The quantum modality of knowledge
Games and decision

1 - An operational notion of knowledge

2 - How is the knowledge acquisition carried out? What modalities? The measurement algebra

3. Some consequences of the quantum modality

4. Non-compatible observables in Nature

5. Games. Shortcomings of Nash equilibrium

6. Quantum games. Examples

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8. Other uses of the quantum paradigm

9. Deterministic, non-deterministic and quantum computation/decision

1. An operational notion of knowledge

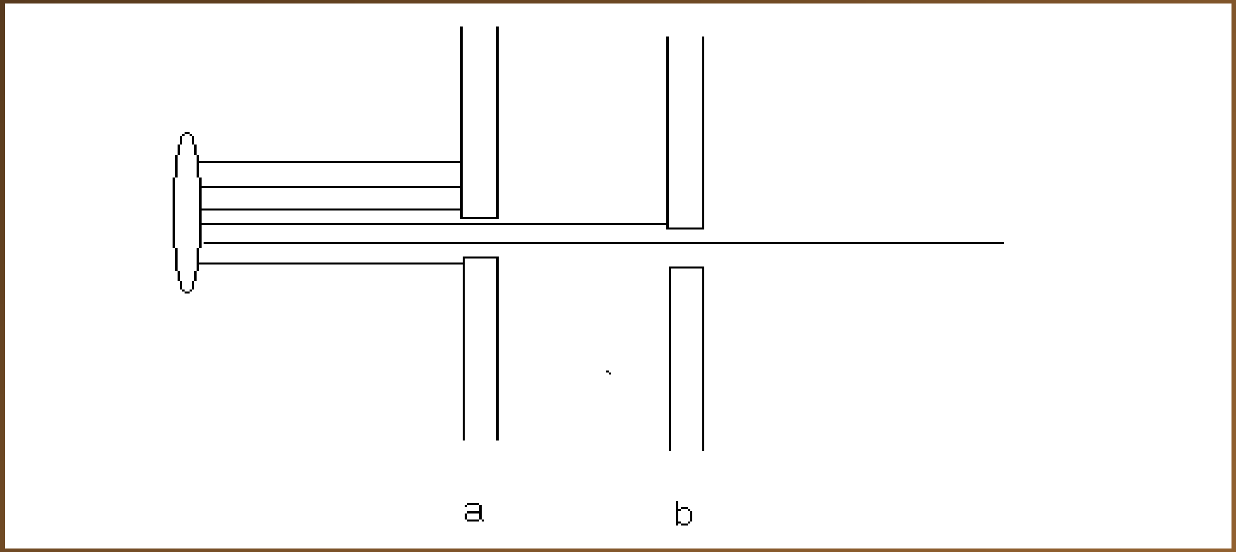
- "Knowledge" = { *Capacity to predict the result of an action* }
- { *Capacity to predict the result of an action* } $\Leftarrow$ { Rules + consequences }
- The dynamic (temporal) nature of human knowledge !
- {Knowledge acquisition} = {1. Questions
  2. Record the answers
  3. Compress the information}
- The questions may be reduced to binary questions **(Yes, No)**

2. How is the knowledge acquisition carried out? What modalities ?

- A tool : The measurement algebra (J. Schwinger)

The measurement algebra (J. Schwinger)

- Instruments ⇔ Observables (**a**)
- Measurements ⇔ Filters applied to a set



- Measurement symbols

  a ⇔ M(a)

  M(a,b) = *Selects for the **a**-properties and then transforms to **b**-properties*

◆ Compatible observables

- $M(a)\ M(b)\ M(a) = M(b)\ M(a)$

  (filtering for one observable does not change the purity of the set concerning the others)

- $M(a)M(b)=M(b)M(a)$ comutativity

- $M(a,b)\ \ M(c,d) = \delta(b,c)\ \ M(a,d)$

$\delta(b,c)$ is a number : 0 ou 1

◆ Non-compatible observables (a and b)

*Measurement of **a** followed by measurement of **b** leads to a set where the value of **a** is no longer well defined*

$M(a)M(b) \neq M(b)M(a)$

$M(a,b)\ M(c,d) = \langle b|c\rangle\ \ M(a,d)$

Two modalities :

- 1) *Classical modality*
  All observables are compatible
  All measurement symbols commute

  $M(a)\ M(b) = \delta(a\ ,\ b)\ M(a)$

- 2) *Quantum modality*
  Some observables are not compatible

  $M(c,a)\ M(b,d) = \langle a|b\rangle\ M(c,d)$

  $\langle a|b\rangle$ must be related to the probability that states prepared for b might be found to have the property a
  Cannot be a probability because of the invariance o f the measurement symbols algebra
  $M(a,b) \rightarrow \lambda(a)\ M(a,b)\ \lambda^{-1}(b)$
  $\langle a|b\rangle \rightarrow \lambda^{-1}(a)\ \langle a|b\rangle\ \lambda(b)$
  The simplest invariant choice is :
  $p(a,b) = \langle a|b\rangle\langle b|a\rangle$
  For $p(a,b)$ to be real $\langle a|b\rangle = \langle b|a\rangle^*$ Complex field

