The geometry of the market

Contents

1 - The Market Geometry

r: return $r(k) = \log(p_t(k)) - \log(p_{t-1}(k))$ **p: price**

The metric

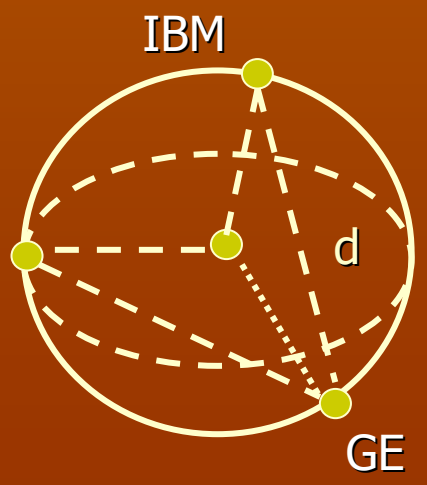
(From the Euclidean distance of the time series)

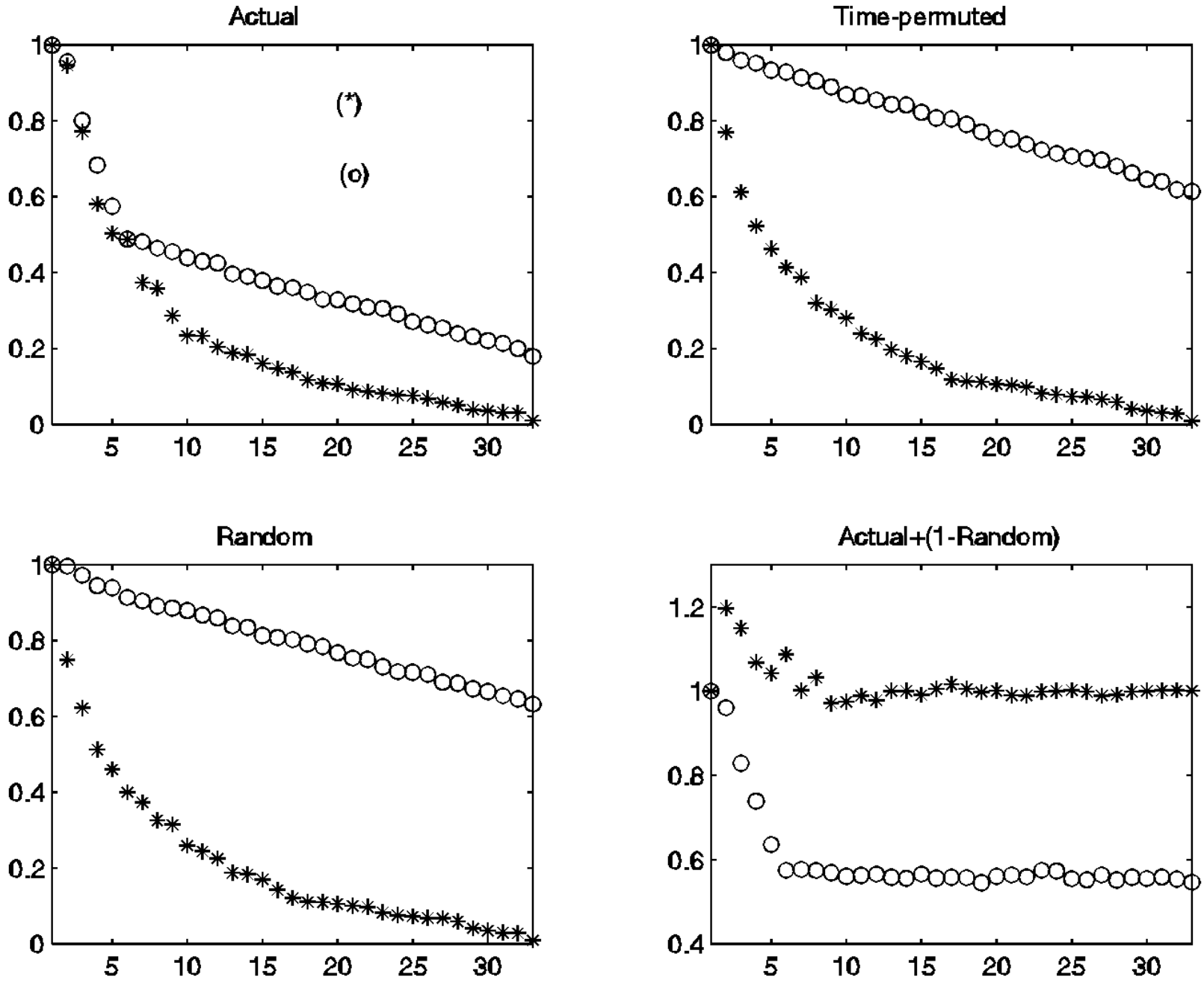
$$dij = \sqrt{2(1-cij)}$$

(Mantegna & Stanley, 1999)

Market space

1. The N stock coordinates are computed from the distances. Embedding space dimension = N-1
2. Find the center of mass $R=\Sigma\, m(k)\, x(k) / \Sigma\, m(k)$
3. Find coordinates in the center of mass system $y(k) = x(k) - R$
4. Construct the inertial tensor $T_{ij} = \Sigma\, m(k)\, y_i(k)\, y_j(k)$
5. Find the eigenvectors and eigenvalues of T : (λ_i, e_i)
6. Compare with the same construction for permuted data




Actual
(*)
(o)
Time-permuted
Random
Actual+(1-Random)

Effective dimensions

What is the number of dimensions of the manifold containing the systematic components of all stocks ?

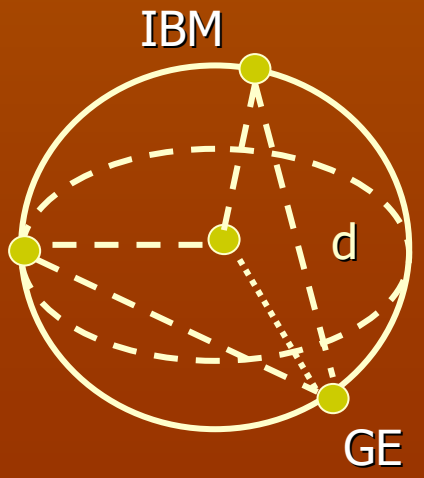
S&P500 and Dow Jones, daily data

- 35 stocks, 10 years
- 70 stocks, 10 years
- 249 stocks, 33 years
- 253 stocks, 35 years
- 253 stocks, 22 years
- 424 stocks, 10 years

