Games and neuroeconomics

Rui Vilela Mendes

Contents

1 - Games

- **Game theory:** Study of multi-person decision problems influencing one another's welfare
- Economics, Biology, Social Sciences, Communication
- **Mechanism**:
  Cooperation or competition to reach a best goal state (from the cooperative or individual point of view)

1 - Games - Nash equilibrium

- $(s_1,s_2,\ldots,s_k,\ldots,s_n)$ is Nash equilibrium if $P(s_1,s_2,\ldots,s_k,\ldots,s_n) > P(s_1,s_2,\ldots,s_k',\ldots,s_n)$ for all s_k'
- No player can improve his payoff by changing his strategy, when the strategies of the other players are fixed
- ***Theor:*** (Nash) *Every N-player game, with finite strategies, has at least one equilibrium, in pure or mixed strategies*
- In economy, Nash equilibrium ⇔ Self-interested rational decisions (Homo Oeconomicus)
- Provides a sound basis for (almost) the whole of (rigorous) economic theory

1 - Games - Nash equilibrium

- Example :
- *Theorem:* Consider the class of abstract economies with
  (i) strategy sets convex and compact,
  (ii) payoff functions continuous and quasi-concave,
  (iii) the feasibility correspondences are continuous and have nonempty convex values.
  Then, the Nash correspondence is the unique solution that satisfies nonemptiness, rationality in one-person games and consistency

  (B. Peleg; Games and Economic Behavior 18 (1997) 277-285)

1 - Games - Nash equilibrium

- Note :
- *"Feasibility correspondences"*
  The feasible strategies of each agent may depend on the other agents strategies
- *Social equilibrium (Debreu) ,*
  a generalization of Nash equilibrium

Nash equilibria. Some examples

- Town or village ?
  Friend or foe ?

	T	V
T	1,1	(2,5)
V	(5,2)	-1,-1

Nash equilibria. Some examples

- The prisioners' dilemma

	C	D
C	1,1	-3,3
D	3,-3	(-1,-1)

Nash equilibria. Some examples

- The battle of sexes

		John	
		Opera	TV
Mary	Opera	(5,2)	1,1
	TV	1,1	(2,5)

However

- *When played by humans, most games do not converge to the Nash equilibrium, consistently*
- *But, in some cases they do !*
- *What shall we do ?*
  - To continue developing and applying classical economic theory, recognizing that it does not apply to humans (or not yet – see evolutionary refs.)
  - To abandon rationality (even bounded rationality) in economics and introduce a large contribution of randomness in economic decisions
  - To modify game theory to account for the experimental results and still use the solid Nash equilibrium framework. Is it possible ?

2 - Deviations from Nash equilibrium in experimental games

- The ultimatum game
- The public goods game (with and without punishment)
- Dictator game
- Gift exchange game
- Third party punishment game
- The trust game

The ultimatum game

			Acceptance	→	(a,c)
					↑
Proposer	a c	→	Responder	⇒	Payoff
					↓
			Non-accept.	→	(0,0)

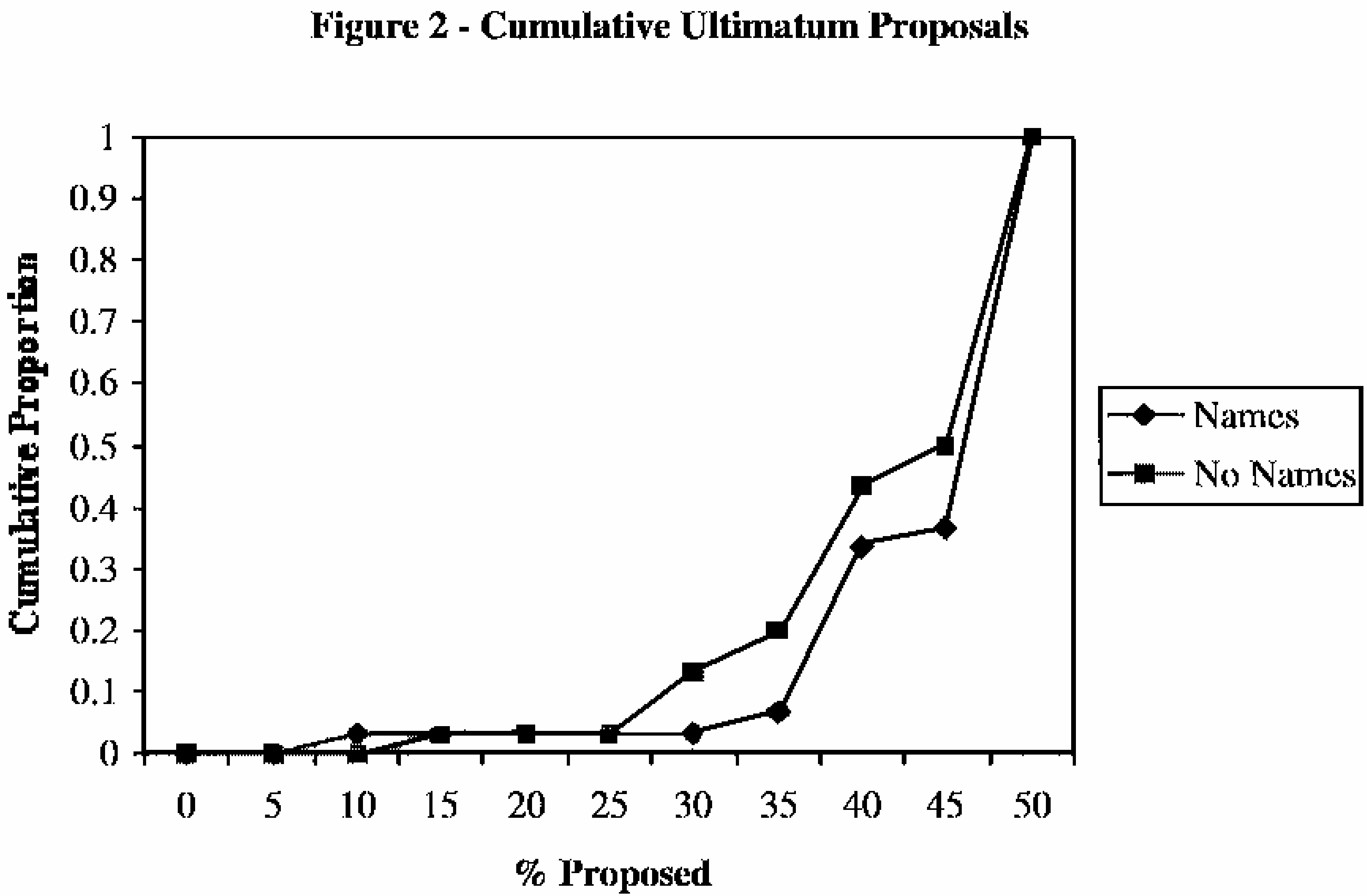
a+c=2b , a>>c,

(Example: a=99, c=1, b=50)