  Conclusion : The quantum theory is the simplest theory compatible with the fact that some variables are incompatible

Codings :

- The Hilbert space formulation is just a coding of the measurement algebra

  $|a\rangle \in V \qquad \langle b| \in V^*$

  $\langle a|b\rangle : V^* \times V \to \mathbf{C}$

  $M(a,b) = |a\rangle \langle b|$

  $p(a,b) = \langle a|b\rangle \langle b|a\rangle = |\langle a|b\rangle|^2$

  Temporal evolution by unitary operators to preserve probability

  $$i\, d/dt\, |\Psi(t)\rangle = H\, |\Psi(t)\rangle$$

- Other codings :

  - Phase space with deformed (Moyal) algebra of functions
  - The tomographic coding

- Whenever there are incompatible variables it is the quantum modality of information processing that must be applied, no matter in which field.
- A similar situation may hold if one is not sure about what variables are incompatible, but some of the quantum modality consequences seem to be present

3.“Strange” (non-obvious) consequences of a simple hypothesis

- ***Superposition***
  If $\Phi \in H$ and $\Psi \in H$ are states then $\Phi + \Psi \in H$ also is

  $\Rightarrow$ wave properties of matter
  cat-type “paradoxes”

- ***Entanglement***
  $\Rightarrow$ Instantaneous non-local effects

  $$|a_1\, b_1\rangle + |a_2\, b_2\rangle$$

  If a measurement is made on the first system and the result is a_1 , then the second system is automatically known to be in state b_1

4 .Non compatible observables in Nature

- In physics
  Position (x) and Momentum (p)
  Position (x) and Angular momentum (J)
  Time (t) and Energy (E)

- In other fields
  Price and Ownership
  Time and National Product

- Notice : Incompatibility is an operational concept meaning simultaneous measurement. One may always “speak” of the energy at a given time, but to actually do such a measurement is a different matter.
  - Identically for momentum and position (after the measurement), etc.

5. Games - Nash equilibrium

- $(s_1,s_2,\ldots,s_k,\ldots,s_n)$ is Nash equilibrium if
  $P(s_1,s_2,\ldots,s_k,\ldots,s_n)>P(s_1,s_2,\ldots,s_k',\ldots,s_n)$
  for all s_k
- No player can improve his payoff by changing his strategy, when the strategies of the other players are fixed
- Every N-player game, with finite strategies, has at least one Nash equilibrium, in pure or mixed strategies

Nash equilibria. Two examples

- The battle of sexes

(John)

		Opera	TV
(Mary)	Opera	(5,2)	1,1
	TV	1,1	(2,5)

The ultimatum game

$$
\begin{array}{lcclcc}
 & & & \text{Acceptance} & \rightarrow & (a,c) \\
 & & & & & \uparrow \\
\text{Proposer} & \begin{matrix} a \\ c \end{matrix} & \rightarrow & \text{Responder} & \Rightarrow & \text{Payoff} \\
 & & & & & \downarrow \\
 & & & \text{Non-accept.} & \rightarrow & (0,0)
\end{array}
$$