In all cases: **6 dimensions are enough !**

The systematic component
is contained in f dimensions << N

Dimensions are not necessarily sectors.
Impact on the choice of portfolios

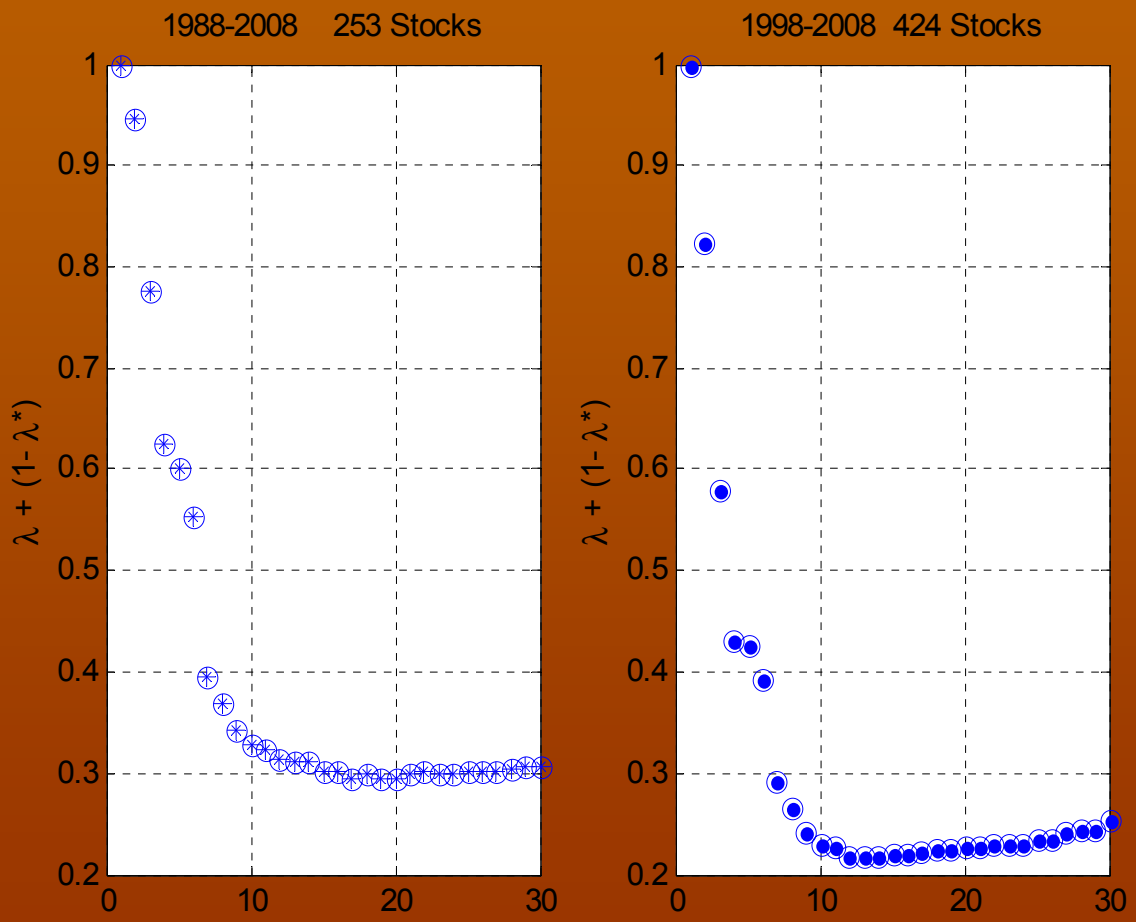


Effective dimensions

Ordered and normalized eigenvalues

