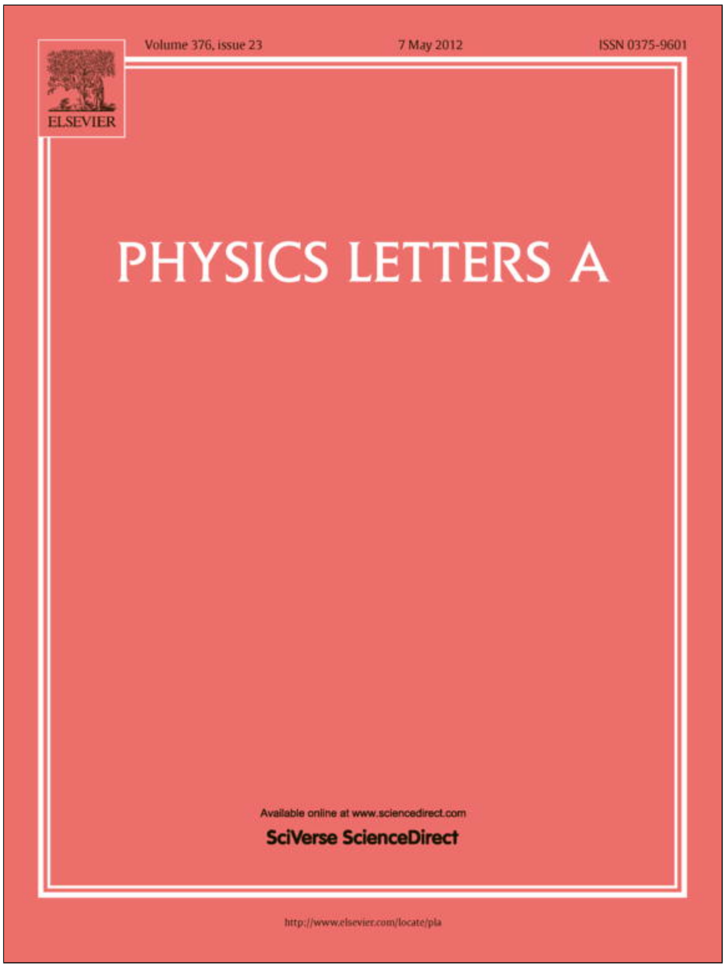
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The deformation-stability fundamental length and deviations from c

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ABSTRACT

A fundamental length (or time) is conjectured in many contexts. The "stability of physical theories principle" provides an unambiguous derivation of the stable structures that Nature might have chosen for its algebraic framework. $1/c$ and $\hbar$ are the deformation parameters that stabilize the Galilean and Poisson algebras. The stability principle applied to the Poincaré–Heisenberg algebra, yields two deformation parameters defining two length (or time) scales. One of the scales is probably related to Planck's length but the other might be much larger. This is used as working hypothesis to compute deviations from c in speed measurements of massless particles.

1. Introduction

The idea of modifying the algebra of the space–time components x_μ in such a way that they become non-commuting operators has appeared many times in the physical literature [1–20], etc. The aim of most of these proposals was to endow space–time with a discrete structure, to be able, for example, to construct quantum fields free of ultraviolet divergences. Sometimes a non-zero commutator is simply postulated, in some other instances the motivation is the formulation of field theory in curved spaces. String theories [21,22] and quantum relativity [23,24] have also provided hints concerning the non-commutativity of space–time at a fundamental level.

A somewhat different point of view has been proposed in [25,26]. There the space–time non-commutative structure is arrived at through the application of the *stability of physical theories principle* (SPT). The rationale behind this principle is the fact that the parameters entering in physical theories are never known with absolute precision. Therefore, robust physical laws with a wide range of validity can only be those that do not change in a qualitative manner under a small change of parameters, that is, *stable* (or *rigid*) theories. The stable-model point of view originated in the field of non-linear dynamics, where it led to the notion of *structural stability* [27,28]. Later on, Flato [29] and Faddeev [30] have shown that the same pattern occurs in the fundamental theories of Nature, namely the transition from non-relativistic to relativistic and from classical to quantum mechanics, may be interpreted as the replacement of two unstable theories by two stable ones. The stabilizing deformations lead, in the first case, from the Galilean to the Lorentz algebra and, in the second one, from the algebra of commutative phase-space to the Moyal–Vey algebra (or equivalently to the Heisenberg algebra). The deformation parameters are $\frac{1}{c}$ (the inverse of the speed of light) and h (the Planck constant). Except for the isolated zero value, the deformed algebras are all equivalent for non-zero values of $\frac{1}{c}$ and h. Hence, relativistic mechanics and quantum mechanics might have been derived from the conditions for stability of two mathematical structures, although the exact values of the deformation parameters cannot be fixed by purely algebraic considerations. Instead, the deformation parameters are fundamental constants to be obtained from experiment and, in this sense, not only is deformation theory the theory of stable theories, it is also the theory that identifies the fundamental constants.

