from large uncertainties on the calculation of the spectral lags, on the statistics of the GRB pulses and on the intrinsic spectral lags. In the framework of the present working hypothesis, of almost simultaneous emission of visible light and PeV photons from SN1054, the result in (12) might be more precise. The uncertainty in the error bars in (12) might result from intrinsic spectral lags at the source, which is improbable given the fast collapse time of supernova or from uncertainties on what was the actual photon energy at the source.

The OPERA[14] experiment has tried to look for eventual deviations from c on the propagation speed of neutrinos. The corrected 2012 OPERA data for 17 GeV neutrinos is

$$\left|\frac{v_\psi - c}{c}\right| = \left(2.7 \pm 3.1(\text{stat})\,{}^{+3.4}_{-3.3}\,(\text{sys})\right) \times 10^{-6}.$$

With the estimate in (12) the value of $\left|\frac{v_\psi - c}{c}\right|$ would be in the range 10^{-10}–10^{-11}, much too small to be detected at OPERA. However, confirmation of a fundamental time on the order of 10^{-31} s, would have a large impact, at least on astrophysical estimates. It would be desirable to repeat an OPERA-type experiment with a much longer baseline and improved precision.

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