	R0	R1
P0	(a,c)	0,0
P1	b,b	0,0

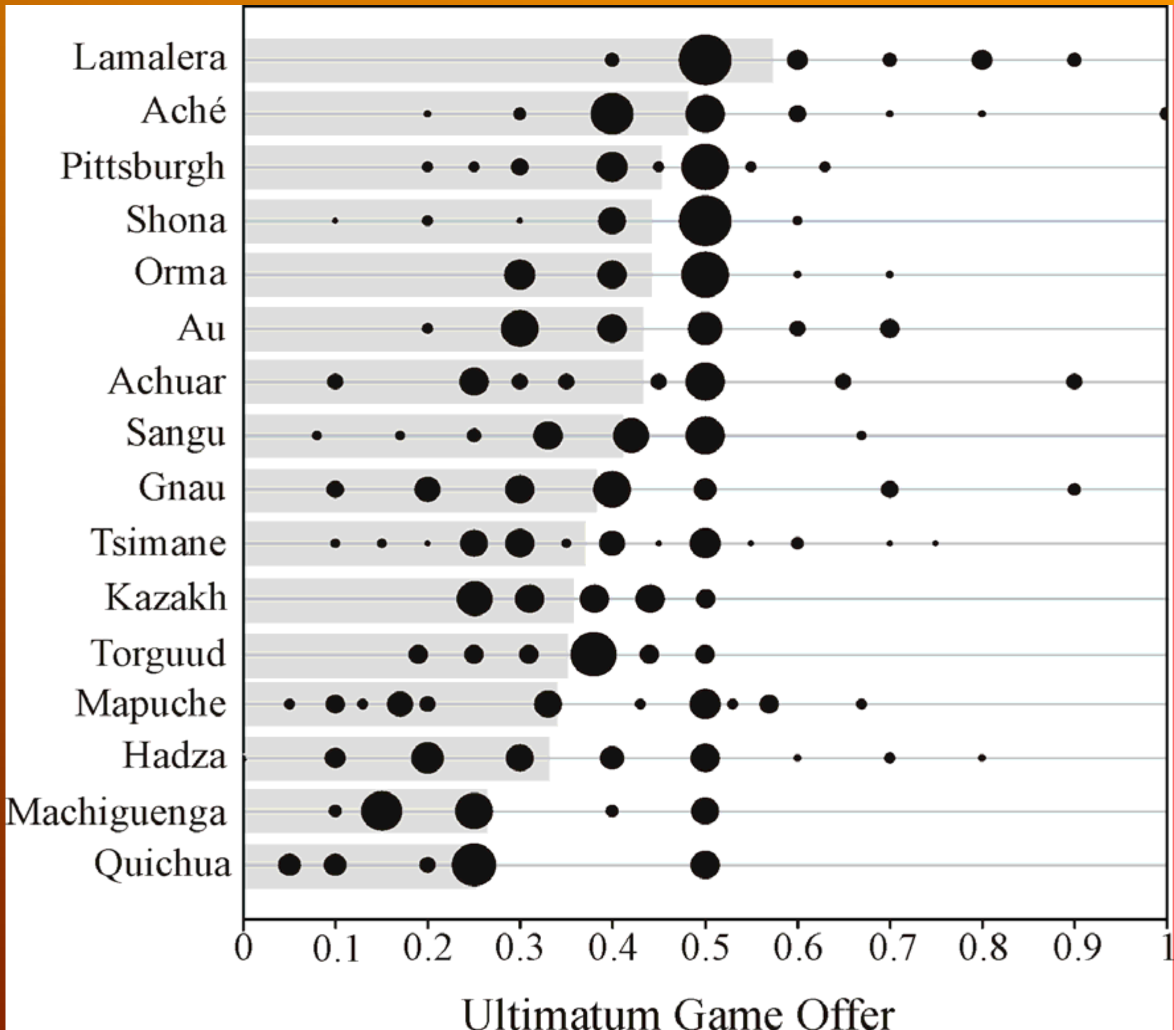
Monopoly pricing of a perishable good

The ultimatum game

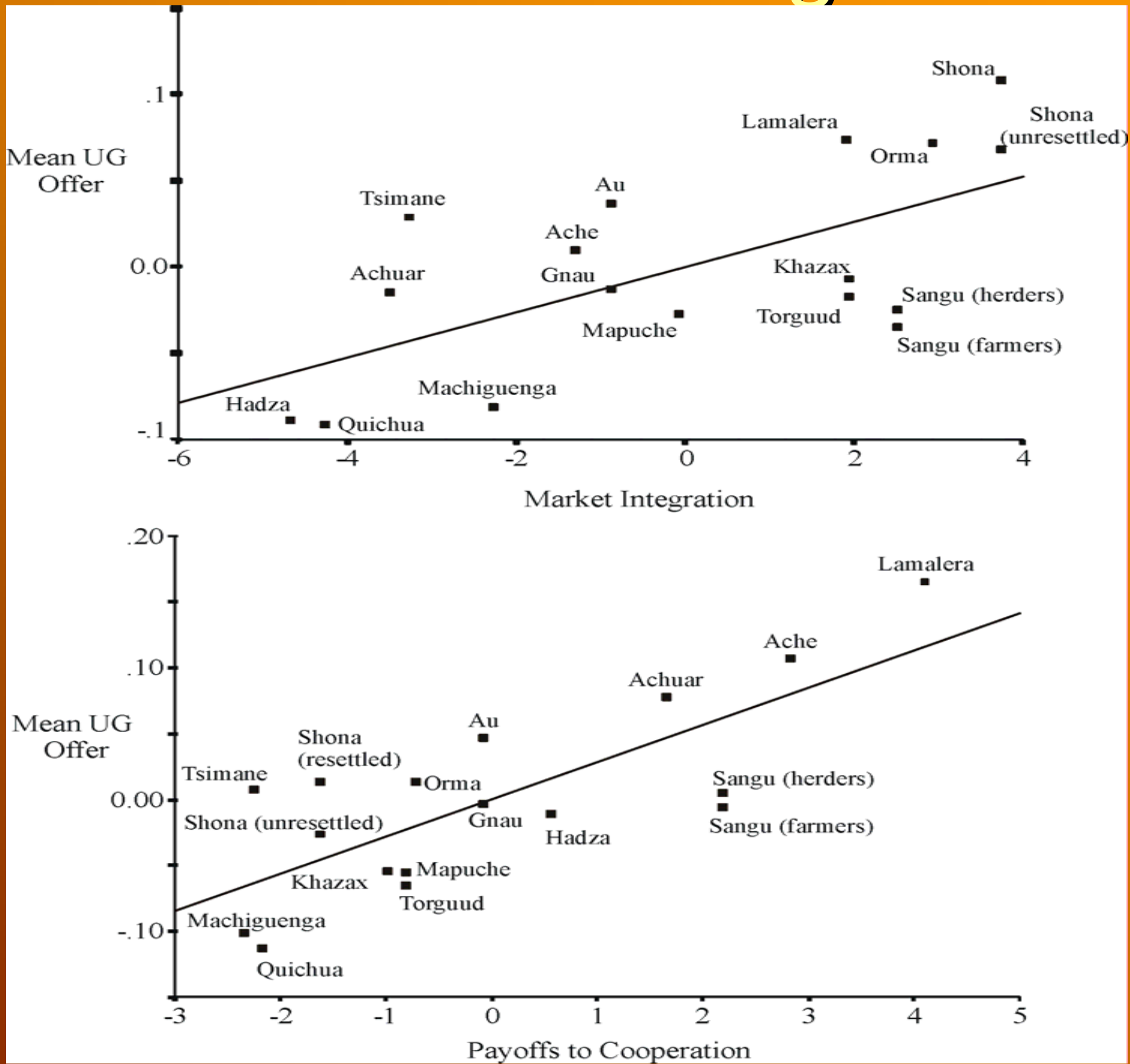


The ultimatum game

The ultimatum game



The ultimatum game



Fair offers correlate with market integration and cooperativeness in everyday life

The ultimatum game

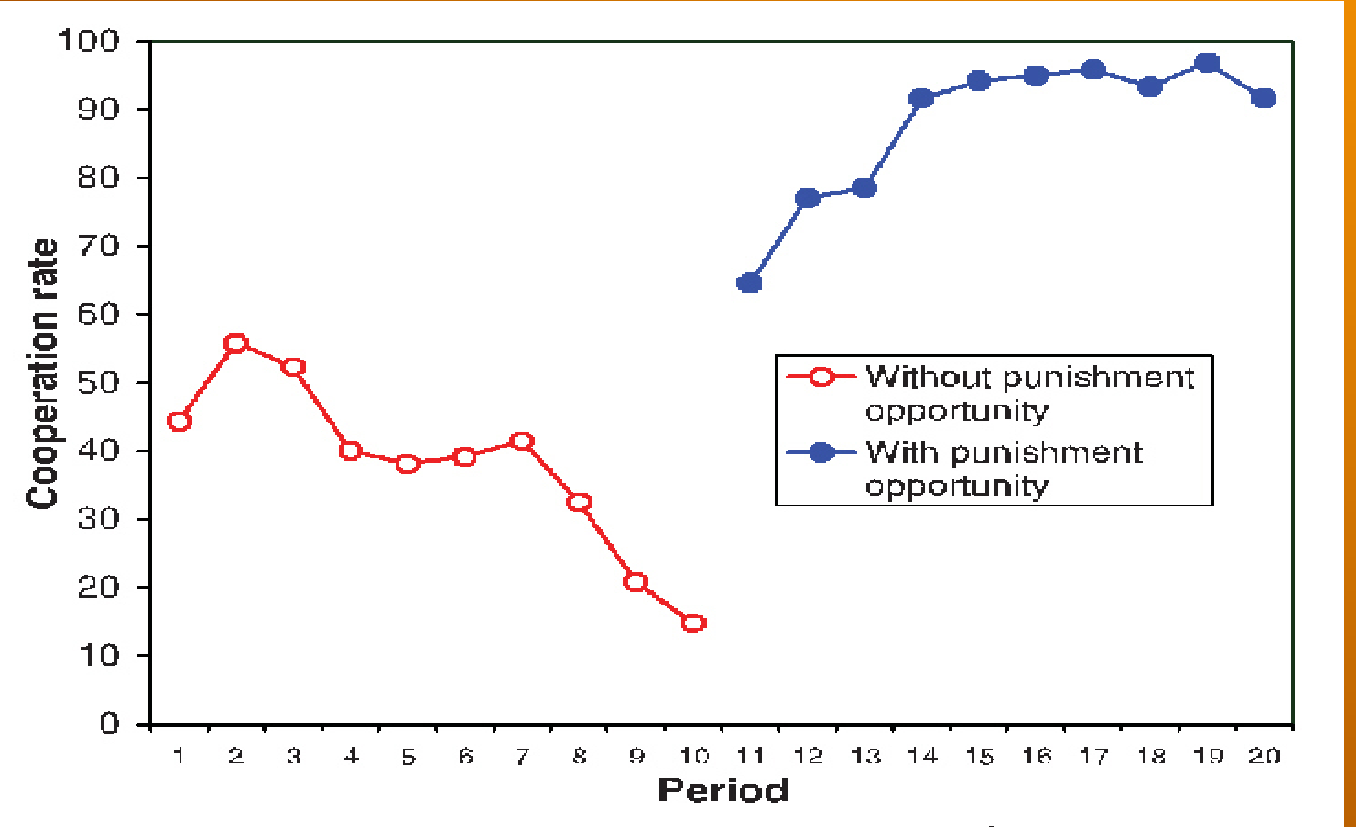
- However with one proposer and several responders or with one responder and two proposers, the results coincide or are close to the Nash equilibrium predictions
- Why ?

The public goods game

n players with endowment y decide on their contribution g, gaining $p_i = y - g_i + m\,\Sigma g_k$

Nash eq. $g_i = 0$ $(m < 1 < mn)$

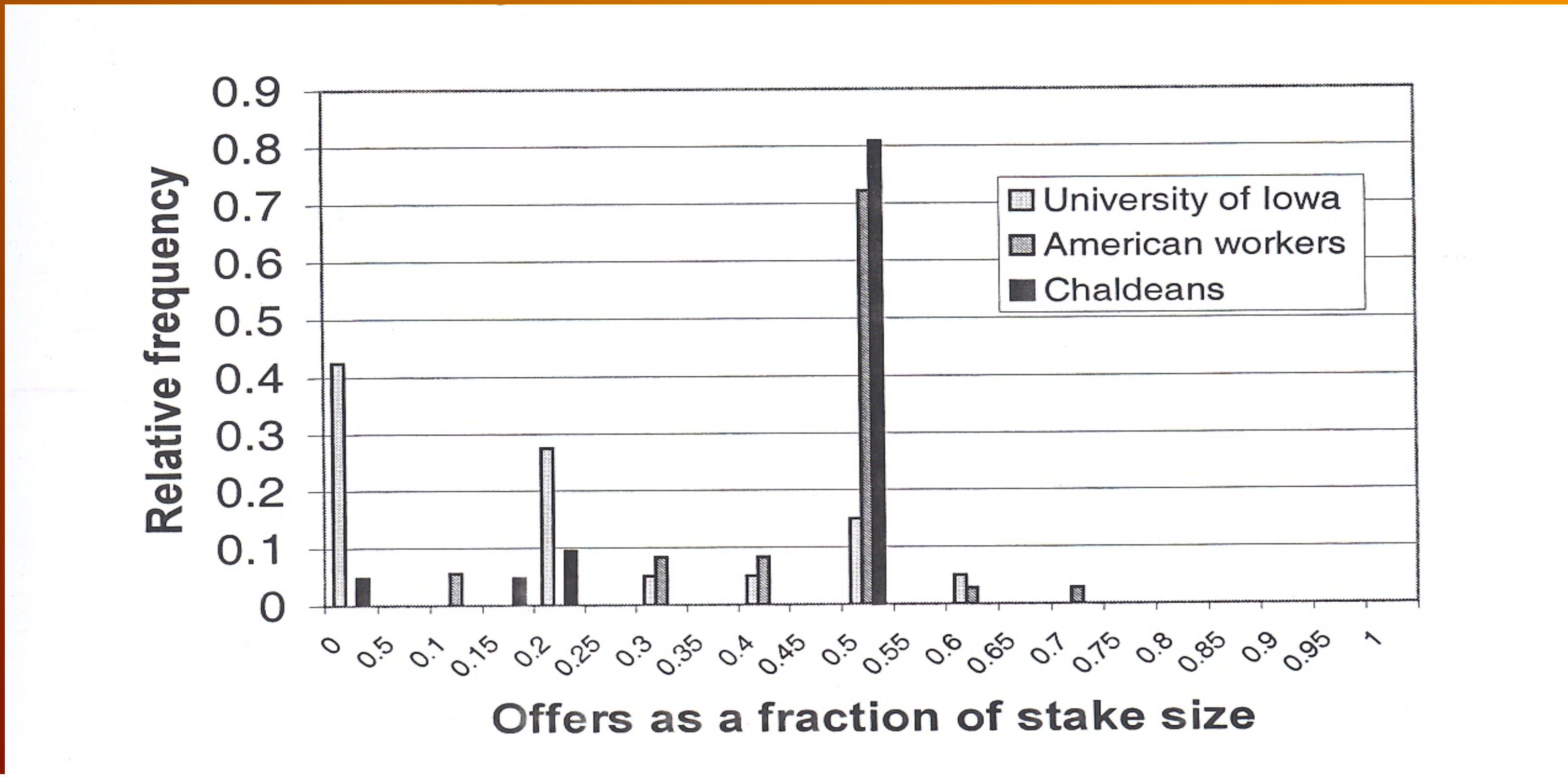
(Cooperative production, use of resources)