The SPT principle is related to the idea that physical theories drift towards simple algebras [31–33], because all simple algebras are stable, although not all stable algebras are simple.

When the SPT principle is applied to the algebra of relativistic quantum mechanics (the Poincaré–Heisenberg algebra)

$$
\begin{aligned}
&[M_{\mu\nu}, M_{\rho\sigma}] = i(M_{\mu\sigma}\eta_{\nu\rho} + M_{\nu\rho}\eta_{\mu\sigma} - M_{\nu\sigma}\eta_{\mu\rho} - M_{\mu\rho}\eta_{\nu\sigma})\\
&[M_{\mu\nu}, p_\lambda] = i(p_\mu\eta_{\nu\lambda} - p_\nu\eta_{\mu\lambda})\\
&[M_{\mu\nu}, x_\lambda] = i(x_\mu\eta_{\nu\lambda} - x_\nu\eta_{\mu\lambda})\\
&[p_\mu, p_\nu] = 0\\
&[x_\mu, x_\nu] = 0\\
&[p_\mu, x_\nu] = i\eta_{\mu\nu}\mathbf{1}
\end{aligned}
\tag{1}
$$

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$\eta_{\mu\nu} = (1, -1, -1, -1)$, $c = \hbar = 1$, it leads [25] to

$$\begin{aligned}
&[M_{\mu\nu}, M_{\rho\sigma}] = i(M_{\mu\sigma}\eta_{\nu\rho} + M_{\nu\rho}\eta_{\mu\sigma} - M_{\nu\sigma}\eta_{\mu\rho} - M_{\mu\rho}\eta_{\nu\sigma})\\
&[M_{\mu\nu}, p_\lambda] = i(p_\mu\eta_{\nu\lambda} - p_\nu\eta_{\mu\lambda})\\
&[M_{\mu\nu}, x_\lambda] = i(x_\mu\eta_{\nu\lambda} - x_\nu\eta_{\mu\lambda})\\
&[p_\mu, p_\nu] = -i\frac{\epsilon'}{R^2}M_{\mu\nu}\\
&[x_\mu, x_\nu] = -i\epsilon\ell^2 M_{\mu\nu}\\
&[p_\mu, x_\nu] = i\eta_{\mu\nu}\Im\\
&[p_\mu, \Im] = -i\frac{\epsilon'}{R^2}x_\mu\\
&[x_\mu, \Im] = i\epsilon\ell^2 p_\mu\\
&[M_{\mu\nu}, \Im] = 0
\end{aligned} \tag{2}$$

Stabilizing (rigidifying) the Poincaré–Heisenberg algebra leads to either $O(1,5)$, $O(2,4)$ or $O(3,3)$ depending on the signs ϵ and ϵ'. When $R \to \infty$ one obtains inhomogeneous $IO(1,4)$ or $IO(2,3)$. The fact that one is using algebraic deformation theory, rather than adding ad-hoc enveloping algebra operators has as a consequence that the representation theory of such algebras is useful in exploring the consequences of the theory (see for example [26] and [39]).

The stabilization of the Poincaré–Heisenberg algebra has been further studied and extended in [34–36]. The essential message from (2) or from the slightly more general form obtained in [34] is that from the unstable Poincaré–Heisenberg algebra $\{M_{\mu\nu}, p_\mu, x_\nu\}$ one obtains a stable algebra with two deformation parameters ℓ and $\frac{1}{R}$. In addition there are two undetermined signs ϵ and ϵ' and the central element of the Heisenberg algebra becomes a non-trivial operator $\Im$. The existence of two continuous deformation parameters when the algebra is stabilized is a novel feature of the deformation point of view, which does not appear in other non-commutative space–time approaches. These deformation parameters may define two different length scales. Of course, once one of them is identified as a fundamental constant, the other will be a pure number.