$\lambda + (1-\lambda^*)$

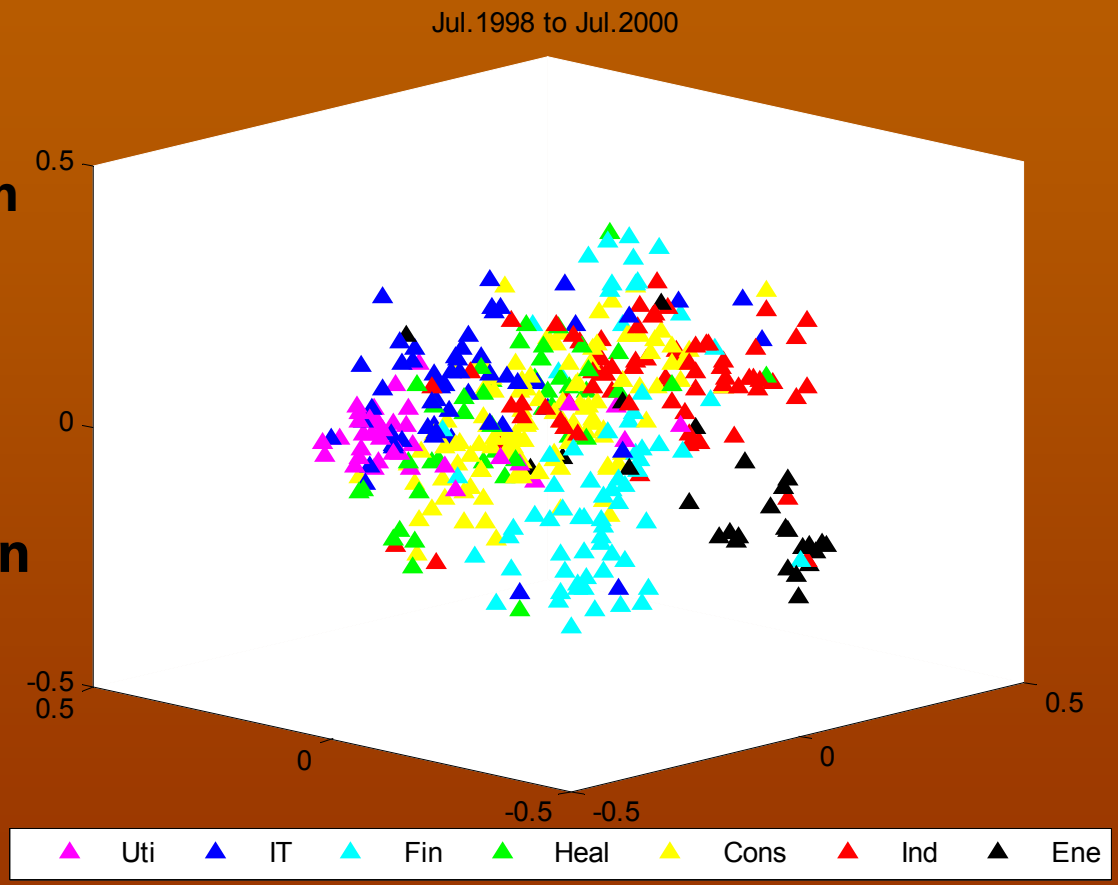
λ : actual
λ^* : random



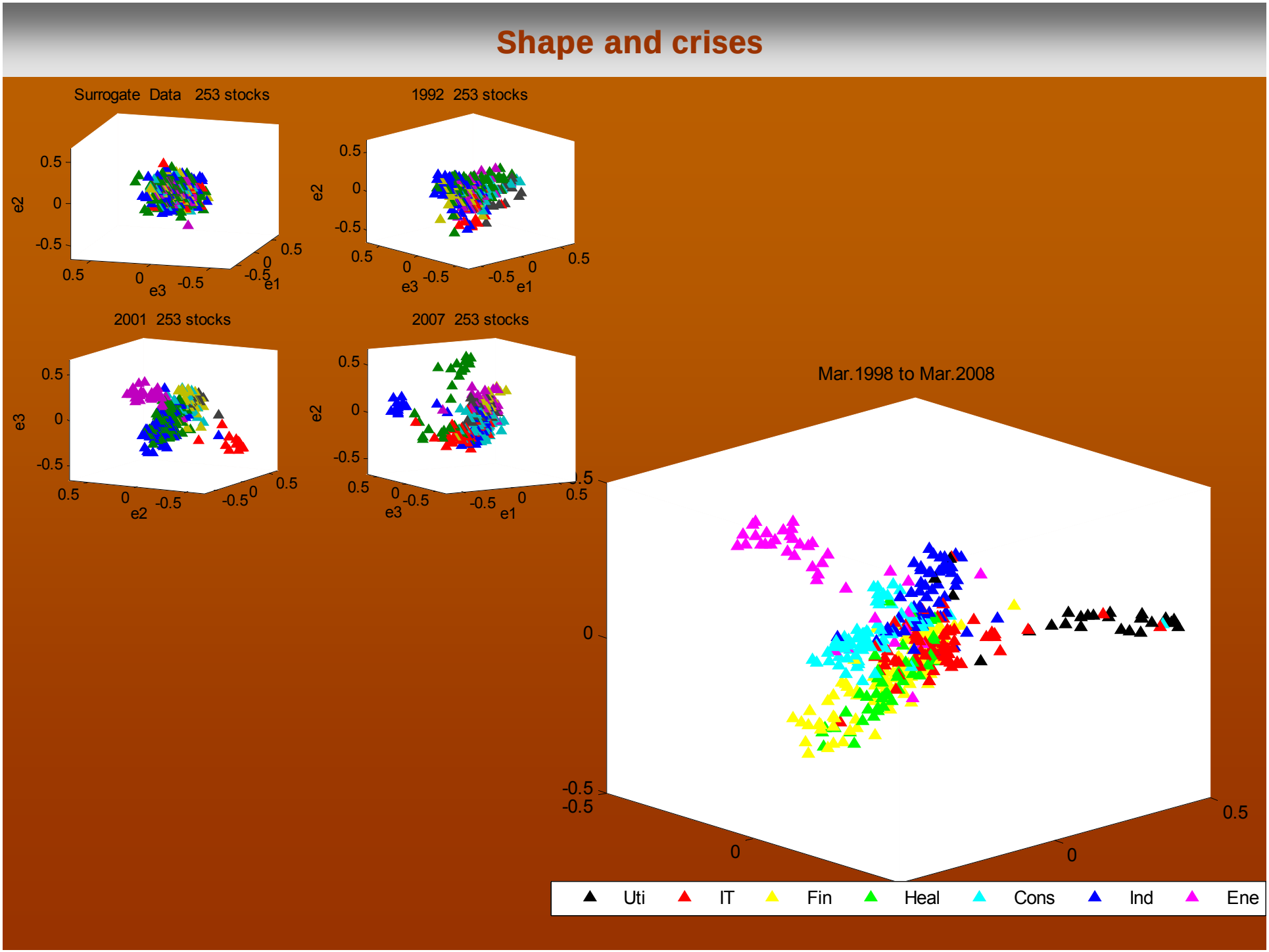
Shape changes in the market space

“Spherical” market
Typical in all projections for “surrogate data” or in “business-as-usual” periods