The dictator game

As in the ultimatum game, but no possibiity of refusal by the responder
Nash eq. x=0

(Giving anonymously to strangers)

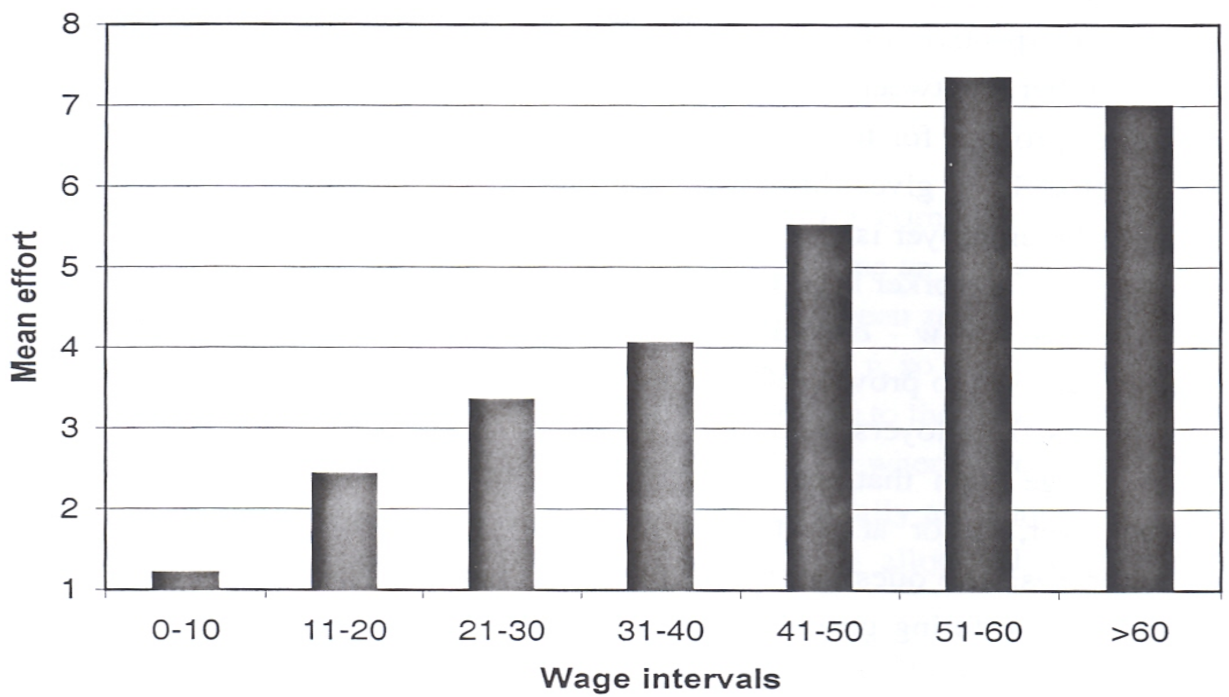


The gift-exchange game

Employer offers the worker a wage W for a minimal effort e. If the worker refuses, payoffs are (0,0)

If he accepts (ke-W , W-C(e)C(e)) $1<k<10$

Nash eq. e=1 , W minimal (Nonenforceability of performance)



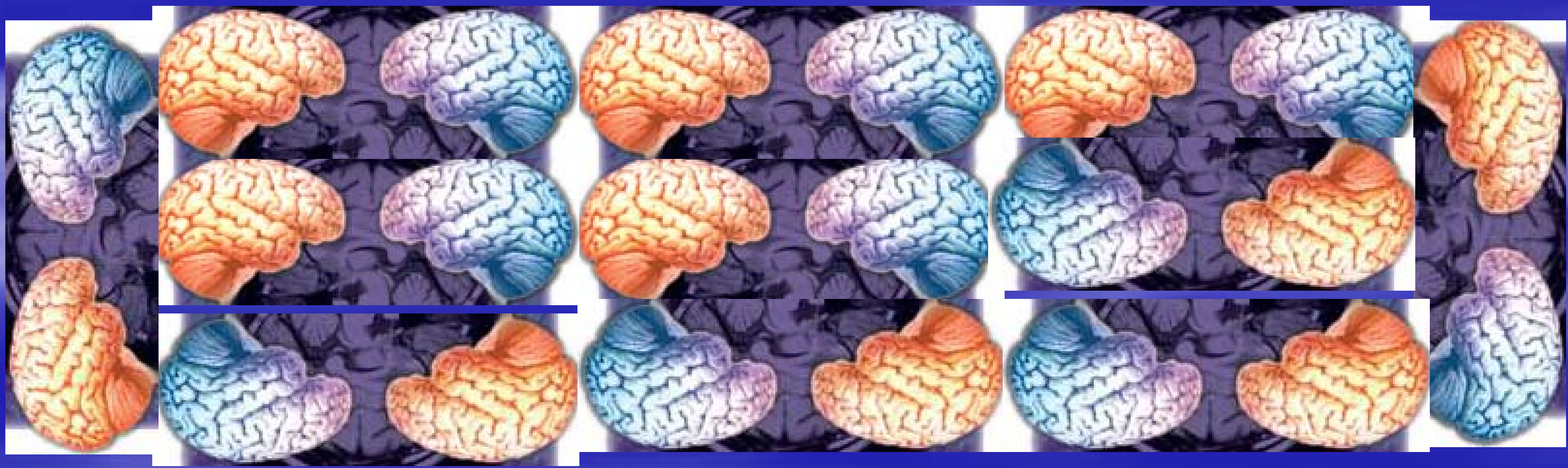
3 - Neurological input

A two-persons game is a two-brains confrontation

3 - Neurological input

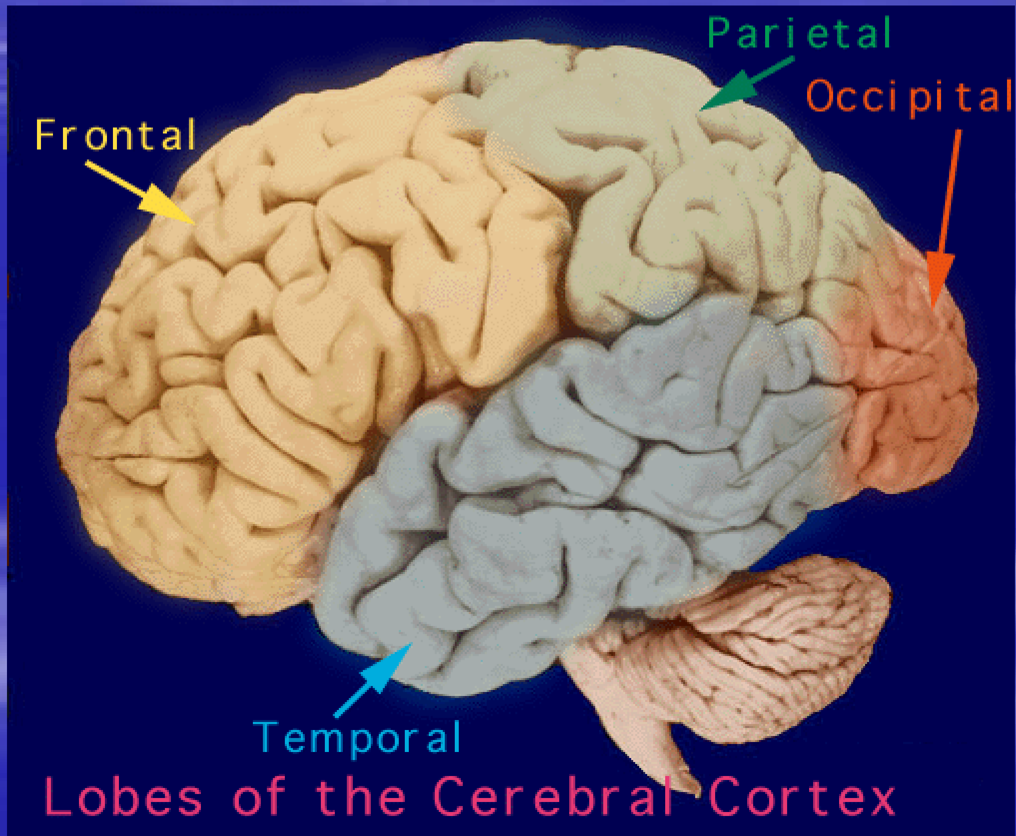
A two-persons game is a two-brains confrontation