Being associated to the non-commutativity of the generators of space–time translations, the parameter $\frac{1}{R}$ may be associated to space–time curvature and therefore might not be relevant for considerations related to the tangent space. It is, of course, very relevant for quantum gravity studies [36]. Already in the past, some authors [30], have associated the non-commutativity of translations to gravitational effects, the gravitation constant being the deformation parameter. Presumably then $\frac{1}{R}$ might be associated to the Planck length scale. However ℓ, the other deformation parameter, defines a completely independent length scale which might be much closer to laboratory phenomena. This will be the working hypothesis to be explored in this Letter. Therefore when $\frac{1}{R}$ is assumed to be very small the deformed algebra may be approximated by

$$\begin{aligned}
&[M_{\mu\nu}, M_{\rho\sigma}] = i(M_{\mu\sigma}\eta_{\nu\rho} + M_{\nu\rho}\eta_{\mu\sigma} - M_{\nu\sigma}\eta_{\mu\rho} - M_{\mu\rho}\eta_{\nu\sigma})\\
&[M_{\mu\nu}, p_\lambda] = i(p_\mu\eta_{\nu\lambda} - p_\nu\eta_{\mu\lambda})\\
&[M_{\mu\nu}, x_\lambda] = i(x_\mu\eta_{\nu\lambda} - x_\nu\eta_{\mu\lambda})\\
&[p_\mu, p_\nu] = 0\\
&[x_\mu, x_\nu] = -i\epsilon\ell^2 M_{\mu\nu}\\
&[p_\mu, x_\nu] = i\eta_{\mu\nu}\Im\\
&[p_\mu, \Im] = 0\\
&[x_\mu, \Im] = i\epsilon\ell^2 p_\mu\\
&[M_{\mu\nu}, \Im] = 0
\end{aligned} \tag{3}$$

Notice that in addition to the space–time non-commutative structure, there is also a new non-trivial operator $\Im$ which replaces the central element of the Heisenberg algebra. In particular this operator corresponds to an additional component in the most general connections compatible with (3) [26].

For future reference this algebra will be denoted $\mathcal{R}_{\ell,\infty}$. Notice that in relation to the more general deformation obtained in [34], we are also considering $\alpha_3 = 0$ (or $\beta = 0$ in [36]). The nature of the sign ϵ has physical consequences. If $\epsilon = +1$ time will have a discrete spectrum, whereas if $\epsilon = -1$ it is when one the space coordinates is diagonalized that discrete spectrum is obtained. In this sense if $\epsilon = +1$, ℓ might be called "the fundamental time" and "the fundamental length" if $\epsilon = -1$.

As far as the commutation relations between the space–time coordinates and their relation to the generators of the Lorentz are concerned, Snyder's [1] non-commutative space–time is similar to the one proposed here. However the full Snyder's model is not a Lie algebra, the commutation relation of coordinates with the momenta being quadratic in the momenta. On the other hand the non-commutative structure is here obtained in a consistent manner by the application of algebraic deformation theory, and by requiring a stability principle for physical theories. This has several important consequences: First, the Lie algebraic setting is preserved, second, two length scales emerge, third, a new operator replaces the trivial center of the Heisenberg algebra which has important consequences for the differential algebra associated to non-commutative space–time (see [26]). In particular this implies that whereas the space–time dimensions are preserved, an additional "dimension" appears in the differential algebra which might have important implications for quantum fields that are connections.

General (non-commutative) geometry properties of the algebra (3) have been studied before [26] as well as some other consequences [37–41]. Here the emphasis will be on the hypothesis that ℓ defines a length scale much larger than Planck's and on its consequences for deviations from c in speed measurements of massless wave packets.

In Appendix A, some explicit representations of the space–time algebra are collected, which are useful for the calculations.

2. Measuring the speed of wave-packets

In the non-commutative context, because the space and the time coordinates cannot be simultaneously diagonalized, speed can only be defined in terms of expectation values, for example

$$v_\psi^i = \frac{1}{\langle\psi_t, \psi_t\rangle}\frac{d}{dt}\langle\psi_t, x^i\psi_t\rangle \tag{4}$$

Here, one considers a normalized state ψ with a small dispersion of momentum around a central value p. At time zero

$$\psi_0 = \int \left|k^0\vec{k}\alpha\right\rangle f_p(k)\, d^3k \tag{5}$$

where $k^0 = \sqrt{|\vec{k}|^2 + m^2}$, α standing for the quantum numbers associated to the little group of k and $f_p(k)$ is a normalized function peaked at $k = p$.

To obtain ψ_t one should apply to ψ_0 the time-shift operator. However this is not p^0 because

$$e^{-iap^0} t e^{iap^0} = t + a\Im \tag{6}$$

follows from

$$[p^0, t] = i\Im \tag{7}$$

whereas a time-shift generator Γ should satisfy

$$[\Gamma, t] = i\mathbf{1} \tag{8}$$

In order $O(\ell^4)$ one has

$$\Gamma = p^0 \Im^{-1} - \frac{\epsilon}{3}\ell^2 (p^0)^3 \Im^{-3} \tag{9}$$

because

$$[\Gamma, t] = i\left(1 - \ell^4 (p^0)^4 \Im^{-4}\right) \tag{10}$$

To obtain this result, use was made of $[t, \Im^{-1}] = -i\epsilon\ell^2 p^0 \Im^{-2}$, which follows from $[t, \Im\Im^{-1}] = 0$.

