Games and neuroeconomics

Rui Vilela Mendes
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1 - Games

- **Game theory**: Study of multi-person decision problems influencing one another's welfare
- Economics, Biology, Social Sciences, Communication
- **Mechanism**: Cooperation or competition to reach a best goal state (from the cooperative or individual point of view)
1 - Games - Nash equilibrium

- $(s_1, s_2, ..., s_k, ..., s_n)$ is Nash equilibrium if $P(s_1, s_2, ..., s_k, ..., s_n) > P(s_1, s_2, ..., s_k', ..., s_n)$ for all $s_k'$
- No player can improve his payoff by changing his strategy, when the strategies of the other players are fixed
- **Theor:** (Nash) Every $N$-player game, with finite strategies, has at least one equilibrium, in pure or mixed strategies
- In economy, Nash equilibrium $\iff$ Self-interested rational decisions (Homo Oeconomicus)
- Provides a sound basis for (almost) the whole of (rigorous) economic theory
1 - Games - Nash equilibrium

- Example:
- **Theorem:** Consider the class of abstract economies with
  (i) strategy sets convex and compact,
  (ii) payoff functions continuous and quasi-concave,
  (iii) the feasibility correspondences are continuous and have nonempty convex values. Then, the Nash correspondence is the unique solution that satisfies nonemptiness, rationality in one-person games and consistency.

*(B. Peleg; Games and Economic Behavior 18 (1997) 277-285)*
1 - Games - Nash equilibrium

- Note:
  - “Feasibility correspondences”
    The feasible strategies of each agent may depend on the other agents strategies
  - Social equilibrium (Debreu),
    a generalization of Nash equilibrium
Nash equilibria. Some examples

- Town or village?
- Friend or foe?

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Nash equilibria. Some examples

- The prisoners’ dilemma

\[
\begin{array}{c|cc}
 & C & D \\
\hline
C & 1,1 & -3,3 \\
D & 3,-3 & (-1,-1)
\end{array}
\]
Nash equilibria. Some examples

- The battle of sexes

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However

- When played by humans, most games do not converge to the Nash equilibrium, consistently
- But, in some cases they do!
- What shall we do?
  - To continue developing and applying classical economic theory, recognizing that it does not apply to humans (or not yet – see evolutionary refs.)
  - To abandon rationality (even bounded rationality) in economics and introduce a large contribution of randomness in economic decisions
  - To modify game theory to account for the experimental results and still use the solid Nash equilibrium framework. Is it possible?
2 - Deviations from Nash equilibrium in experimental games

- The ultimatum game
- The public goods game (with and without punishment)
- Dictator game
- Gift exchange game
- Third party punishment game
- The trust game
The ultimatum game

Acceptance $\rightarrow (a,c)$

Proposer $\frac{a}{c} \rightarrow$ Responder $\Rightarrow$ Payoff

Non-accept. $\rightarrow (0,0)$

$a+c=2b$, $a \gg c$,
(Example: $a=99$, $c=1$, $b=50$)

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Monopoly pricing of a perishable good
The ultimatum game

Figure 2 - Cumulative Ultimatum Proposals

- Names
- No Names
The ultimatum game
The ultimatum game
The ultimatum game

Fair offers correlate with market integration and cooperativeness in everyday life
The ultimatum game

However with one proposer and several responders or with one responder and two proposers, the results coincide or are close to the Nash equilibrium predictions

Why?
The public goods game

$n$ players with endowment $y$ decide on their contribution $g$, gaining $p_i = y - g_i + m \sum g_k$

Nash eq. $g_i = 0$ \hspace{1cm} ($m < 1 < mn$)

(Cooperative production, use of resources)
The dictator game

As in the ultimatum game, but no possibility of refusal by the responder
Nash eq. $x=0$

(Giving anonymously to strangers)
The gift-exchange game

Employer offers the worker a wage $W$ for a minimal effort $e$. If the worker refuses, payoffs are $(0,0)$. If he accepts $(ke-W, W-C(e)C(e))$ with $1<k<10$. Nash eq. $e=1$, $W$ minimal. (Nonenforceability of performance)
3 - Neurological input