and an economy is a large brain crowd



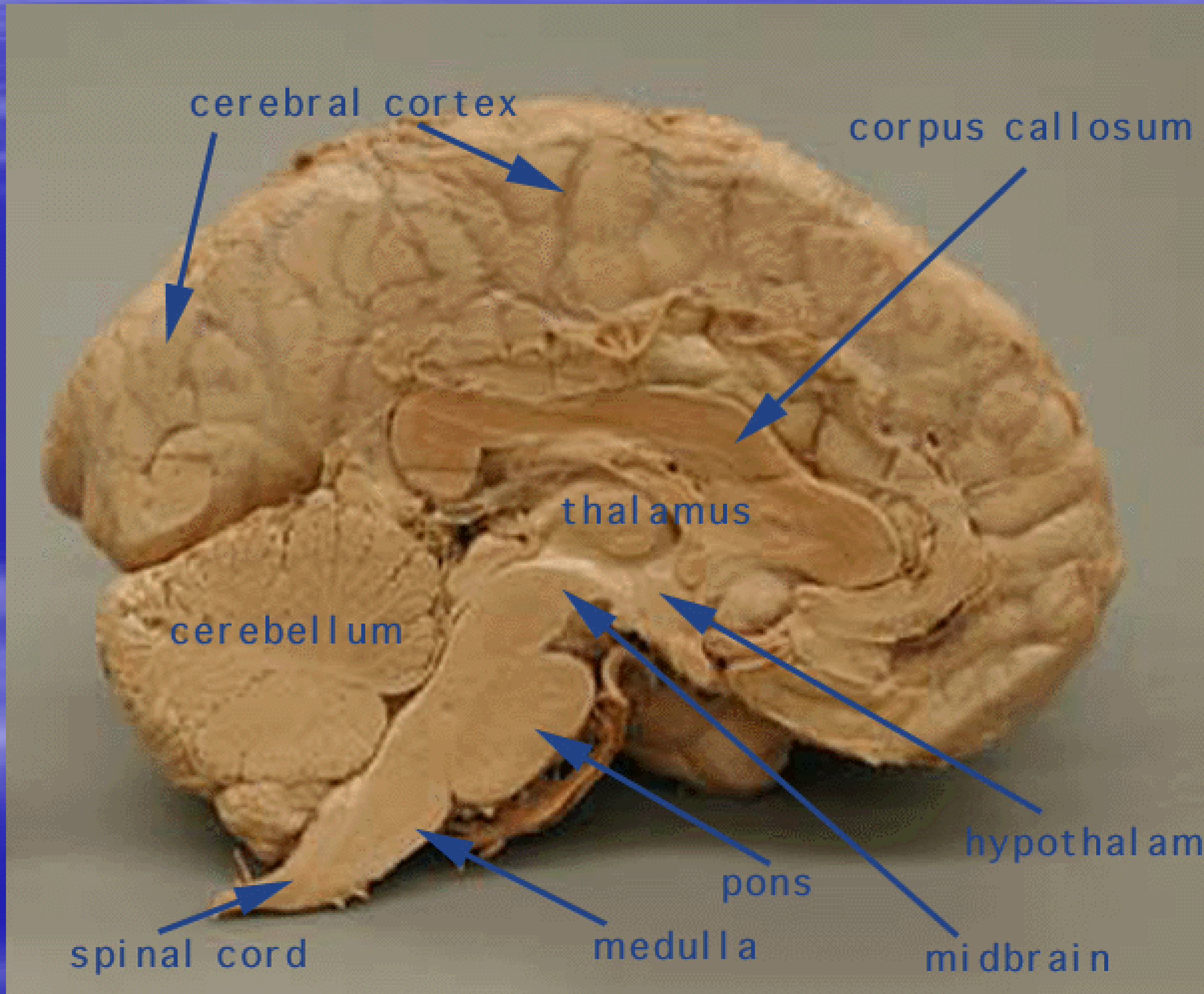
Neurological input

and a brain is not only a cortex (left) frontal lobe,



Neurological input

- it has many other specialized parts, with different evolutionary ages



Neurological input

- Forebrain
  - Cortex → *Thought, action, perception of stimuli* (*Young*)
  - Thalamus
  - Hypothalamus
  - Amygdala
  - Hippocampus

  Thalamus, Hypothalamus, Amygdala, Hippocampus: *Limbic System*, "*the emotional brain*" (*Old*)

- Midbrain
  - Tectum
  - Tegmentum
- Hindbrain
  - Pons
  - Medulla
  - Cerebellum → "The little brain" (*Movement, posture, balance*)(*Old*)

Tectum, Tegmentum, Pons, Medulla: *Brain Stem (vital life functions: breathing, heartbeat, blood pressure*)

Neurological input

- Many brain activities are automatic, rapid processes which typically occur without awareness. No intervention of the deliberative cortex and the language module.
- Experiment (Whalen et al. J. Neuroscience 8 (1998) 411): 40 ms flashes of happy and angry faces activate the amygdala, without people being aware of what they saw.
- Conscient actions are preceded by a much earlier reaction of the limbic system, which modulates the conscient deliberation.
- Beware also of “irrationality” of the rational modules :

  · Experiments with music and light flashes. Tendency to mentalise a synchronization where there is none.

  · The hindsight bias : Overwriting of what was previously believed. Past events may seem predictable. Erroneous attribution of intentions to other agents.

Techniques to “see” the brain in action

- EEG

  Electrical activity outside the brain. Good time resolution (~1ms), spatial resolution poor and does not record interior brain activity

- PET

  Measures blood flow in the brain. Better spatial resolution than EEG, but poorer time resolution. Limited to short tasks, because radioactivity decays.

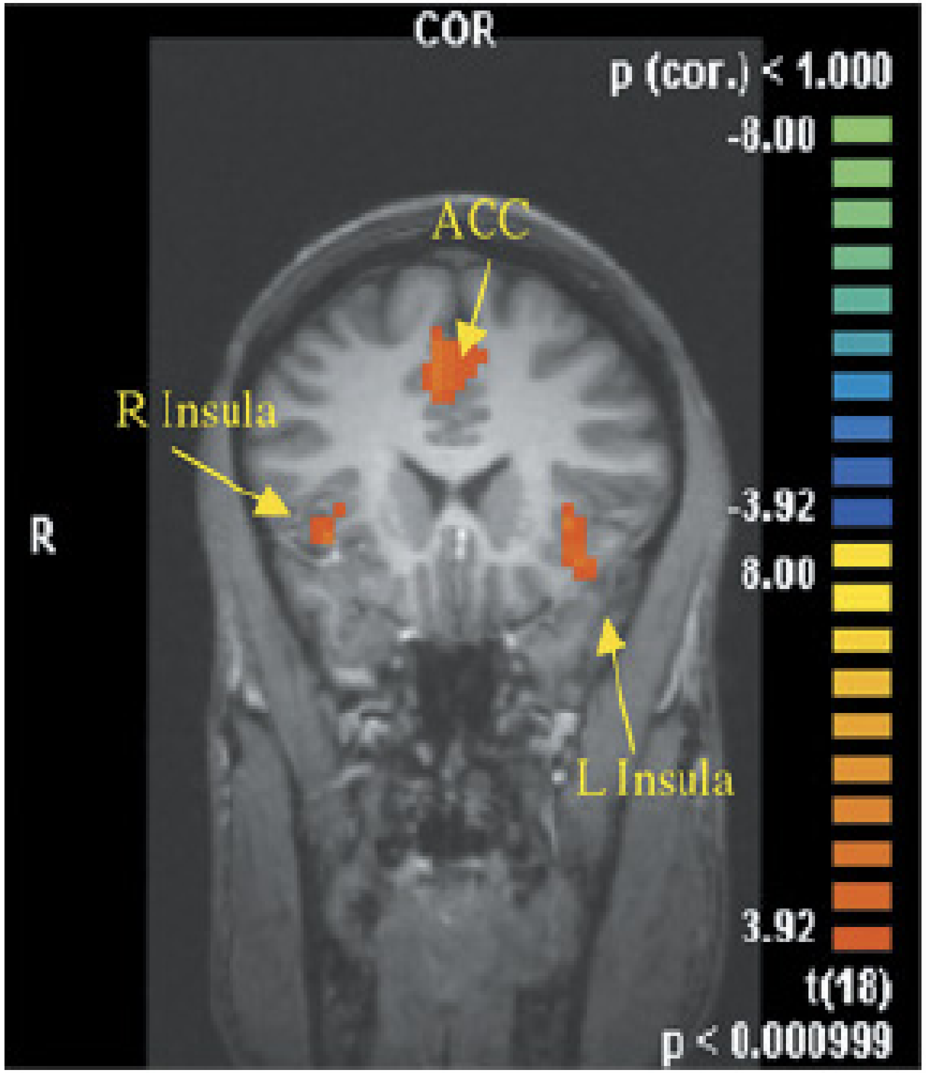
- fMRI

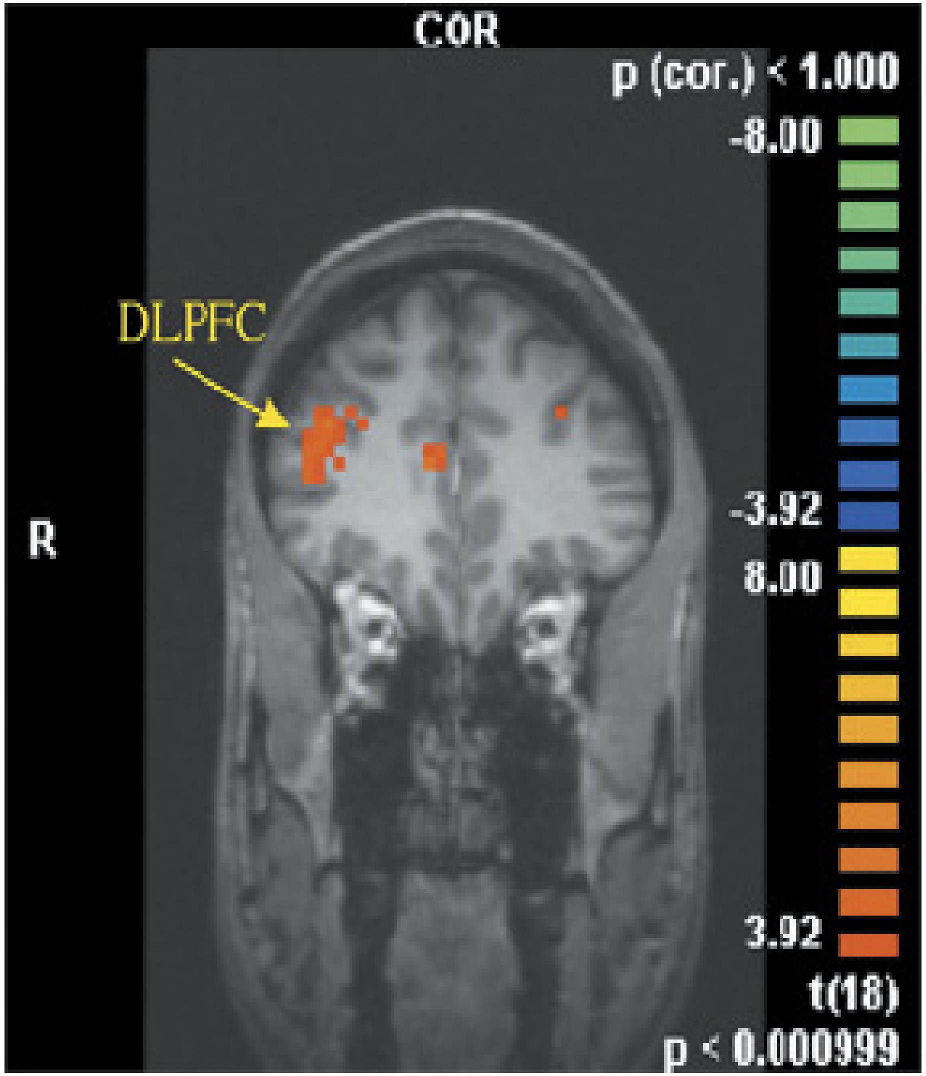
  Measures changes in the blood oxygenation. Weak signal, better spatial resolution than PET (~3mm)