Now use a basis where the set $(p^\mu, \Im)$ is diagonalized and define

$$\tilde{p}^\mu = \frac{p^\mu}{\Im} \tag{11}$$

$\tilde{p}^\mu$ is the momentum in units of $\Im$.

Therefore, in the same $O(\ell^4)$ order

$$\psi_t = \int \exp\left(-it\left(\tilde{p}^0 - \frac{\epsilon}{3}\ell^2 (\tilde{p}^0)^3\right)\right) |\tilde{k}^0 \tilde{k}^i \alpha\rangle f_p(\tilde{k})\, d^3\tilde{k} \tag{12}$$

To compute the expectation value of x^i one notices that from (18)

$$x^\mu = i\left(\epsilon\ell^2 p^\mu \frac{\partial}{\partial \Im} - \Im \frac{\partial}{\partial p_\mu}\right) \tag{13}$$

using $\frac{\partial}{\partial \Im} = -\frac{p^\nu}{\Im^2}\frac{\partial}{\partial \tilde{p}^\nu}$ one obtains

$$x^\mu = -i\left(\frac{\partial}{\partial \tilde{p}_\mu} + \epsilon\ell^2 \left\{\tilde{p}^\mu \tilde{p}^\nu \frac{\partial}{\partial \tilde{p}^\nu}\right\}_S\right) \tag{14}$$

$\{\}_S$ meaning symmetrization of the operators.

Now the expectation value of this operator in the state ψ_t is computed and taking the time derivative one obtains for the wave packet speed in order ℓ^2

$$v_\psi = \frac{\tilde{p}}{\tilde{p}^0}\left(1 - \epsilon\ell^2 (p^0)^2\right) - \epsilon\ell^2 \left(\tilde{p} p^0 + (\tilde{p})^2 \frac{\tilde{p}}{\tilde{p}^0}\right) \tag{15}$$

ℓ^2 being small, this deviation from $\frac{\tilde{p}}{\tilde{p}^0}$ may be difficult to detect for massive particles given the uncertainty on the values of the masses. However, for massless particles the deviation from c $(=1)$

$$\Delta v_\psi = -3\epsilon\ell^2 (p^0)^2 \tag{16}$$

might already be possible to detect accurately with present experimental means. Such deviation above or below the speed c (depending on the sign of ϵ) would not imply any modification of the relativistic deformation constant ($\frac{1}{c}$), nor a breakdown of relativity.

To have a quantitative idea of the intensity of the effects that might be expected from (16) let $p^0 = 20$ GeV and $\ell = 3 \times 10^{-18}$ cm (or equivalently $\ell \simeq 10^{-28}$ s). Then

$$\left(\frac{v_\psi - c}{c}\right)_{20} = (-\epsilon) \cdot 2.77 \times 10^{-5}$$

However, the effect is strongly dependent on energy. With the same ℓ but $p^0 = 3$ GeV, it would only be

$$\left(\frac{v_\psi - c}{c}\right)_{3} = (-\epsilon) \cdot 6.2 \times 10^{-7}$$

and for $p^0 = 2.5$ eV (visible light)

$$\left(\frac{v_\psi - c}{c}\right)_{25} = (-\epsilon) \cdot 4.3 \times 10^{-25}$$

Of course, with ℓ of the order of Planck's length (1.6×10^{-33} cm), in all cases, the effect would much too small to be observable at the present time. The deviation from c of the "effective" velocity of massless particles would be negative or positive depending on the $\epsilon = \pm$ sign of the deformation class.

Already in the past, either as a probe of the quantum nature of space–time in the framework of quantum gravity [42–44] or in the q-deformation context [45], other authors have suggested deviations from c for massless particles. Also such deviations have been suggested as an explanation for the spectral time lags in gamma ray bursts [46]. However, in all cases this involved violation of Lorentz invariance. In contrast, the deformed algebra (3) is consistent with Lorentz invariance, the deviations from c originating instead from the non-commutativity of the time and space coordinates.