During crises there is distortion and reduction of volume



Shape and crises



A structure index

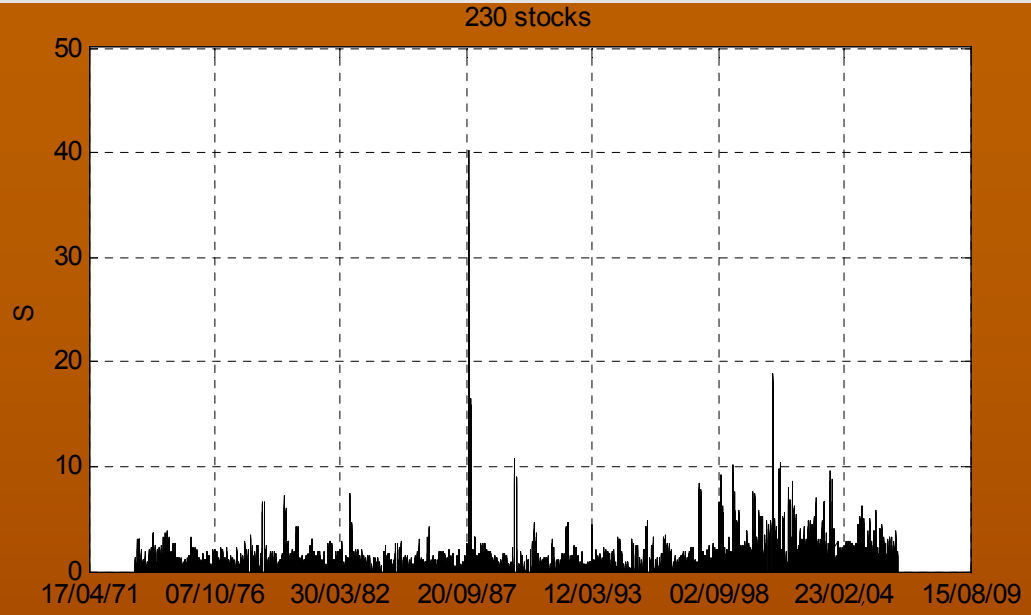
Structure index

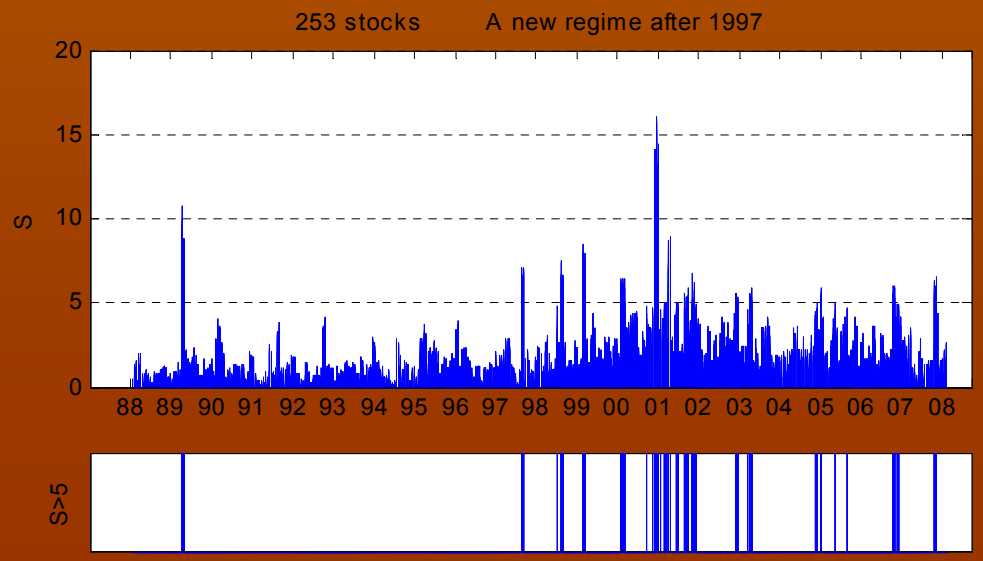
λ : actual
λ': random

$$S_t = \sum_{i=1}^{6} \left(\frac{\lambda_t'(i)}{\lambda_t(i)} - 1 \right)$$

After 1997 deviations from sphericity become more frequent.

What happened ?



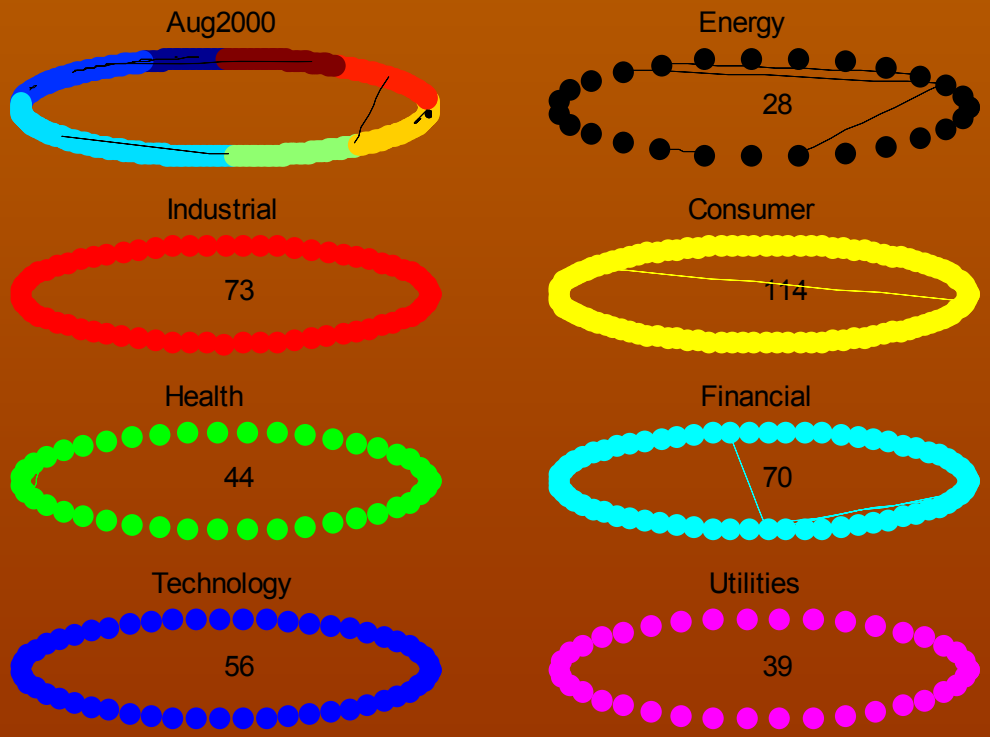


Market networks: From a fully connected network to a sparse one

1. minimal spanning tree
2. L_D^6 = smallest distance in R^6 that guarantees the network connectivity
3. hierarchical clustering

$$d^6_{i,j} \le \frac{L_{D^6}}{2} \Rightarrow b_{i,j} = 1$$

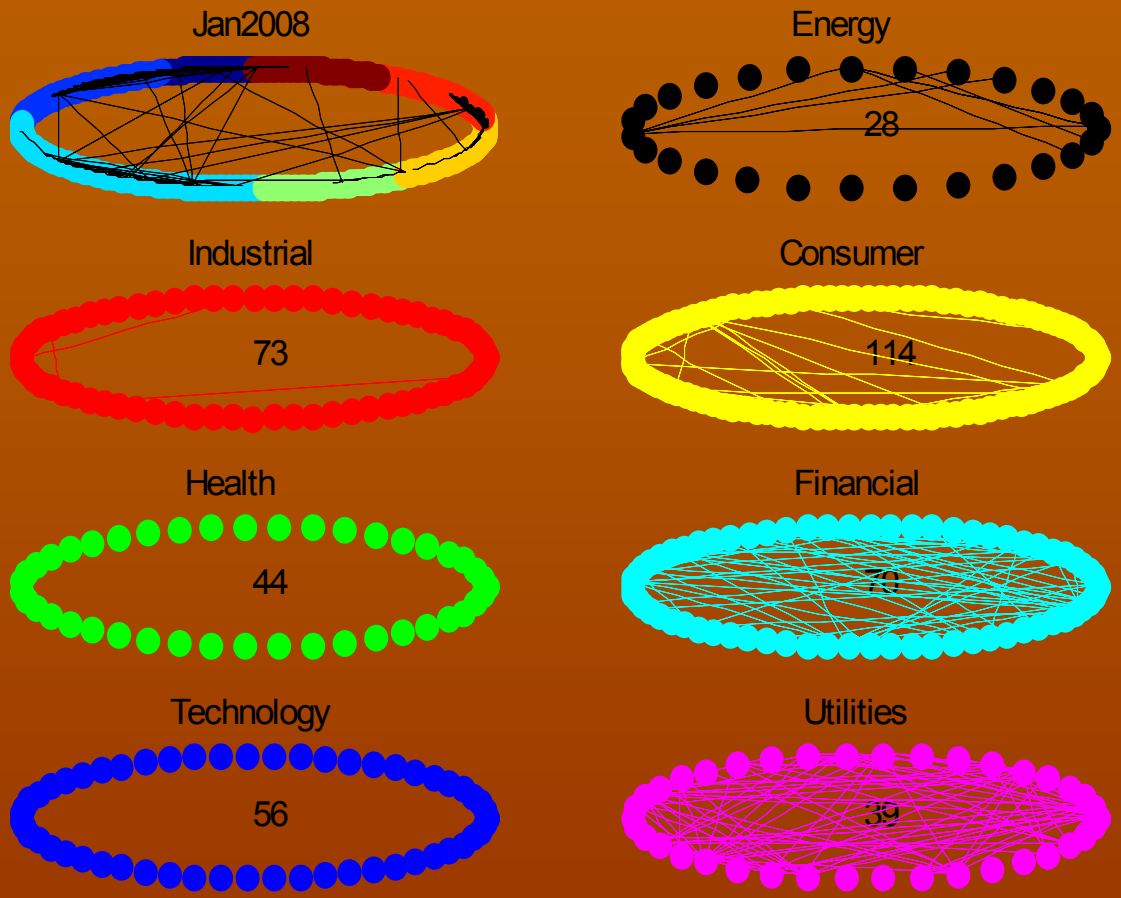
$$d^6_{i,j} > \frac{L_{D^6}}{2} \Rightarrow b_{i,j} = 0$$