- Single neuron measurements
- Psychopathology
- Brain damage in humans

Unfair offers in the ultimatum game

(Sanfey et al., Science 300 (2003) 1755)





Unfair offers in the ultimatum game

fMRI – Unfair offers activate prefrontal cortex (PFC), anterior cingulate (ACC) and the insula

Interpretation:

- ACC (executive function area) struggles to resolve the conflict between wanting money (PFC) and disliking being treated unfairly (Insula)
- Insula activated for pain, disgust, etc.

The neural basis of strong reciprocity

Strong reciprocity = altruistic punishment

Punishing unfair behavior (or social norm violators) even at a cost to himself

- The trust game with punishment

(de Quervain et al. Science 305 (2004)1254)

Player A and B receive 10 MU

Player A may transfer to B either 0 or 10

Transferred quantity is multiplied by 4

Player B decides to transfer half or nothing

- In case of no transfer, player A has 1 minute to decide whether to punish or not (p up to 20 points)
- Punishment costs p to A and 2xp to B
- Player A is PET scanned during the decision minute

The neural basis of strong reciprocity

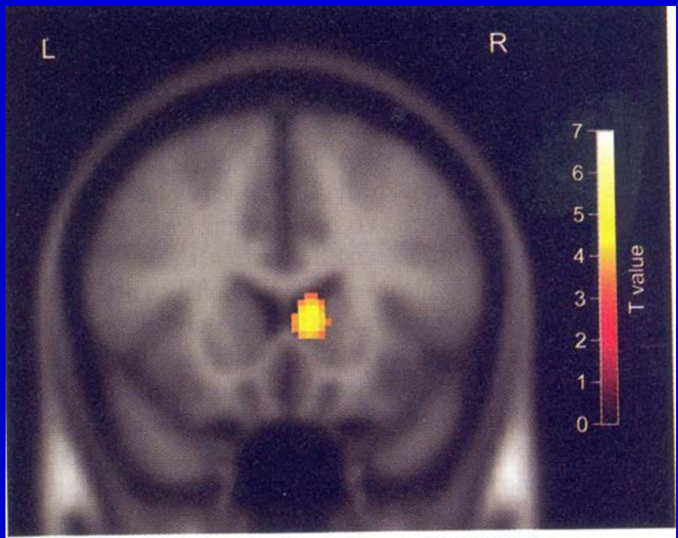
- Several protocols

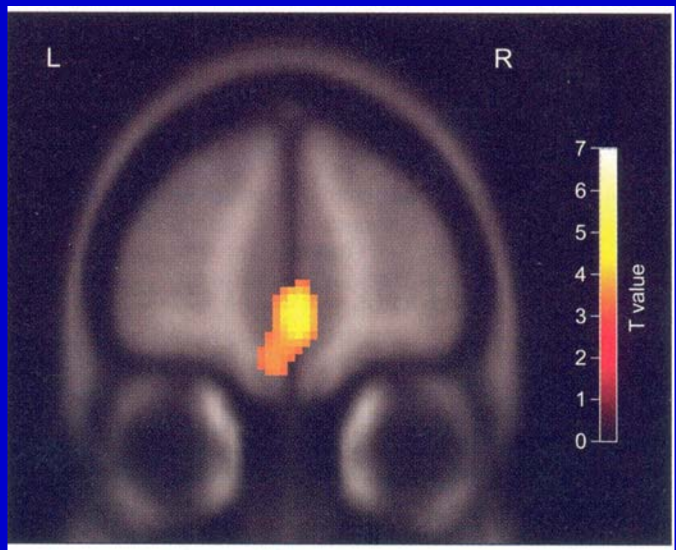
IF – Intentional and free

IC – Intentional and costly

IS – Intentional and symbolic

NC – Non-intentional and costly





The neural basis of strong reciprocity

Interpretation :

- Activation of the caudate nucleus (region associated with reward processing, anticipation of pleasure)
- Activation of prefrontal cortex when punishment is costly (integration of the benefits and costs of punishing)

- The same caudate region is activated when people rewards cooperators.

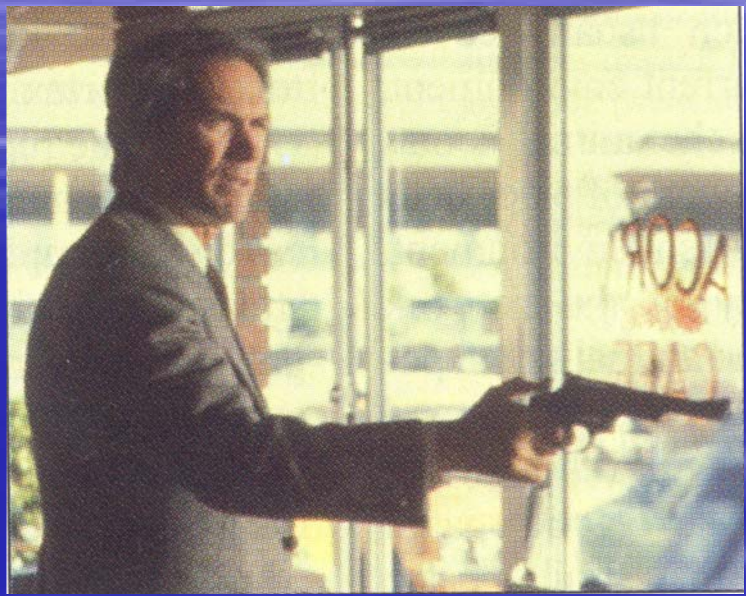
  (Rilling et al. Neuron 35 (2002) 395)

-Punishment of defectors is altruistic in the biological sense but not in the psychological sense

- Conclusion : a satisfying social outcome is sweet, but revenge is sweet too

The neural basis of strong reciprocity

Brian Knutson illustration of altruistic punishment:



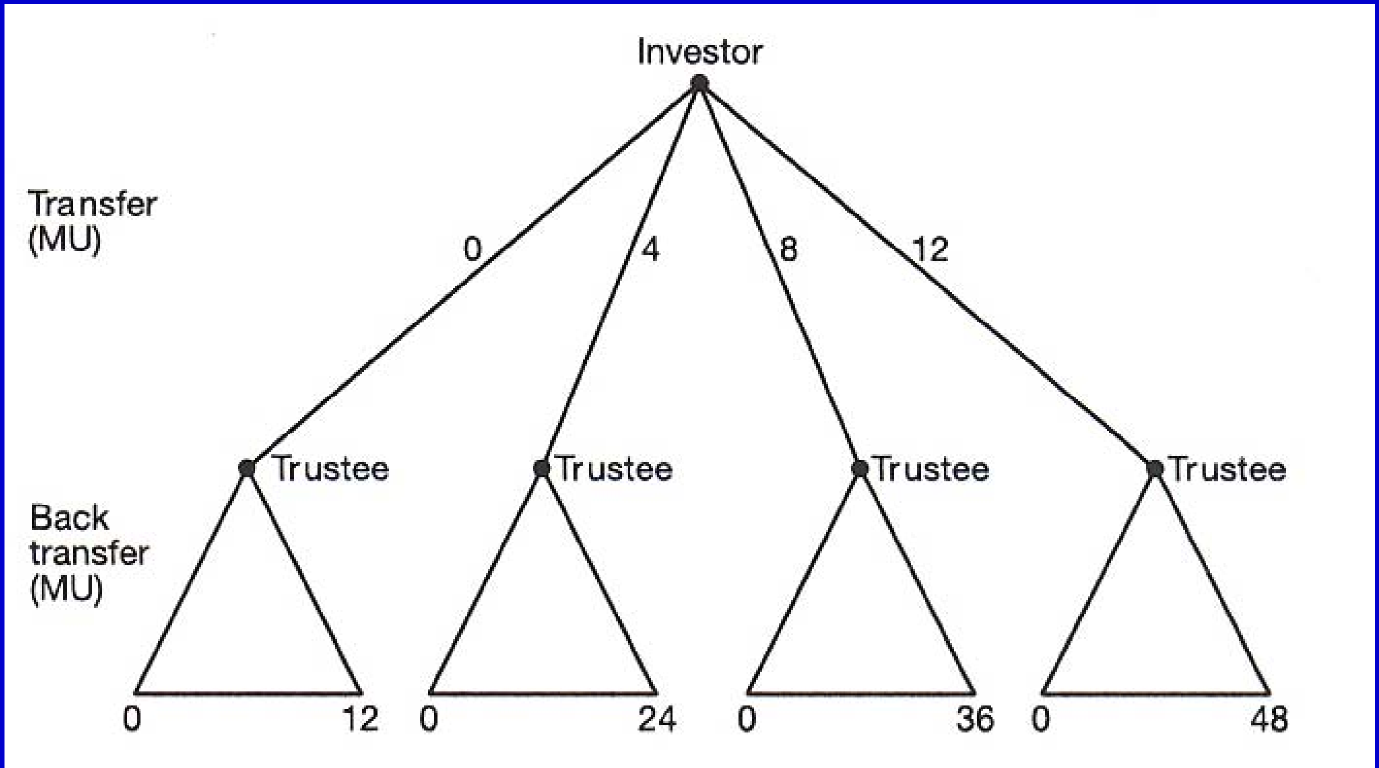
Go ahead, make my day !