The ultimatum game

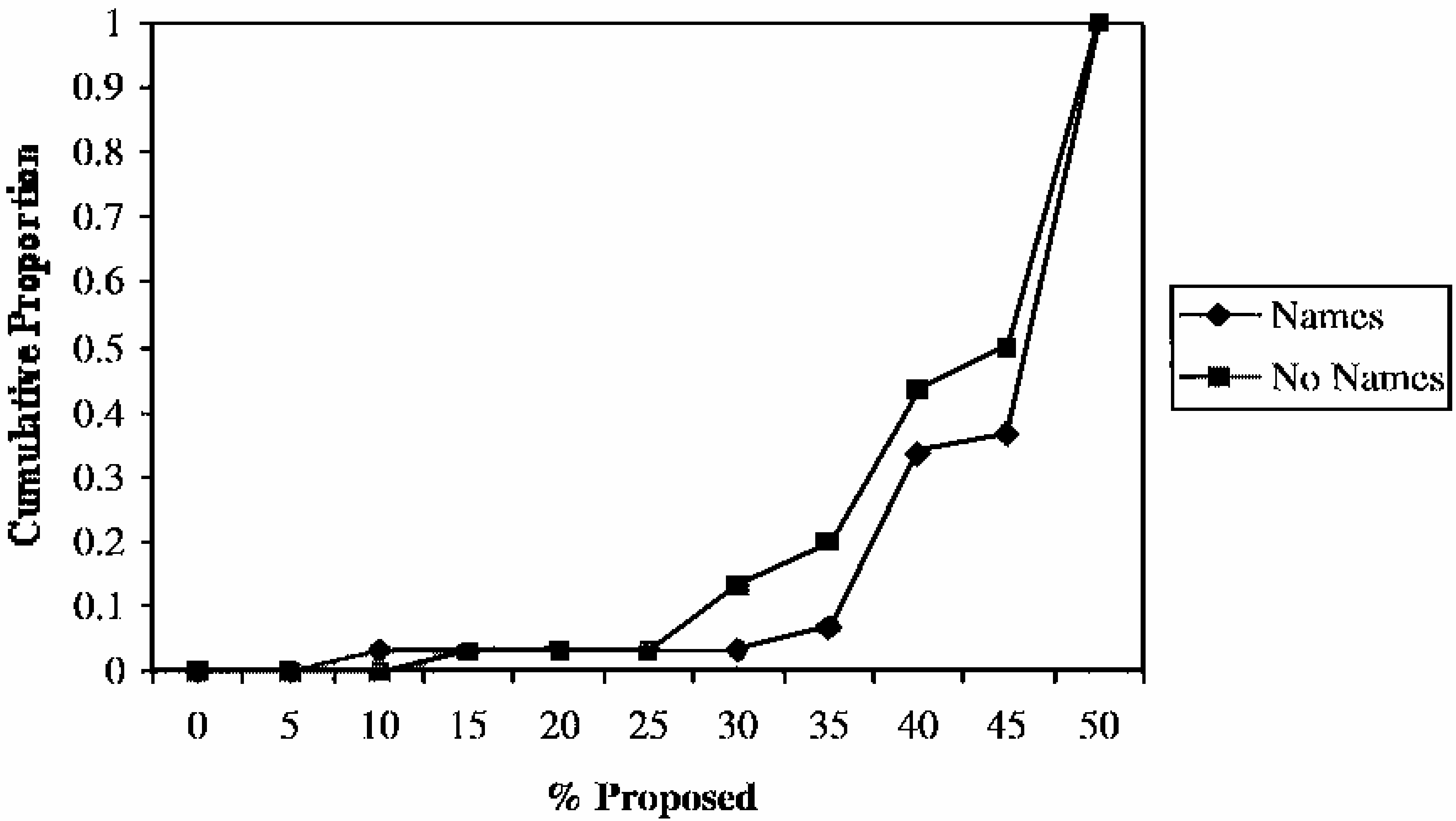
- a+c=2b , a>>c, (Example: a=99, c=1, b=50)

	R0	R1
P0	(a,c)	0,0
P1	b,b	0,0

Experimental ultimatum game

- University students

Figure 2 - Cumulative Ultimatum Proposals



Experimental ultimatum game

- Similar experiments in small-scale societies
- Nash equilibrium rejected in all cases
- The player's behavior is strongly correlated with existing social norms in their societies and market structure
- Human decision problems involve a mixture of self-interest and a background of (internalized) social norms
- Could the internalized social norms be codified by quantum games ?

6. Quantum games

- In a classical game the space of strategies is a discrete space or a simplex (for mixed strategies)
- In a quantum game the space of strategies is a linear space (Hilbert space)
- Given an initial state the players' moves are unitary operations in the (linear) space of strategies

6. Quantum games. An example

The battle of sexes (Classical)

		John	
		O(0)	T(1)
Mary	O(0)	(α,β)	(γ,γ)
	T(1)	(γ,γ)	(β,α)

$$\alpha > \beta > \gamma$$

Mixed strategies :

Mary	O$\rightarrow p$	,	T$\rightarrow (1-p)$
John	O$\rightarrow q$	,	T$\rightarrow (1-q)$

3 classical Nash equilibria :

$$p=1, q=1 \qquad (\alpha,\beta)$$

$$p=0, q=0 \qquad (\beta,\alpha)$$

$$p=\frac{\alpha-\gamma}{\alpha+\beta-2\gamma}, q=\frac{\beta-\gamma}{\alpha+\beta-2\gamma} \qquad (P',P')$$

$$\alpha > \beta > P' = \frac{\alpha\beta-\gamma^2}{\alpha+\beta-2\gamma} > \gamma$$

6. Quantum games. An example

Quantum version

Initial state (initial strategy): Any linear combination of $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$$J|00\rangle \qquad \text{with} \qquad J \in SU(2)$$

Allowed moves: $A, B \in \left\{ I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ or } \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}$

Factorized strategies (factorized initial state)

Same results as in the classical case

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Factorized strategies (factorized initial state)

Same results as in the classical case

Entangled strategy $\qquad J|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Three Nash equilibria as before, but

$$\begin{array}{ll} p_I = 1, q_I = 1 & \left(\frac{\alpha+\beta}{2}, \frac{\alpha+\beta}{2}\right) \\ p_I = 0, q_I = 0 & \left(\frac{\alpha+\beta}{2}, \frac{\alpha+\beta}{2}\right) \\ p_I = \frac{1}{2}, q_I = \frac{1}{2} & \left(\frac{\alpha+\beta+2\gamma}{4}, \frac{\alpha+\beta+2\gamma}{4}\right) \end{array}$$

The best solution occurs when both players play the same move (I or σ_x)

Because $\frac{\alpha+\beta}{2} > \beta$, this Nash equilibrium is better than the third solution of the classical case. (*Entanglement as a Contract*).