A two-persons game is a two-brains confrontation
3 - Neurological input

A two-persons game is a two-brains confrontation

and an economy is a large brain crowd
Neurological input

and a brain is not only a cortex (left) frontal lobe,
Neurological input
it has many other specialized parts, with different evolutionary ages
Neurological input

Forebrain
- Cortex $\rightarrow$ *Thought, action, perception of stimuli* (Young)
- Thalamus
- Hypothalamus
- Amygdala
- Hippocampus

Midbrain
- Tectum
- Tegmentum

Hindbrain
- Pons
- Medulla
- Cerebellum $\rightarrow$ *"The little brain"* (Movement, posture, balance) (Old)

Limbic System, *"the emotional brain"* (Old)

Brain Stem  *(vital life functions: breathing, heartbeat, blood pressure)*
Neurological input

- Many brain activities are automatic, rapid processes which typically occur without awareness. No intervention of the deliberative cortex and the language module.


- Conscient actions are preceded by a much earlier reaction of the limbic system, which modulates the conscient deliberation.

- Beware also of “irrationality” of the rational modules:
  - Experiments with music and light flashes. Tendency to mentalise a synchronization where there is none.
  - The hindsight bias: Overwriting of what was previously believed. Past events may seem predictable. Erroneous attribution of intentions to other agents.
Techniques to “see” the brain in action

- EEG
  Electrical activity outside the brain. Good time resolution (≈1ms), spatial resolution poor and does not record interior brain activity

- PET
  Measures blood flow in the brain. Better spatial resolution than EEG, but poorer time resolution. Limited to short tasks, because radioactivity decays.

- fMRI
  Measures changes in the blood oxygenation. Weak signal, better spatial resolution than PET (≈3mm)

- Single neuron measurements
- Psychopathology
- Brain damage in humans
Unfair offers in the ultimatum game

(Sanfey et al., Science 300 (2003) 1755)
Unfair offers in the ultimatum game

fMRI – Unfair offers activate prefrontal cortex (PFC), anterior cingulate (ACC) and the insula

Interpretation:
- ACC (executive function area) struggles to resolve the conflict between wanting money (PFC) and disliking being treated unfairly (Insula)
- Insula activated for pain, disgust, etc.
The neural basis of strong reciprocity

*Strong reciprocity = altruistic punishment*

Punishing unfair behavior (or social norm violators) even at a cost to himself

- The trust game with punishment
  
  (de Quervain et al. Science 305 (2004)1254)
  
  Player A and B receive 10 MU
  
  Player A may transfer to B either 0 or 10
  
  Transferred quantity is multiplied by 4
  
  Player B decides to transfer half or nothing
  
  - In case of no transfer, player A has 1 minute to decide whether to punish or not (p up to 20 points)
  
  - Punishment costs p to A and 2xp to B
  
  - Player A is PET scanned during the decision minute
The neural basis of strong reciprocity

- Several protocols
  IF – Intentional and free
  IC – Intentional and costly
  IS – Intentional and symbolic
  NC – Non-intentional and costly
The neural basis of strong reciprocity

*Interpretation*:

- Activation of the caudate nucleus (region associated with reward processing, anticipation of pleasure)
- Activation of prefrontal cortex when punishment is costly (integration of the benefits and costs of punishing)

- The same caudate region is activated when people reward cooperators.
- Punishment of defectors is altruistic in the biological sense but not in the psychological sense
- Conclusion: a satisfying social outcome is sweet, but revenge is sweet too
The neural basis of strong reciprocity

Brian Knutson illustration of altruistic punishment:

Go ahead, make my day!