"Dirty" Harry informs a norm violator that he anticipates pleasure from inflicting punishment

Oxytocin increases trust

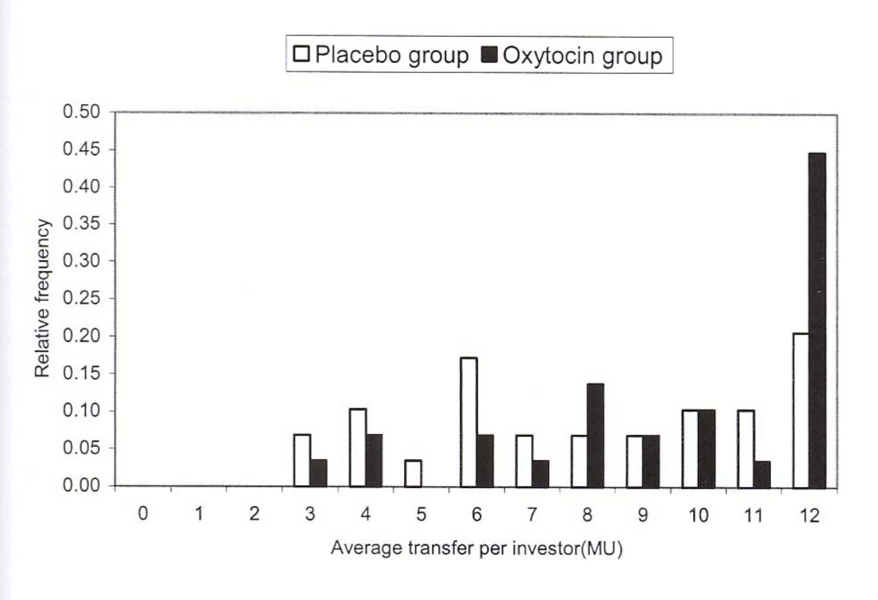
(Kosfeld et al. Nature 435 (2005) 673)

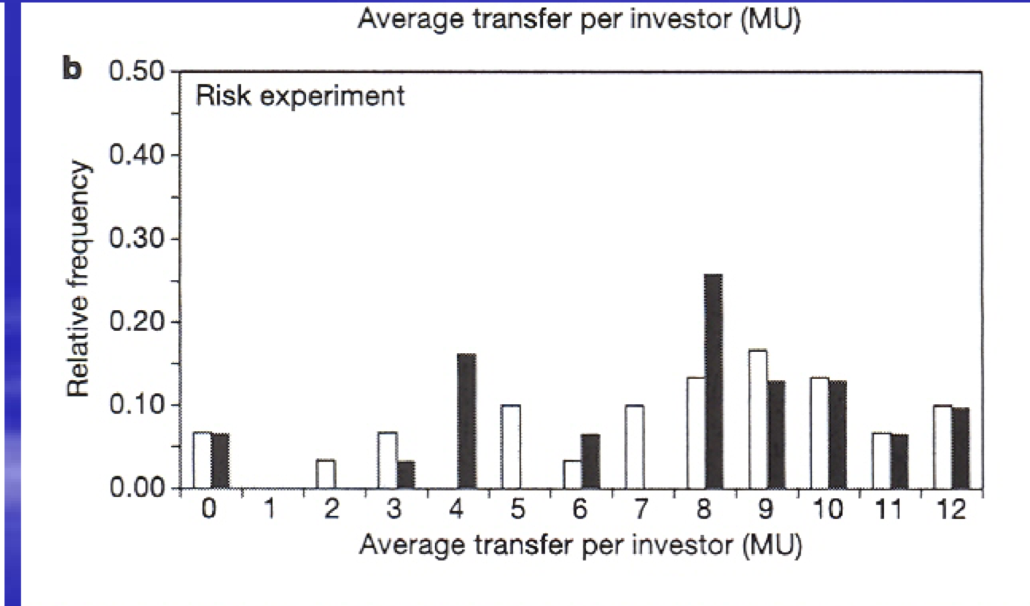
- Oxytocin – a small peptide produced naturally in the hypothalamus
- Acts on some functional targets (inducing labour and lactation) and in brain regions (amygdala, nucleus accumbens). It facilitates approach behavior
- The trust game



Oxytocin increases trust

- The players receive a intranasal dose either of oxytocin or of a placebo
- The game is either played with an human trustee (trust) or with a computer (risk)

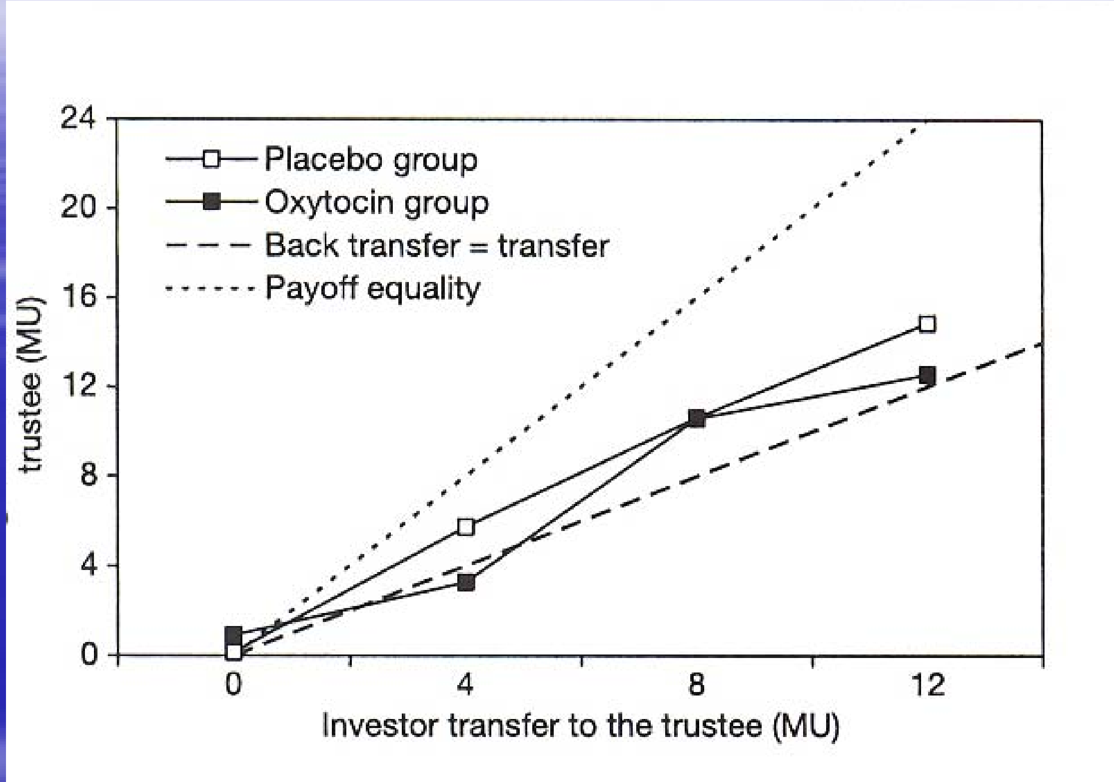




Trust

Risk

Oxytocin increases trust



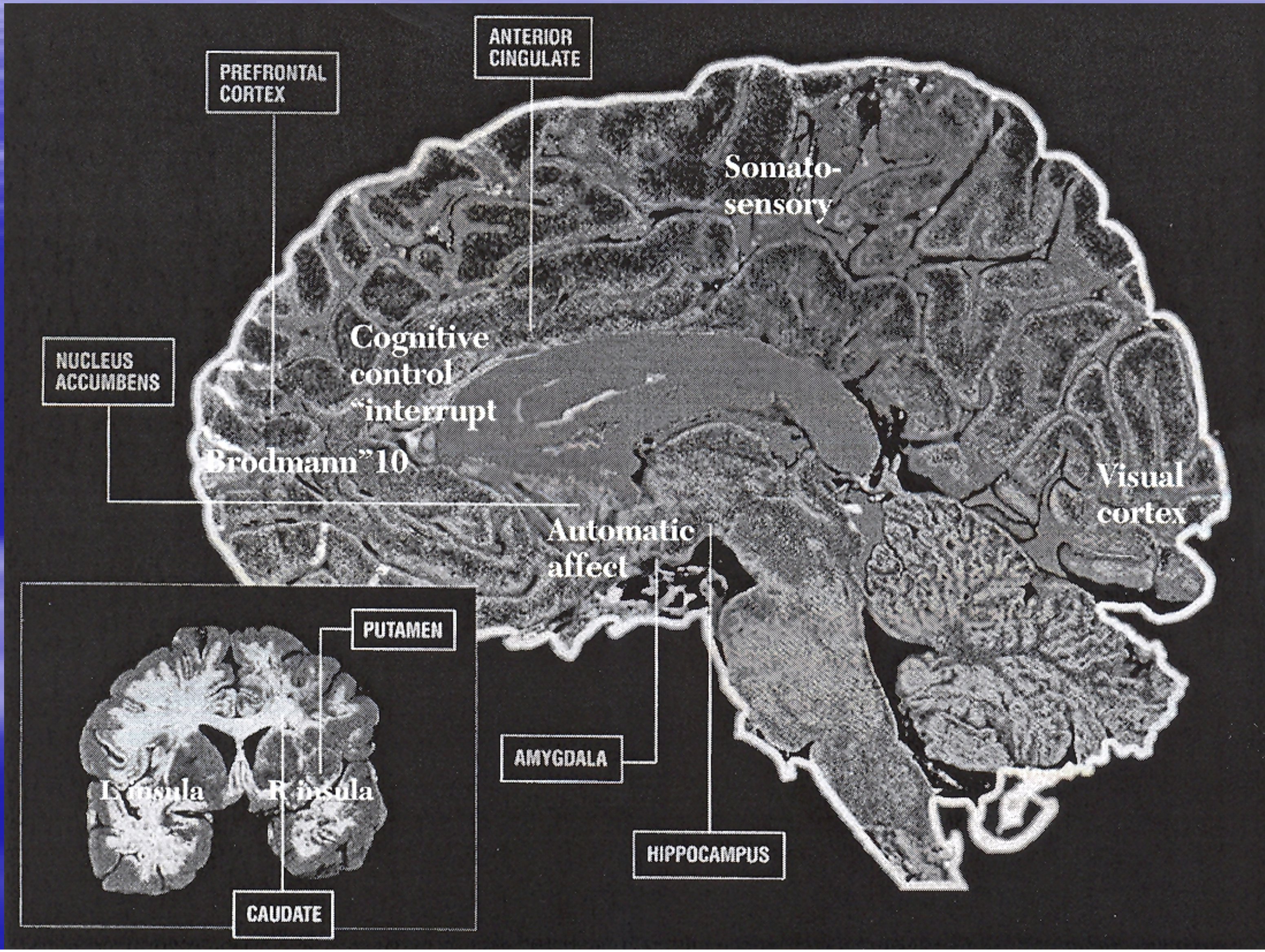
- Strong effect on investors (in the trust experiment) but not on trustees
- Reciprocity not affected by oxytocin