7. The quantum ultimatum game

	R_0	R_1
P_0	$\|00\rangle$ a; c	$\|01\rangle$ $0; 0$
P_1	$\|10\rangle$ b; b	$\|11\rangle$ $0; 0$

$a \gg c$, $a + c = 2b$ (for example $a = 99; c = 1; b = 50$)

The unique classical Nash equilibrium is $|00\rangle$, $\longrightarrow$ the greedy proposal $(a; c)$

7. The quantum ultimatum game

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$a \gg c$, $a + c = 2b$ (for example $a = 99$; $c = 1$; $b = 50$)

The unique classical Nash equilibrium is $|00\rangle$, $\longrightarrow$ the greedy proposal $(a; c)$

The 2 options ($|0\rangle$ or $|1\rangle$) of each player are a basis for 2 two-dimensional linear spaces $\mathcal{H}_P$ and $\mathcal{H}_R$.

The *space of the game* is $\mathcal{H} = \mathcal{H}_P \otimes \mathcal{H}_R$ with basis $\{|00\rangle ; |01\rangle ; |10\rangle ; |11\rangle\}$

7. The quantum ultimatum game

Classical case : the *game outcome* ψ is one of these four states (pure strategies) or a point in the simplex (mixed strategies).

Quantum case : the *game outcome* is any linear combination with unit norm

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$
$$|c_{00}|^2 + |c_{01}|^2 + |c_{10}|^2 + |c_{11}|^2 = 1.$$

Quantum game setting :

An initial vector $|\phi\rangle$ defines the *game environment*.

Then the players apply their *allowed moves* to the $|\phi\rangle$ state transforming it into some other state $|\psi\rangle$ (the *game outcome*).

The payoffs are computed by projection on the basis states $|\langle ij|\,\psi\rangle|^2$, weighted by the entries of the classical payoff matrix

$$\begin{aligned}\mathbb{P}_P &= a\,|c_{00}|^2 + 0\,|c_{01}|^2 + b\,|c_{10}|^2 + 0\,|c_{11}|^2\\ \mathbb{P}_R &= c\,|c_{00}|^2 + 0\,|c_{01}|^2 + b\,|c_{10}|^2 + 0\,|c_{11}|^2\end{aligned}$$

7. The quantum ultimatum game

Allowed moves : Two cases

Classical moves

Unitary moves.

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Unitary moves.

Classical moves : $M_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $M_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

or probabilistic combinations thereof (mixed strategies). That is, permutations of the basis states. (Coincide with the classical operations. However the effect may be different because they operate in the full Hilbert space)

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Unitary moves : the full set of unitary matrices in his part of the space.

Most general case : the full set of completely positive trace-preserving maps

7. The quantum ultimatum game

Restricted Quantum Game

Quantum environment + classical moves = *re-stricted quantum game* (RQG)

7. The quantum ultimatum game

Restricted Quantum Game

Quantum environment + classical moves = *restricted quantum game* (RQG)

Three different types of $|\phi\rangle$ states :

(i) $|\phi\rangle = |i\rangle \otimes |j\rangle = |ij\rangle$, $i, j = 0$ or 1.

$|\phi\rangle$ is one of the basis states of $\mathcal{H} = \mathcal{H}_P \otimes \mathcal{H}_R$.

By the classical moves the players may always convert $|\phi\rangle$ into any one of the basis states. $\Longrightarrow$ a unique Nash equilibrium $|00\rangle$

This game coincides with the classical game with $\mathbb{P}_P = a$, $\mathbb{P}_R = c$.

7. The quantum ultimatum game

(ii) $|\phi\rangle$ is a *factorized state*, but not one of the basis states. Factorized states are states that may be written as a product

$$|\phi\rangle = \{a_0 |0\rangle + a_1 |1\rangle\} \otimes \{b_0 |0\rangle + b_1 |1\rangle\}$$

Let $(\mu, 1-\mu)$ and $(\nu, 1-\nu)$ be the probabilities for proposer and responder to use moves M_0 and M_1. The payoffs are

$$\begin{aligned}
\mathbb{P}_P &= \begin{array}{l} \mu(a-b)\left(|a_0|^2 - |a_1|^2\right)\left(\nu|b_0|^2 + (1-\nu)|b_1|^2\right) \\ +\nu\left(a|a_1|^2 + b|a_0|^2\right)\left(|b_0|^2 - |b_1|^2\right) + \left(a|a_1|^2 + b|a_0|^2\right)|b_1|^2 \end{array} \\
\mathbb{P}_R &= \mathbb{P}_P\{a \to c\}
\end{aligned}$$

Maxime $\mathbb{P}_P$ in μ for fixed ν and then $\mathbb{P}_R$ in ν for that same μ, $\Longrightarrow$ there is, in all cases, a unique Nash equilibrium for pure strategies.