“Dirty” Harry informs a norm violator that he anticipates pleasure from inflicting punishment.
Oxytocin increases trust


- Oxytocin – a small peptide produced naturally in the hypothalamus
- Acts on some functional targets (inducing labour and lactation) and in brain regions (amygdala, nucleus accumbens). It facilitates approach behavior
- The trust game
Oxytocin increases trust

- The players receive an intranasal dose either of oxytocin or of a placebo.
- The game is either played with a human trustee (trust) or with a computer (risk).
Oxytocin increases trust

- Strong effect on investors (in the trust experiment) but not on trustees
- Reciprocity not affected by oxytocin
Other experiments

- Normal versus prefrontal cortex damage decisions (Bechara et al. Science 275 (1997) 1293)
- Brain activity during monetary incentive task (Knutson et al. NeuroImage 12 (2000) 20)
- Neural responses to monetary gains and losses (Breiter et al., Neuron 30 (2001) 619)
- Gains versus losses activity in the OFC (O’Doherty et al., Nature Neuroscience 4 (2001) 95)
- Mirror neuron activation (Keysers Neuron 31 (2001) 155)
- Neural basis for social cooperation (Rilling et al., Neuron 35 (2002) 395)
- Gains versus losses activity in the cortex (Dickhaut et al. PNAS 100 (2003) 3536)
- Social cognition and self-referential thought (Mitchell et al., J. Cognitive Neuroscience 17 (2005) 1306)
- Uncertainty in decision-making (Hsu et al., Science 310 (2005) 1680)
- Responses of the cingulate cortex in economic exchanges (Tomlin et al., Science 312 (2006) 1047)
- Neural coding of reward (Dreher et al., Cerebral Cortex 16 (2006) 561)
- Neurons encoding economic value (Padoa-Schioppa et al., Nature 441 (2006) 223)
- etc. etc.
4 - Modelling neuroeconomics

- The economic brain
Modelling neuroeconomics

- Some detailed neurological reaction circuits

  The limbic system relates to everything (Damasio 98)

  Somatic activation and decision making (Bechara 2005)
Modelling neuroeconomics

- Some detailed neurological reaction circuits

- Reward processing (Schultz 2000)
Modelling neuroeconomics

- Are there any simple modelling concepts amenable to mathematical treatment?

- Several concepts:
  - Inequality aversion (Fehr, Schmidt)
  - Reciprocity (Bowles, Gintis, Falk, Fischbacher)
  - Beliefs, preferences and constraints (Heinrich, Boyd, Bowles, Camerer, Fehr, Gintis)

- From payoffs to utility functions

- Game theory with operators
Inequality aversion. The INA operator

- $\otimes M_i = \text{space of payoffs and utilities, N players}$
- $g_i = \text{payoff of player i } ; \quad s = \{s_{ia}\} \quad \text{strategies of player i } ;$
- $s = \text{set of all strategies}$
- $\otimes g_i \otimes M_i \quad \rightarrow \quad \otimes u_i \otimes M_i$

\[ u_i(s) = g_i(s) - \left( \frac{1}{N} - 1 \right) \sum_{k \neq i} (g_k - g_i) \varepsilon_{\alpha\beta}(g_k - g_i) \]

- with $\varepsilon_{\alpha\beta}(x) = \alpha \quad (x>0)$ and $\varepsilon_{\alpha\beta}(x) = -\beta \quad (x<0)$
- Typically $\alpha > \beta$

After the application of the INA operator, Nash equilibrium is found for the $u$’s

- This operator leads to an utility function identical to the one of Fehr and Schmidt
Inequality aversion. The INA operator

- Example: Ultimatum game

\[ g_P \]

\[ \begin{array}{ccc}
4:1 & 4 \times & 0 \\
3:2 & 3 & 0 \\
2:3 & 2 & 0 \\
1:4 & 1 & 0 \\
\end{array} \]

\[ g_R \]

\[ \begin{array}{ccc}
4:1 & 1 \times & 0 \\
3:2 & 2 & 0 \\
2:3 & 3 & 0 \\
1:4 & 4 & 0 \\
\end{array} \]
Inequality aversion. The INA operator

- After the application of INA

- \( \alpha = 0.6 \quad \beta = 0.4 \)