Other experiments

- Normal versus prefrontal cortex damage decisions (Bechara et al. Science 275 (1997) 1293)
- Brain activity during monetary incentive task (Knutson et al. NeuroImage 12 (2000) 20)
- Neural responses to monetary gains and losses (Breiter et al., Neuron 30 (2001) 619)
- Gains versus losses activity in the OFC (O' Doherty et al., Nature Neuroscience 4 (2001) 95)
- Mirror neuron activation (Keysers Neuron 31 (2001) 155
- Neural basis for social cooperation (Rilling et al., Neuron 35 (2002) 395)
- Gains versus losses activity in the cortex (Dickhaut et al. PNAS 100 (2003) 3536)
- Social exclusion (Eisenberger et al. Science 302 (2003) 290)
- Social cognition and self-referencial thought (Mitchell et al., J. Cognitive Neuroscience 17 (2005) 1306))
- Uncertainty in decision-making (Hsu et al., Science 310 (2005) 1680)
- Responses of the cingulate cortex in economic exchanges (Tomlin et al., Science 312 (2006) 1047)
- The effects of ketamine (Corlett et al., Arch. Gen. Psychiatry 63 (2006) 611)
- Neural coding of reward (Dreher et al., Cerebral Cortex 16 (2006) 561)
- Neurons encoding economic value (Padoa-Schioppa et al., Nature 441 (2006) 223)
- etc. etc.

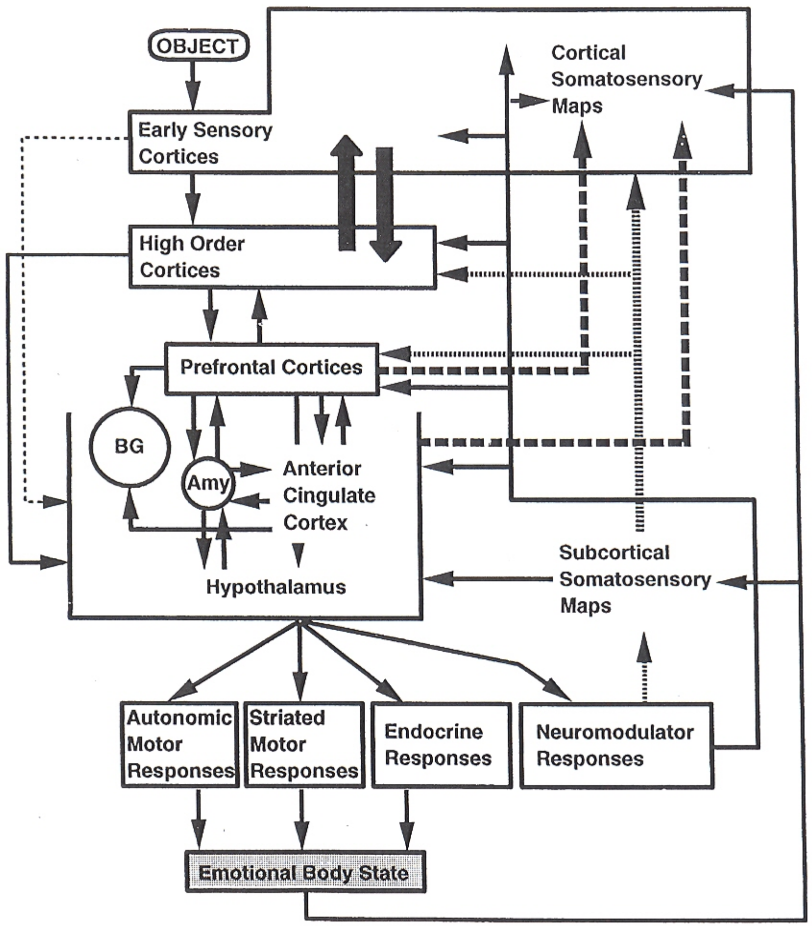
4 - Modelling neuroeconomics

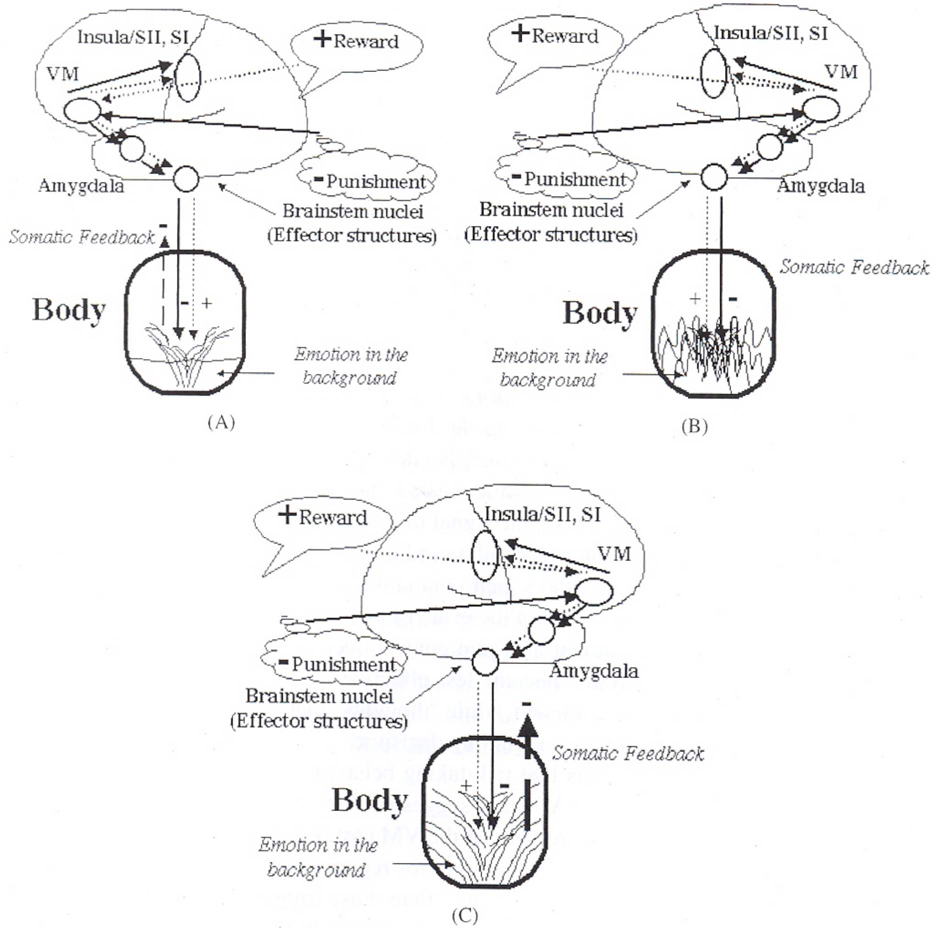
- The economic brain



Modelling neuroeconomics

- Some detailed neurological reaction circuits



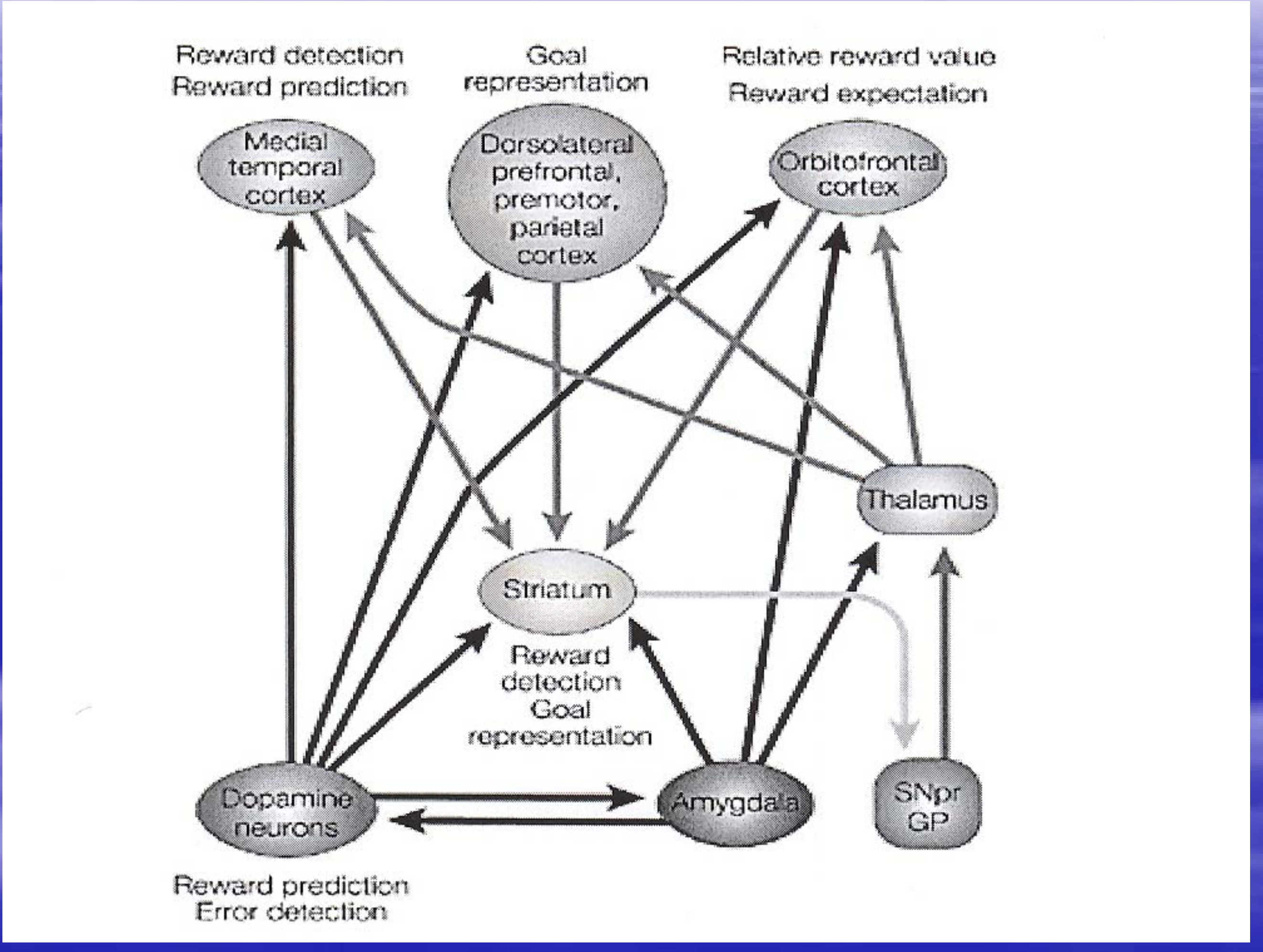


- The limbic system relates to everything (Damasio 98)