7. The quantum ultimatum game

The values of the Nash equilibrium $\mu's$ and $\nu's$:

	$\|b_0\|^2 > \|b_1\|^2$	$\|b_0\|^2 < \|b_1\|^2$
$\|a_0\|^2 > \|a_1\|^2$	$\mu = 1, \nu = 1$	$\mu = 1, \nu = 0$
$\|a_0\|^2 < \|a_1\|^2$	$\mu = 0, \nu = 1$	$\mu = 0, \nu = 0$

The equilibrium payoffs :

$$\mathbb{P}_P = \left\{a\,|a_{\max}|^2 + b\left(1 - |a_{\max}|^2\right)\right\}|b_{\max}|^2$$

$$\mathbb{P}_R = \mathbb{P}_P\{a \to c\}$$

$|a_{\max}|^2$ and $|b_{\max}|^2$ are the larger of $|a_i|^2$ and $|b_i|^2$.

Conclusion : *although obtained for pure strategies, the payoffs are substantially different from those of the classical game.*

7. The quantum ultimatum game

(iii) $|\phi\rangle$ is an *entangled state*.

A general state

$$|\phi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

is entangled if and only if

$$|c_{00}|^2 |c_{11}|^2 + |c_{01}|^2 |c_{10}|^2 - 2Re\{c_{00}c_{10}^{*}c_{11}c_{01}^{*}\} \neq 0$$

Analyze the cases

$$|\phi_1\rangle = \alpha_1|00\rangle + \beta_1|11\rangle$$

and

$$|\phi_2\rangle = \alpha_2|01\rangle + \beta_2|10\rangle$$

for $\alpha_i, \beta_i \neq 0$ are entangled. Furthermore

$$|\alpha_i|^2, |\beta_i|^2 > \frac{c}{b+c}$$

7. The quantum ultimatum game

(a) For the ϕ_1^- state,

$(\mu, 1-\mu)$ and $(\nu, 1-\nu)$ are the probabilities for proposer and responder to use moves M_0 and M_1

Payoffs

$$\mathbb{P}_P = \mu(a-b)\left(\nu - |\beta_1|^2\right) + \nu\left(b|\alpha_1|^2 - a|\beta_1|^2\right) + a|\beta_1|^2$$
$$\mathbb{P}_R = \mathbb{P}_P\{a \to c\}$$

Let $\nu > |\beta_1|^2$. Then the proposer best reply is $\mu = 1$ but then, for that μ, the responder best reply is $\nu = 0$, which contradicts $\nu > |\beta_1|^2$.

Conclusion : there is no equilibrium in pure strategies.

However, there is an equilibrium for mixed strategies at

$$\mu = \frac{b|\alpha_1|^2 - c|\beta_1|^2}{b-c}, \quad \nu = |\beta_1|^2$$

with payoffs

$$\mathbb{P}_P = |\alpha_1|^2\left(1 - |\alpha_1|^2\right)(a+b)$$
$$\mathbb{P}_R = |\alpha_1|^2\left(1 - |\alpha_1|^2\right)(c+b)$$

7. The quantum ultimatum game

(b) For the ϕ_2^- state

The analysis is identical, with mixed strategy equilibrium at

$$\mu = \frac{b|\alpha_2|^2 - c|\beta_2|^2}{b-c}$$
$$\nu = |\alpha_2|^2$$

Payoffs

$$\mathbb{P}_P = |\alpha_2|^2 \left(1 - |\alpha_2|^2\right)(a+b)$$
$$\mathbb{P}_R = |\alpha_2|^2 \left(1 - |\alpha_2|^2\right)(c+b)$$

7. The quantum ultimatum game

Conclusions :

1) A range of different equilibrium points is obtained.

An even wider range of possibilities and payoff structures may be obtained by increasing the number of possible proposer offers.

2) In all cases the self-interest mechanism of payoff maximization leads to a solution, but the solution strongly depends on the game environment coded by the ϕ^{-}state.

3) How does one relate the coding ϕ-state to the deviations from classical Nash equilibria in experimental games ?

7. The quantum ultimatum game

How do the players' environment constraints (or preferences) are related to particular features of the initial state ?

