The geometry of the market
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The metric
(From the Euclidean distance of the time series)

\[ d_{ij} = \sqrt{2(1 - c_{ij})} \]  
(Mantegna & Stanley, 1999)

Market space
1. The N stock coordinates are computed from the distances. Embedding space dimension = N-1
2. Find the center of mass \( R = \sum m(k) \cdot x(k) / \sum m(k) \)
3. Find coordinates in the center of mass system \( y(k) = x(k) - R \)
4. Construct the inertial tensor \( T_{ij} = \sum m(k) \cdot y_i(k) \cdot y_j(k) \)
5. Find the eigenvectors and eigenvalues of \( T : (\lambda_i, e_i) \)
6. Compare with the same construction for permuted data
What is the number of dimensions of the manifold containing the systematic components of all stocks?

S&P500 and Dow Jones, daily data

- 35 stocks, 10 years
- 249 stocks, 33 years
- 253 stocks, 22 years
- 70 stocks, 10 years
- 253 stocks, 35 years
- 424 stocks, 10 years

In all cases: **6 dimensions are enough!**

The systematic component is contained in f dimensions << N

Dimensions are not necessarily sectors. Impact on the choice of portfolios
Ordered and normalized eigenvalues

\[ \lambda + (1 - \lambda^*) \]

\( \lambda \): actual
\( \lambda^* \): random

Effective dimensions

1988-2008  253 Stocks

1998-2008  424 Stocks
Shape changes in the market space

“Spherical” market
Typical in all projections for “surrogate data” or in “business-as-usual” periods

During crises there is distortion and reduction of volume

Shape and crises


Surrogate Data 253 stocks

1992 253 stocks

2001 253 stocks

2007 253 stocks

253 stocks
Structure index

\[ S_t = \sum_{i=1}^{6} \left( \frac{\lambda'_t(i)}{\lambda_t(i)} - 1 \right) \]

After 1997 deviations from sphericity become more frequent.

What happened?
Market networks: From a fully connected network to a sparse one

1. minimal spanning tree
2. \( L_{D}^{6} = \) smallest distance in \( R^{6} \) that guarantees the network connectivity
3. hierarchical clustering

\[
\begin{align*}
    d^{6}_{i,j} & \leq \frac{L_{D}^{6}}{2} \Rightarrow b_{i,j} = 1 \\
    d^{6}_{i,j} & > \frac{L_{D}^{6}}{2} \Rightarrow b_{i,j} = 0
\end{align*}
\]

Few connections in normal periods
Market networks

The economic agents are much more correlated during market crises.

An increased number of connections, mostly inside sectors.
Another structure index

Ratio of weak and strong links

\[ R_t = \frac{\sum_{d_t^6(i, j) \leq L_{D^6}} d_t^6(i, j)}{\sum_{d_t^6(i, j) > L_{D^6}} d_t^6(i, j)} \]

Synchronization

\[ s_i = 1 \iff \exists \ d_t^6(i, j) \leq \frac{L D^6}{2} \]
\[ s_i = 0 \text{ otherwise} \]

The state depends on the existence of a strong link inside or outside the sector.

Mar1998

Sep2001
Synchronization

Aug2000

Sep2000

Dec2007

Jan2008

Feb2008

Legend:
- Uti
- IT
- Fin
- Heal
- Cons
- Ind
- Ene
Market shape as a precursor of crises and indicator of return to normality

- **Characterizing shape:**
- **Volume, asymmetry and tails**
- **Mardia’s Skewness** $K_3$ and **Kurtosis** $K_4$

\[
K_3 = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left\{ (X_i - \overline{X}) S^{-1} (X_j - \overline{X}) \right\}^3
\]

\[
K_4 = \frac{1}{n} \sum_{i=1}^{n} \left\{ (X_i - \overline{X}) S^{-1} (X_i - \overline{X}) \right\}^2
\]

- **The volume is the product of the first 6 eigenvalues**
- **$K_3$ and $K_4$ are computed after the dimensions are renormalized to have the same volume**
Market shape as a precursor of crises and indicator of return to normality
Market shape as a precursor of crises and indicator of return to normality
- From some of the cases studied so far it looks as if the shape of the market might provide some precursors of impending crises and would also be an indicator of when normality has returned.
- However this is just work in progress. More to come.