Deviations from c in the effective speed, arising from the non-commutativity, would be felt by all massless particles. Therefore they should also be taken into account on the calculation of the spectral lags of gamma ray bursts (GRB). However, because of the spectral evolution occurring in the prompt phase of the GRB's, it would be difficult to untangle the effects. In this sense careful precision experiments with neutrino beams are still to be encouraged. Notice that the non-commutative deviation may be positive or negative depending on the plus or minus sign ϵ. This sign is physically very relevant in the sense that if it is positive, it is time that has a discrete spectrum, whereas for negative sign the spectral discreteness lies on the space coordinates.

3. Remarks and conclusions

(1) The most relevant point of the stability approach to non-commutative space–time is the emergence of two deformation parameters, which might define different length scales. This led to the conjecture that one of them might be much larger than the Planck length and therefore already detectable with contemporary experimental means.

(2) The deviation from c when measuring the velocity of wavepackets of massless (or near massless) particles, does not implies any violation of relativity nor does it imply a modification of the value of the deformation parameter $\frac{1}{c}$. What it perhaps implies is that c should not be called the speed of light.

(3) Other effects arising from the deformation-stability non-commutative structure are explored in Ref. [47]. Both the effects explore here and in [47] are rather conservative in the sense that they explore well-known physical observables. In the deformation setting, that underlies the calculations in this Letter, the physical space–time would still have only 4 dimensions. What gains an extra dimension is the differential algebra and that may have an effect on the interactions of gauge fields because they are connections. Hence, a more speculative aspect of the non-commutative structure concerns the physical relevance of the extra derivation ∂_4 in the geometrical structure. This includes new fields associated to gauge interactions which may lead to effective mass terms for otherwise massless particles (see [26] for more details).

Appendix A. Representations of the deformed algebra and its subalgebras

For explicit calculations of the consequences of the non-commutative space–time algebra (2) (with $\epsilon' = 0$) it is useful to have at our disposal functional representations of this structure.

Such representations on the space of functions defined on the cone C^4 ($\epsilon = -1$) or $C^{3,1}$ ($\epsilon = +1$) have been described in [26]. Here one collects a few other useful representations of the full algebra and some subalgebras.

1. As differential operators in a 5-dimensional commutative manifold $M_5 = \{\xi_\mu\}$ with metric $\eta_{aa} = (1, -1, -1, -1, \epsilon)$

$$p_\mu = i\frac{\partial}{\partial\xi^\mu}$$
$$\Im = 1 + i\ell\frac{\partial}{\partial\xi^4}$$
$$M_{\mu\nu} = i\left(\xi_\mu\frac{\partial}{\partial\xi^\nu} - \xi_\nu\frac{\partial}{\partial\xi^\mu}\right)$$
$$x_\mu = \xi_\mu + i\ell\left(\xi_\mu\frac{\partial}{\partial\xi^4} - \epsilon\xi^4\frac{\partial}{\partial\xi^\mu}\right) \tag{17}$$

2. Another global representation is obtained using the commuting set $(p^\mu, \Im)$, namely

$$x_\mu = i\left(\epsilon\ell^2 p_\mu\frac{\partial}{\partial\Im} - \Im\frac{\partial}{\partial p^\mu}\right)$$
$$M_{\mu\nu} = i\left(p_\mu\frac{\partial}{\partial p^\nu} - p_\nu\frac{\partial}{\partial p^\mu}\right) \tag{18}$$

3. Representations of subalgebras.

Because of non-commutativity only one of the coordinates can be diagonalized. Here, consider the restriction to one space dimension, namely the algebra of $\{p^0, \Im, p^1, x^0, x^1\}$.

For $\epsilon = +1$ define hyperbolic coordinates in the plane $(p^1, \Im)$ and polar coordinates in the plane $(p^0, \Im)$. Then, from it follows from (18)

$$p^1 = \frac{r}{\ell}\sinh\mu$$
$$p^0 = \frac{\gamma}{\ell}\sin\theta$$
$$\Im = r\cosh\mu = \gamma\cos\theta$$
$$x^1 = i\ell\frac{\partial}{\partial\mu}$$
$$x^0 = -i\ell\frac{\partial}{\partial\theta} \tag{19}$$

For $\epsilon = -1$ with polar coordinates in the plane $(p^1, \Im)$ and hyperbolic coordinates in the plane $(p^0, \Im)$,

$$p^1 = \frac{r}{\ell}\sin\theta$$
$$p^0 = \frac{\gamma}{\ell}\sinh\mu$$
$$\Im = \gamma\cosh\mu = r\cos\theta$$
$$x^1 = i\ell\frac{\partial}{\partial\theta}$$
$$x^0 = -i\ell\frac{\partial}{\partial\mu} \tag{20}$$

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