3.1 - For factorized states one has a *measure of uncertainty* for proposer and responder given by

$$\mathbb{S}_{\mathsf{P}} = -|\mathsf{a}_0|^2 \log |\mathsf{a}_0|^2 - |\mathsf{a}_1|^2 \log |\mathsf{a}_1|^2$$
$$\mathbb{S}_{\mathsf{R}} = -|\mathsf{b}_0|^2 \log |\mathsf{b}_0|^2 - |\mathsf{b}_1|^2 \log |\mathsf{b}_1|^2$$

and the compound uncertainty $\mathbb{S}_{\mathsf{P}} + \mathbb{S}_{\mathsf{R}}$.

Even if the solution corresponds to an equilibrium pure strategy, the game environment is equivalent to a compulsion to fluctuating decisions by the players.

7. The quantum ultimatum game

3.2- In factorized states, each player has no effect on measurements made on the space of the other player.

That is not the case for entangled states. The constraints are much stronger.

Entanglement (for a two-player game) is quantified by the entropy of the reduced density matrix.

Computing

$$\lambda\pm = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4\left|c_{00}\right|^2 \left|c_{11}\right|^2 - 4\left|c_{01}\right|^2 \left|c_{10}\right|^2 + 8Re\left\{c_{00}c_{10}^{*}c_{11}c_{01}^{*}\right\}}$$

the *entanglement measure* is

$$\mathbb{E} = -\lambda_+ \log \lambda_+ - \lambda_- \log \lambda_-$$

Conclusion : the players uncertainties $(\mathbb{S}_P, \mathbb{S}_R)$ and the entanglement measure $(\mathbb{E})$ may be used to quantify some of the deviations from classical Nash equilibrium in experimental games.

7. The quantum ultimatum game

The unitary and trace-preserving games

- In a *unitary game*, the players are allowed to operate on the ϕ^{-}state with arbitrary unitary transformations in $\mathcal{H}_P$ and $\mathcal{H}_R$,

$$\begin{pmatrix} e^{i(\alpha+\beta)/2}\cos\frac{\theta}{2} & e^{i(\alpha-\beta)/2}\sin\frac{\theta}{2} \\ e^{i(\beta-\alpha)/2}\sin\frac{\theta}{2} & e^{i(\alpha+\beta)/2}\cos\frac{\theta}{2} \end{pmatrix}$$

and global phase transformations.

1) If ϕ is a factorized state,

The proposer may, by an unitary operation, transform it to a state $|0\rangle\,|\zeta\rangle$ and then the best reply of the responder leads to $|0\rangle\,|0\rangle$.

Conclusion : for factorized ϕ^{-}states, the unitary game is equivalent to the classical game.

7. The quantum ultimatum game

2) The situation is different for entangled states.
Let ϕ be a (maximally) entangled state

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

The payoffs

$$\mathbb{P}_P = \frac{1}{2}\left\{\begin{array}{c} a\left|e^{i\frac{\beta_P+\beta_R}{2}}\cos\frac{\theta_P}{2}\cos\frac{\theta_R}{2} - e^{-i\frac{\beta_P+\beta_R}{2}}\sin\frac{\theta_P}{2}\sin\frac{\theta_R}{2}\right|^2 \\ +b\left|e^{i\frac{-\alpha_P+\alpha_R+\beta_P+\beta_R}{2}}\sin\frac{\theta_P}{2}\cos\frac{\theta_R}{2} - e^{i\frac{\alpha_P+\alpha_R+\beta_P-\beta_R}{2}}\cos\frac{\theta_P}{2}\sin\frac{\theta_R}{2}\right|^2 \end{array}\right\}$$

$$\mathbb{P}_R = \mathbb{P}_P\{a \rightarrow c\}$$

7. The quantum ultimatum game

Given any responder strategy, the proposer may always choose α_P, β_P to reduce his payoff to

$$\mathbb{P}_P = \frac{1}{2}\left\{a\cos^2\frac{1}{2}(\theta_P+\theta_R) + b\sin^2\frac{1}{2}(\theta_P+\theta_R)\right\}$$

$$\mathbb{P}_R = \mathbb{P}_P\{a \to c\}$$

and then, choosing $\theta_P = \theta_R$, obtain his best reply.

But then, the responder spoils this situation by choosing $\theta_P + \theta'_R = 0$, and so on.

Conclusion :for the unitary game with maximally entangled states there is no Nash equilibrium in pure strategies.

7. The quantum ultimatum game

For mixed strategies, we no longer have unitary operations, but rather fall in the framework of completely positive trace-preserving maps

(*Mixed "unitary" strategies are simply the case where the Kraus operators are proportional to a unitary one*).