Few connections in normal periods

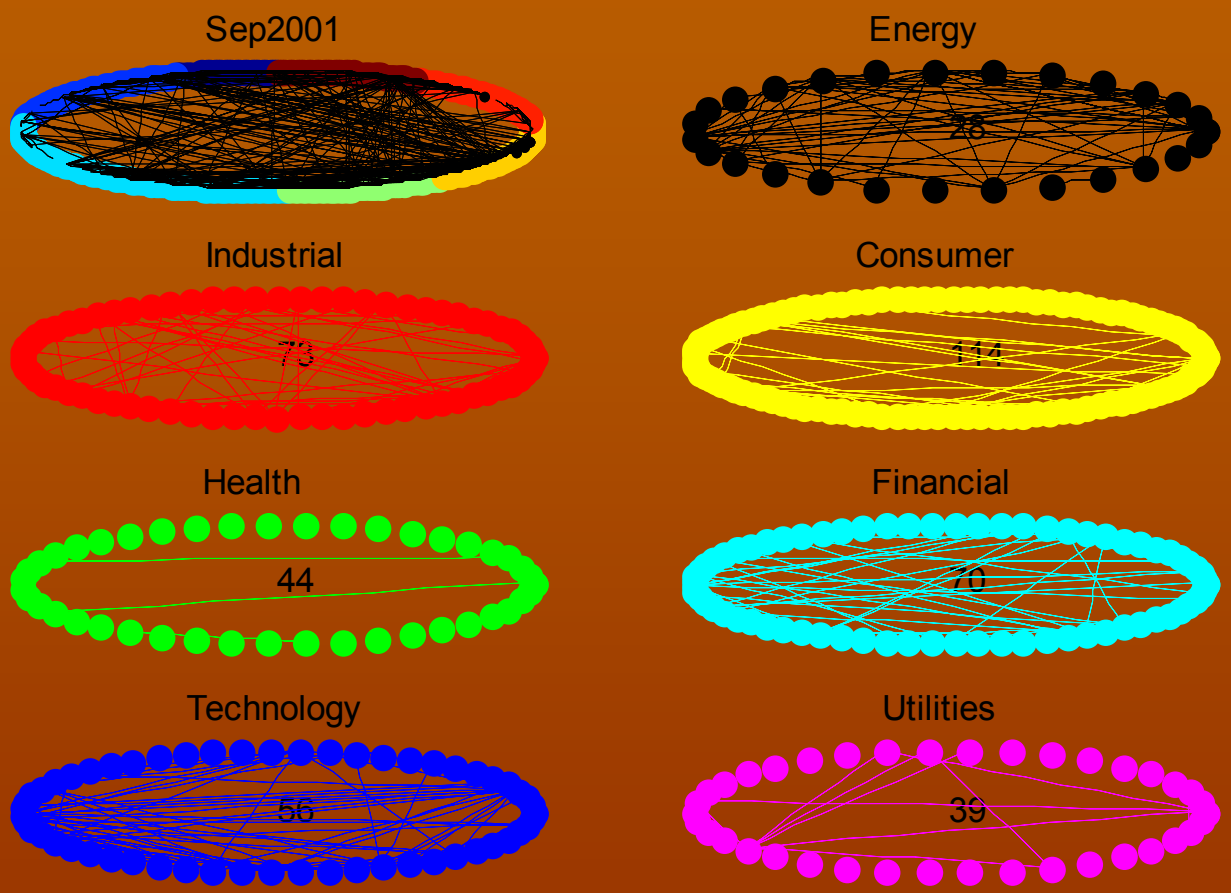
Market networks

The economic agents are much more correlated during market crises



An increased number of connections, mostly inside sectors

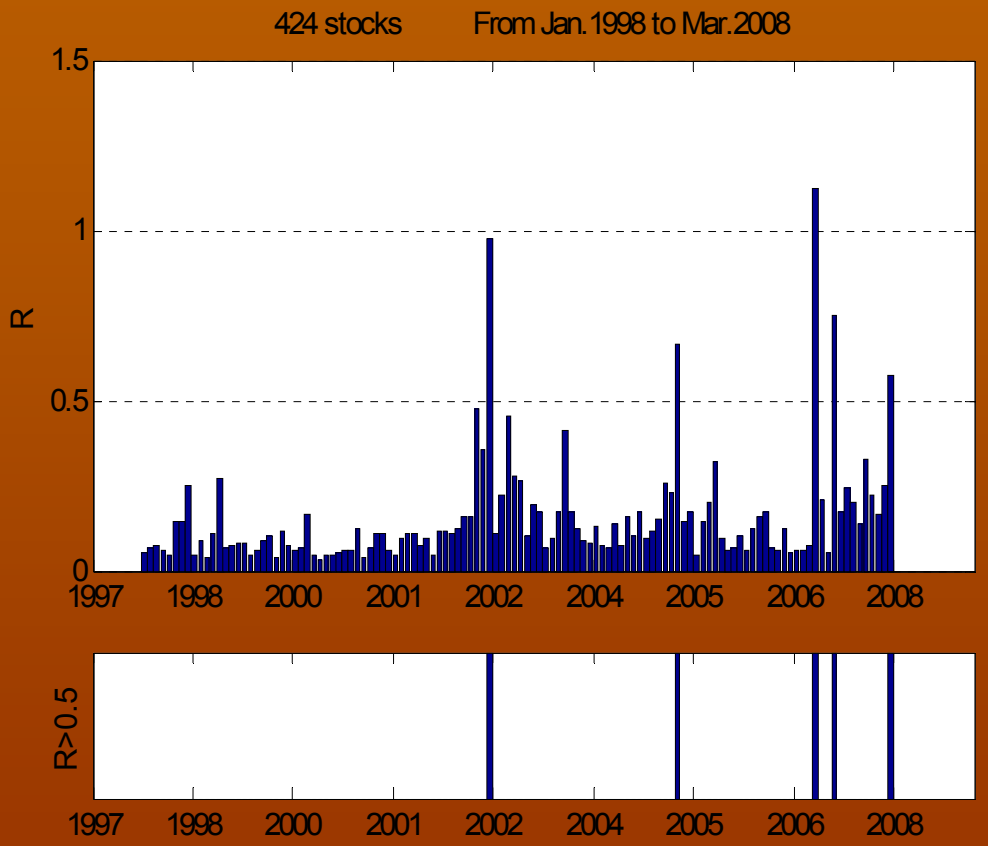
Market networks



Another structure index

Ratio of weak and strong links

$$R_t = \frac{\sum_{d_t^6(i,j) \le L_{D^6}} d_t^6(i,j)}{\sum_{d_t^6(i,j) > L_{D^6}} d_t^6(i,j)}$$