Somatic activation and decision making (Bechara 2005)

Modelling neuroeconomics

- Some detailed neurological reaction circuits



- Reward processing (Schultz 2000)

Modelling neuroeconomics

- Are there any simple modelling concepts amenable to mathematical treatment ?
- Several concepts :
  - *Inequality aversion* (Fehr, Schmidt)
  - *Reciprocity* (Bowles, Gintis, Falk, Fischbacher)
  - *Beliefs, preferences and constraints* (Heinrich, Boyd, Bowles, Camerer, Fehr, Gintis)
- From payoffs to utility functions
- Game theory with operators

Inequality aversion. The INA operator

- $\otimes M_i$ = space of payoffs and utilities, N players
- g_i = payoff of player i ; $s_i = \{s_{ia}\}$ strategies of player i ; s = set of all strategies
- $\otimes g_i \in \otimes M_i \xrightarrow[INA]{} \otimes u_i \in \otimes M_i$
- $u_i(s) = g_i(s) - (1/N-1) \sum_{k \neq i} (g_k - g_i)\, \varepsilon_{\alpha\beta}(g_k - g_i)$
- with $\varepsilon_{\alpha\beta}(x) = \alpha \ (x>0)$ and $\varepsilon_{\alpha\beta}(x) = -\beta \ (x<0)$
- Typically $\alpha > \beta$
- After the application of the INA operator, Nash equilibrium is found for the u's
- This operator leads to an utility function identical to the one of Fehr and Schmidt

Inequality aversion. The INA operator

- Example : Ultimatum game

	A	R
$4:1$	$4*$	0
$3:2$	3	0
$2:3$	2	0
$1:4$	1	0

g_P

	A	R
$4:1$	$1*$	0
$3:2$	2	0
$2:3$	3	0
$1:4$	4	0

g_R

Inequality aversion. The INA operator

- After the application of INA
- $\alpha = 0.6$ $\beta = 0.4$ $\alpha = 0.7$ $\beta = 0.6$

u_P ($\alpha = 0.6$):

	A	R
4:1	2.8	0*
3:2	2.6	0
2:3	1.4	0
1:4	–0.8	0

u_R ($\beta = 0.4$):

	A	R
4:1	–0.8	0*
3:2	1.4	0
2:3	2.6	0
1:4	2.8	0

u_P ($\alpha = 0.7$):

	A	R
4:1	2.2	0
3:2	2.4*	0
2:3	1.3	0
1:4	–1.1	0

u_R ($\beta = 0.6$):

	A	R
4:1	–1.1	0
3:2	1.3*	0
2:3	2.4	0
1:4	2.2	0

- Experimental results : offers < 0.2 3.8%
  0.4 < offers < 0.5 71%

Inequality aversion. The INA operator

- The INA operator seems OK
- But is not !
- Example : $\alpha = 0.7$ $\beta = 0.4$

Inequality aversion. The INA operator

- The INA operator seems OK
- But is not !
- Example : α= 0.7 β=0.4

First case, g_P and g_R (each followed by INA →, giving u_P and u_R):

g_P:

	A	R
8 : 2	8 *	0
5 : 5	5	0

g_R:

	A	R
8 : 2	2 *	0
5 : 5	5	0

$\xrightarrow{INA}$

u_P:

	A	R
8 : 2	5.6	0 *
5 : 5	5	0

u_R:

	A	R
8 : 2	– 2.2	0 *
5 : 5	5	0

Second case:

g_P:

	A	R
8 : 2	8 *	0
10 : 0	10	0

g_R:

	A	R
8 : 2	2 *	0
10 : 0	0	0

$\xrightarrow{INA}$

u_P:

	A	R
8 : 2	5.6	0 *
10 : 0	6	0 *

u_R:

	A	R
8 : 2	– 2.2	0 *
10 : 0	– 7	0 *

- The result is the same in both cases. The utility of the 8:2 proposal for the responder is the same in both cases
- But one knows that experimentally that proposal is accepted more often in the second case

Reciprocity. The RECI operator

- A simplified version of the ideas of Falk and Fischbacher
- Kindness of j towards i
  $K_{ji}(s_1 \ldots s_j \ldots s_N) = g_i(s) - (1/n_{s_j}) \sum g(s_1 \ldots s_j \ldots s_N)$
- n_{s_j} = number of strategies of player j

- $u_i(s) = g_i\ (s) - (1/N\text{-}1) \sum_j (K_{ij} - K_{ji})\ \varepsilon_{\alpha\beta}(K_{ij} - K_{ji})$

- Nonlocal operator in strategy space
- Instead of using the differential of payoffs to compute the utility, the kindness differential is used
- Takes care of intentions

Reciprocity. The RECI operator

- Apply to the previous example :
- α= 0.7 β=0.4

First case (g_P, g_R $\xrightarrow{RECI}$ u_P, u_R):

g_P:

	A	R
8:2	8*	0
5:5	5	0

g_R:

	A	R
8:2	2*	0
5:5	5	0

$\xrightarrow{RECI}$

u_P:

	A	R
8:2	5.8	−2.8
5:5	4.6	−1.75

u_R:

	A	R
8:2	−1.85	−1.6
5:5	4.3	−1

Second case:

g_P:

	A	R
8:2	8*	0
10:0	10	0

g_R:

	A	R
8:2	2*	0
10:0	0	0

$\xrightarrow{RECI}$

u_P:

	A	R
8:2	6.8	−2.8
10:0	7.6	−3.5

u_R:

	A	R
8:2	−0.1	−1.6
10:0	−4.2	−2

- Now the responder utility for the 8:2 offer is different in the two cases
- p = probability of 8:2 offer
- a = acceptance probability
- The Nash equilibrium in the first case (p=0.95, a=0.46)
- The Nash equilibrium in the second case (p=0.59, a=0.46)
- However, for the conditional probabilities : if p=1 then a=0 in the first case and a=1 in the second

Calculation of Nash equilibria for mixed strategies

	A	R
8:2	5.8	−2.8
5:5	4.6	−1.75

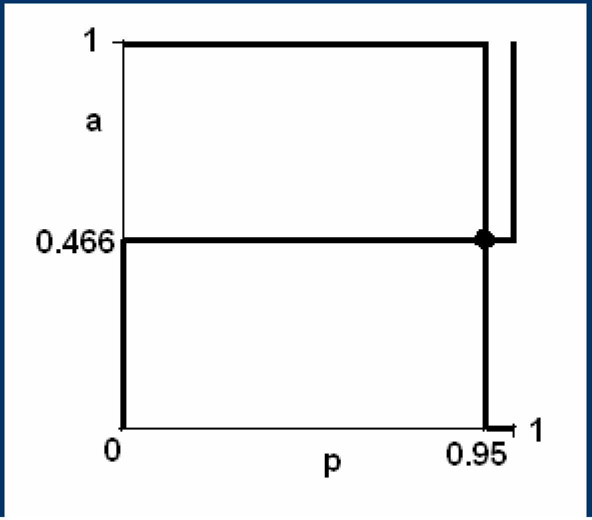
u_P

	A	R
8:2	−1.85	−1.6
5:5	4.3	−1

u_R

$$X_P = p(2.25a - 1.05) + 6.35a - 1.75$$

$$X_R = a(-5.55p + 5.3) - 0.6p - 1$$



	A	R
8:2	6.8	−2.8
10:0	7.6	−3.5

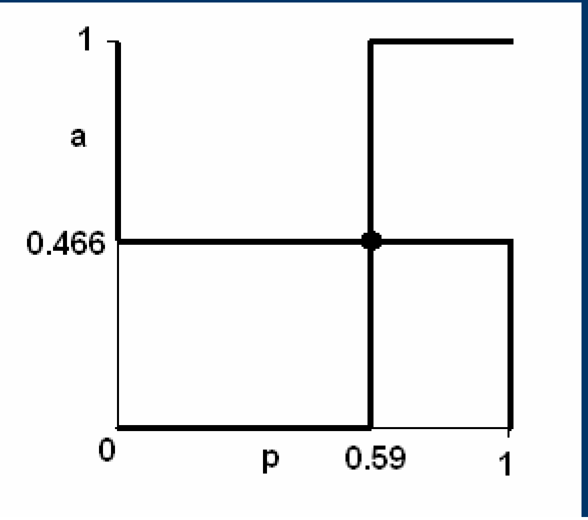
u_P

	A	R
8:2	−0.1	−1.6
10:0	−4.2	−2

u_R

$$X_P = p(-1.5a + 0.7) + 11.1a - 3.5$$

$$X_R = a(3.7p - 2.2) + 0.4p - 2$$



Concluding questions

- “Game theory with operators” is simply a way to systematize and unify the computation of utility functions
- Is the RECI operator sufficiently universal ?
- Insofar as the feasible strategies of each player are the simplex of all pure strategies, existence of equilibrium points for the utilities is guaranteed by the Nash theorem
- In the case of social equilibrium (Debreu) the operators should not spoil contractibility of the set of feasible strategies
- If more than one operator is needed, what about commutativity properties ?
- Etc.

Some additional references

- *Neuroeconomics :*
  C. Camerer, G. Loewenstein and D. Prelec; J. of Economic Literature 43 (2005) 9-64
  A. Bechara and A. R. Damasio; Games and Economic Behavior 52 (2005) 336-372
- *Inequality aversion :*
  Fehr and Schmidt; Quarterly J. of Economics 114 (1999) 817-868
- *Reciprocity :*
  Falk and Fischbacher; Games and Econ. Behavior 54 (2006) 293-315
- *Evolutionary aspects of strong reciprocity :*
  S. Bowles and H. Gintis; Theor. Pop. Bio. 65 (2004) 17-28.
  R V M; Advances in Complex Systems 7 (2004) 357-368
- *An alternative to model deviations from classical Nash equilibrium :*
  R V M; Quantum Information Proc. 4 (2005) 1-12