In this case we have transformations

$$|\phi\rangle\langle\phi| \to \sum_{\mu,\nu} K_\mu^{(P)} \otimes K_\nu^{(R)} |\phi\rangle\langle\phi| K_\nu^{(P)\dagger} \otimes K_\mu^{(R)\dagger}$$

with the (Kraus) operators, satisfying

$$\sum_\mu K_\mu^\dagger K_\mu = 1$$

Choosing a basis of hermitean operators

$$K_\mu^{(P)} = \sum_\nu k_{\mu\nu}^{(P)} \sigma_\nu$$
$$K_\mu^{(R)} = \sum_\nu k_{\mu\nu}^{(R)} \sigma_\nu$$

7. The quantum ultimatum game

Defining

$$p_{\nu\nu'} = \sum_\mu k_{\mu\nu}^{(P)} k_{\mu\nu'}^{(P)*}$$
$$r_{\nu\nu'} = \sum_\mu k_{\mu\nu}^{(R)} k_{\mu\nu'}^{(R)*}$$

the payoffs are

$$\mathbb{P}_P = \sum_{\mu\nu\alpha\beta} p_{\mu\nu} r_{\alpha\beta} \left\{ \begin{array}{l} a \langle 00 | \sigma_\mu^{(P)} \otimes \sigma_\alpha^{(R)} | \phi \rangle \langle \phi | \sigma_\beta^{(R)} \otimes \sigma_\nu^{(P)} | 00 \rangle \\ + b \langle 11 | \sigma_\mu^{(P)} \otimes \sigma_\alpha^{(R)} | \phi \rangle \langle \phi | \sigma_\beta^{(R)} \otimes \sigma_\nu^{(P)} | 11 \rangle \end{array} \right\}$$

$$\mathbb{P}_R = \mathbb{P}_P \{ a \rightarrow c \}$$

$|p_{\nu\nu'}|^2 \leq 1, |r_{\nu\nu'}|^2 \leq 1$. Then, compactness and convexity of the sets $\{p_{\nu\nu'}\}$ and $\{r_{\nu\nu'}\}$ and the multilinearity of the payoff implies, by Kakutani's theorem, the existence of Nash equilibria for the trace-preserving game.

8. Other uses of the quantum decision paradigm

- *The quantum treatment of public goods economics*
  Entanglement avoids “tragedy of the commons”
- *Quantum finance*
  The importance of trading (change of ownership) in determining the value of a security.
  Volatility related to the trading probability

9. A computational view of deterministic, non-deterministic and quantum computation (or decision)

Turing machine M with k work tapes, alphabet Γ and one input tape with alphabet Σ.
The configuration c of the machine is :

- the contents of the k work tapes
- the $\mathsf{k}+1$ tape pointers
- the current state.

$\mathcal{C}(\mathsf{x})$ is the set of all possible configurations on input x. N is the cardinality of $\mathcal{C}(\mathsf{x})$.
Q is the set of states

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DETERMINISTIC COMPUTATION

The transition function

$$\delta : Q \times \Sigma \times \Gamma^k \times Q \times \Gamma^k \times \{L, R\}^{k+1} \longrightarrow \{0, 1\}$$

defines a $N \times N$ transition matrix T

If it is possible to go from c_i to c_j in one step then $T(c_i, c_j) = 1$, otherwise $T(c_i, c_j) = 0$.

Only one element in each row is different from zero.

$T^k(c_i, c_j)$ is the number of paths of length k leading from c_i to c_j.

PROBABILISTIC COMPUTATION

δ is allowed to take nonbinary values.

For a probabilistic Turing machine

$$\delta : Q \times \Sigma \times \Gamma^k \times Q \times \Gamma^k \times \{L, R\}^{k+1} \to [0, 1]$$

with the condition

$$\sum_{q_2, b_1 \cdots b_k, p_0, p_1 \cdots p_k} \delta \left(q_1, s, a_1 \cdots a_k, q_2, b_1 \cdots b_k, p_0, p_1 \cdots p_k\right) = 1$$

for all values $q_1, s, a_1 \cdots a_k$ of the initial state and the currently read symbols in the input and work tapes.

The entries in the matrix are between zero and one and the rows add up to one (Stochastic matrices).

They are $\mathcal{L}1$ norm preserving ($\mathcal{L}1\,(v) = \mathcal{L}1\,(Tv)$ for an arbitrary $N-$vector v).

QUANTUM COMPUTATION

δ may to take arbitrary real (positive or negative) or even complex values

The acceptance probabilities (after k computational steps) will now be defined by

$\left|T^k\left(c_i, c_j\right)\right|^2$

with

$$\sum_{q_2, b_1 \cdots b_k, p_0, p_1 \cdots p_k} \left|\delta\left(q_1, s, a_1 \cdots a_k, q_2, b_1 \cdots b_k, p_0, p_1 \cdots p_k\right)\right|^2 = 1$$