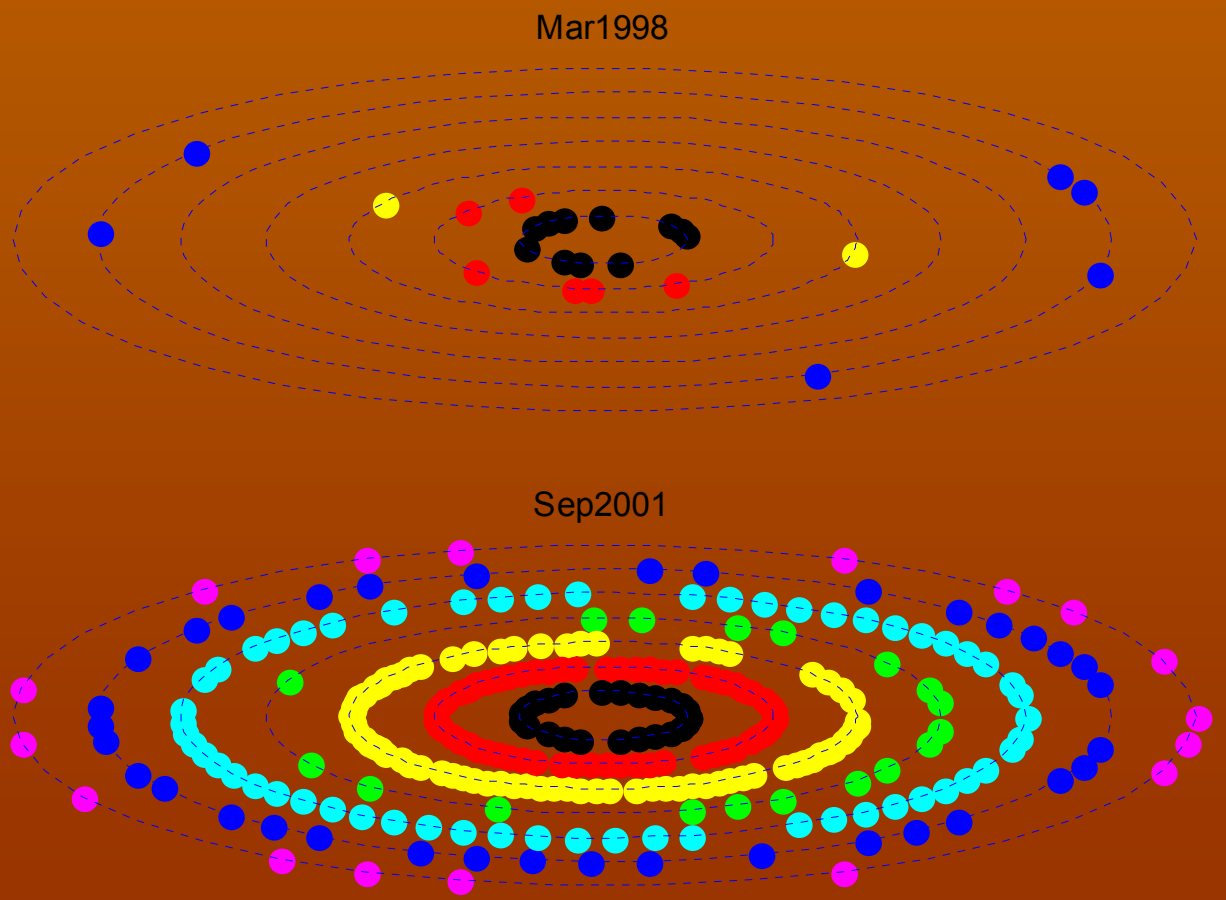


Synchronization

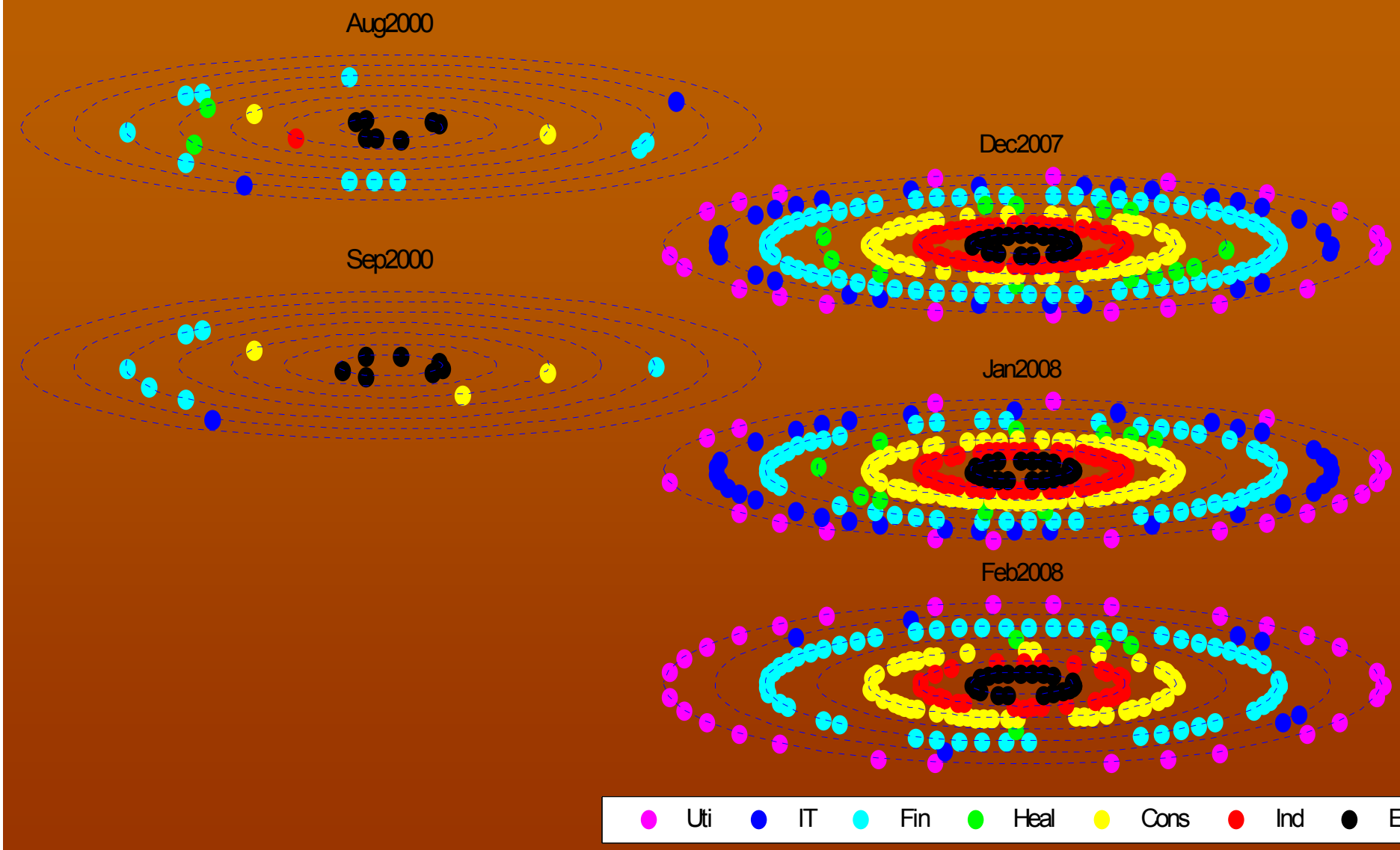
$$s_i = 1 \Leftrightarrow \exists \quad d_t^6(i,j) \leq \frac{L_{D^6}}{2}$$