- \( \alpha = 0.7 \quad \beta = 0.6 \)

\[
\begin{array}{c|cc|c|cc|c|cc}
\hline
& A & R & & A & R & & A & R \\
\hline
4:1 & 2.8 & 0* & 4:1 & -0.8 & 0* & 4:1 & 2.2 & 0 \\
3:2 & 2.6 & 0 & 3:2 & 1.4 & 0 & 3:2 & 2.4* & 0 \\
2:3 & 1.4 & 0 & 2:3 & 2.6 & 0 & 2:3 & 1.3 & 0 \\
1:4 & -0.8 & 0 & 1:4 & 2.8 & 0 & 1:4 & -1.1 & 0 \\
\hline
\end{array}
\]

- Experimental results:
  - Offers < 0.2: 3.8%
  - 0.4 < offers < 0.5: 71%
Inequality aversion. The INA operator

- The INA operator seems OK
- But is not!
- Example: \( \alpha = 0.7 \quad \beta = 0.4 \)
Inequality aversion. The INA operator

- The INA operator seems OK
- But is not!
- Example: $\alpha = 0.7$ $\beta = 0.4$

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$\Rightarrow$ INA

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$\Rightarrow$ INA

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<td>6</td>
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$g_P$ $g_R$ $u_P$ $u_R$

- The result is the same in both cases. The utility of the 8:2 proposal for the responder is the same in both cases.
- But one knows that experimentally that proposal is accepted more often in the second case.
Reciprocity. The RECI operator

- A simplified version of the ideas of Falk and Fischbacher
- Kindness of $j$ towards $i$
  \[ K_{ji}(s_1 \ldots s_j \ldots s_{N}) = g_i(s) - \left(\frac{1}{n_{sj}}\right) \sum g(s_1 \ldots s_j \ldots s_{N}) \]
  \[ n_{sj} = \text{number of strategies of player } j \]
- \[ u_i(s) = g_i(s) - \left(\frac{1}{N-1}\right) \sum_j (K_{ij} - K_{ji}) \varepsilon^{\alpha \beta} (K_{ij} - K_{ji}) \]
- Nonlocal operator in strategy space
- Instead of using the differential of payoffs to compute the utility, the kindness differential is used
- Takes care of intentions
Reciprocity. The RECI operator

- Apply to the previous example:
  - $\alpha = 0.7$  $\beta = 0.4$

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$g_P$ $g_R$ → $RECI$ $RECI$

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<td>10:0</td>
<td>7.6</td>
<td>-3.5</td>
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- $p =$ probability of 8:2 offer
- $a =$ acceptance probability

The Nash equilibrium in the first case: (p=0.95, a=0.46)
The Nash equilibrium in the second case: (p=0.59, a=0.46)

However, for the conditional probabilities: if $p=1$ then $a=0$ in the first case and $a=1$ in the second
Calculation of Nash equilibria for mixed strategies

\[ u_P \]

\[ u_R \]

\[ X_P = p(-1.5a + 0.7) + 11.1a - 3.5 \]

\[ X_R = a(3.7p - 2.2) + 0.4p - 2 \]
Concluding questions

- “Game theory with operators” is simply a way to systematize and unify the computation of utility functions.

- Is the RECI operator sufficiently universal?

- Insofar as the feasible strategies of each player are the simplex of all pure strategies, existence of equilibrium points for the utilities is guaranteed by the Nash theorem.

- In the case of social equilibrium (Debreu) the operators should not spoil contractibility of the set of feasible strategies.

- If more than one operator is needed, what about commutativity properties?

- Etc.
Some additional references

- **Neuroeconomics**:  
  C. Camerer, G. Loewenstein and D. Prelec; J. of Economic Literature 43 (2005) 9-64  
  A. Bechara and A. R. Damasio; Games and Economic Behavior 52 (2005) 336-372

- **Inequality aversion**:  
  Fehr and Schmidt; Quarterly J. of Economics 114 (1999) 817-868

- **Reciprocity**:  
  Falk and Fischbacher; Games and Econ. Behavior 54 (2006) 293-315

- **Evolutionary aspects of strong reciprocity**:  
  R V M; Advances in Complex Systems 7 (2004) 357-368

- **An alternative to model deviations from classical Nash equilibrium**:  
  R V M; Quantum Information Proc. 4 (2005) 1-12