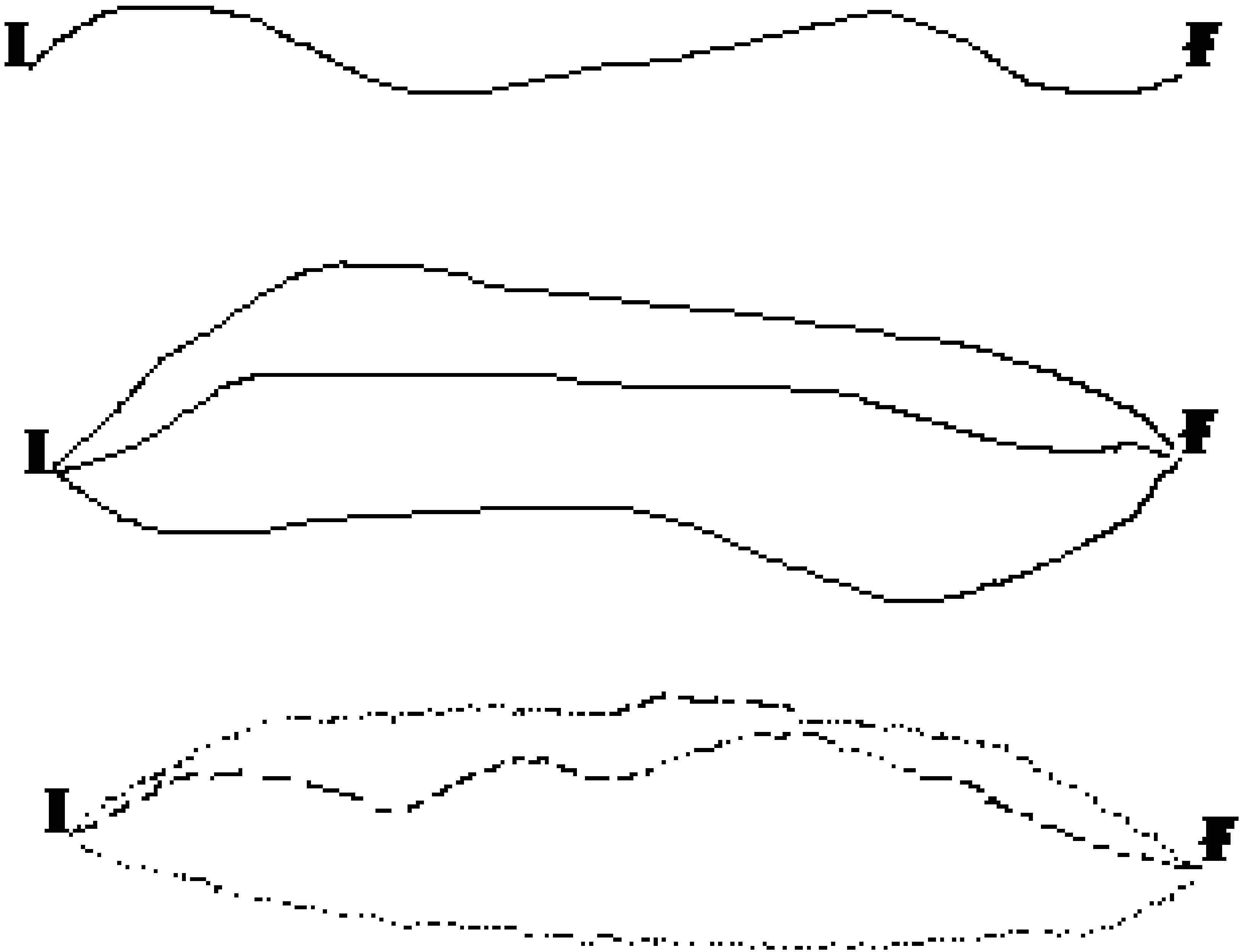
Equivalently,

$$\mathcal{L}2\left(v\right) = \mathcal{L}2\left(Tv\right) = \sum_{i=1}^{N} \left|v_i\right|^2$$

The transition matrix is required to be unitary.

In all cases the transition probabilities between any two initial and final states are positive, well defined and normalized. The difference between these different computational models is the method through which the final outcome is obtained. In this way a unified framework is obtained that, in each case, captures the power of deterministic, nondeterministic, probabilistic or unitary (quantum) computation.

E. Bernstein and U. Vazirani; Quantum complexity theory, SIAM Journal on Computing 26 (1997) 1411-1473.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- Penrose and Hameroff claimed that quantum effects in the brain that might occur, for example, in the cytoskeletal microtubules of the neurons would be responsible for the human consciousness. They have been criticized on the basis that the cognitive processes in the brain are mediated by the axonal action potentials which are well modeled by classical physics.
- However we need not have physical quantum effects to have a computational phenomena that is better described by the quamtum computation paradigm. Does one know exactly how we finally take a decision when faced with the inner feeling that there is a multitude of alternatives?
- Do we follow a deterministic path, a probabilistic one or are there interference-like interactions between the alternatives?

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