$$s_i = 0 \quad otherwise$$

The state depends on the existence of a strong link inside or outside the sector



Synchronization



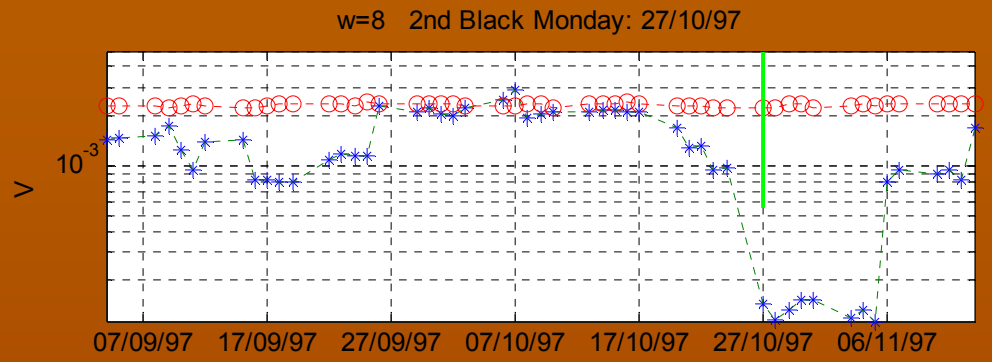
- **Characterizing shape:**
- **Volume, asymmetry and tails**
- **Mardia's Skewness K_3 and Kurtosis K_4**

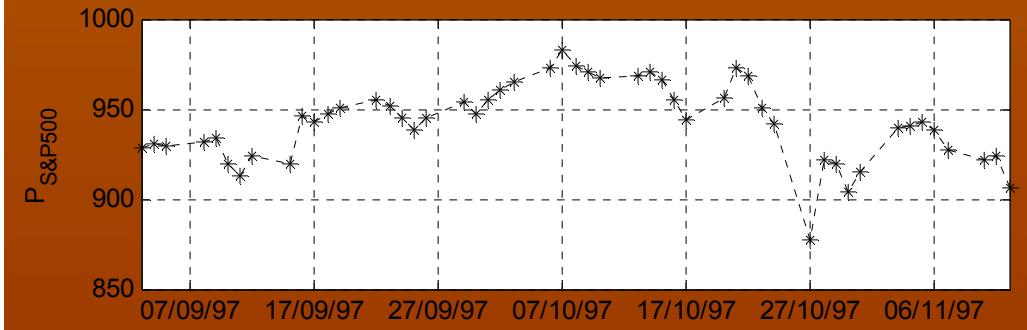
$$K_3 = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left\{ \left(X_i - \overline{X} \right)' S^{-1} \left(X_j - \overline{X} \right) \right\}^3$$

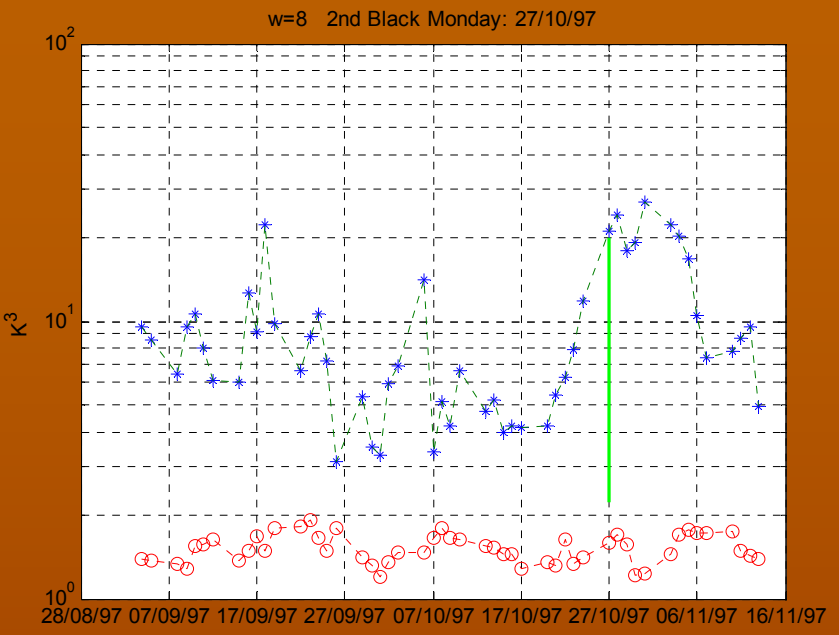
$$K_4 = \frac{1}{n} \sum_{i=1}^{n} \left\{ \left(X_i - \overline{X} \right)' S^{-1} \left(X_i - \overline{X} \right) \right\}^2$$

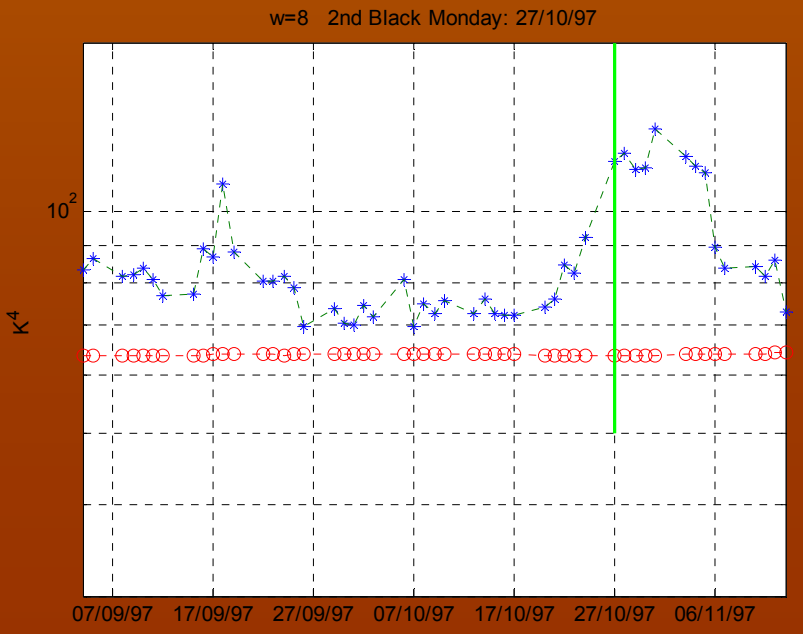
- **The volume is the product of the first 6 eigenvaues**
- **K_3 and K_4 are computed after the dimensions are renormalized to have the same volume**

Market shape as a precursor of crises and indicator of return to normality

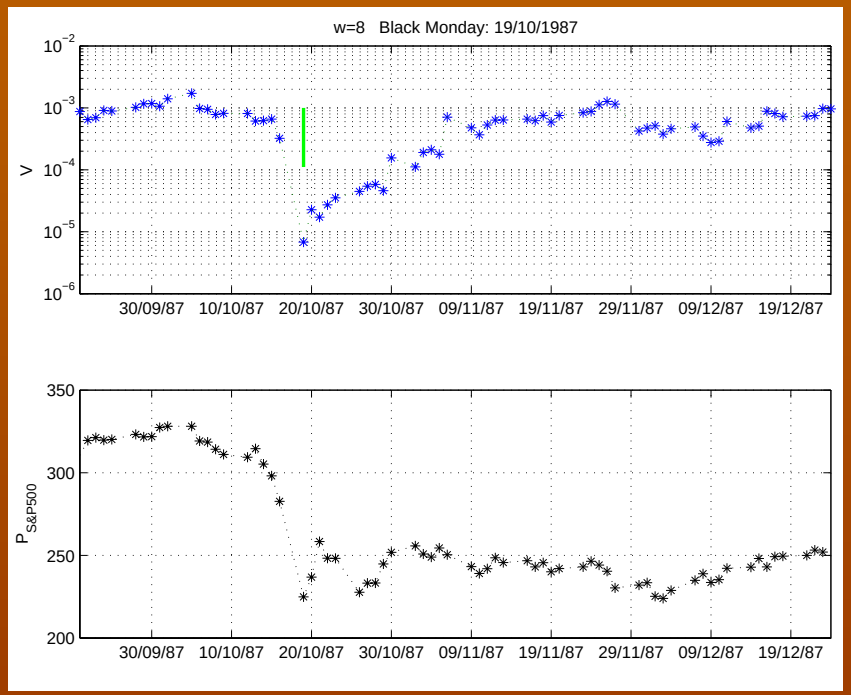


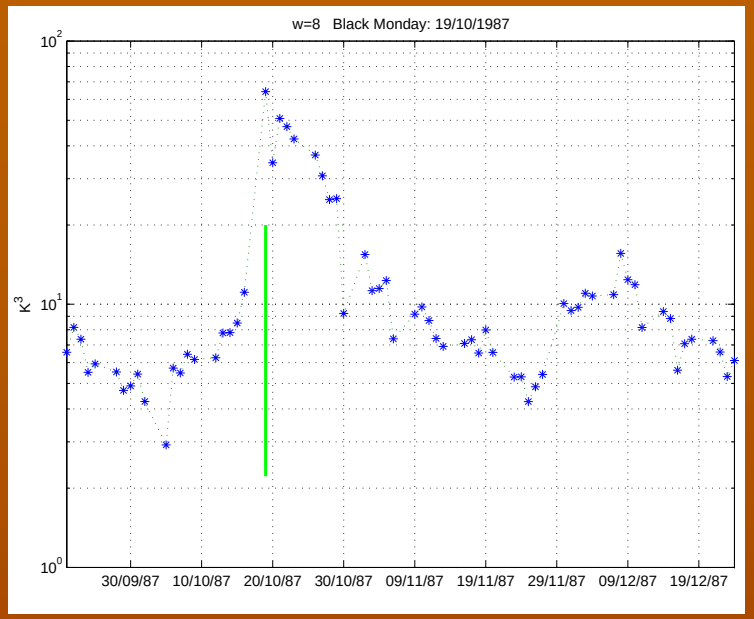


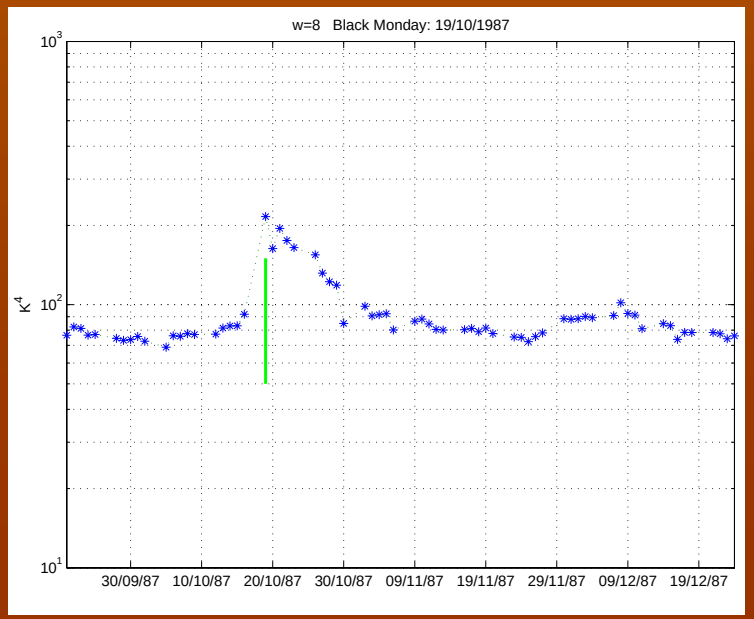




Market shape as a precursor of crises and indicator of return to normality







Market shape as a precursor of crises and indicator of return to normality

- From some of the cases studied so far it looks as if the shape of the market might provide some precursors of impending crises and would also be an indicator of when normality has returned.
- However this is just work in progress. More to come.

References

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